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"In a vacant leaf of an old prayer-book, in which Dudley Emerson, the father of our Author, had registered his marriage, and the births of several of his children, it is written, William Emerson was born on Wednesday, May 14th, at one o'clock in the morning and forty minutes, and baptized June 10th, 1701."

His father, Dudley, who was possessed of but a small estate, at that time, taught a school, and seems to have thought himself of some little consequence in the world, for there is a paper written by himself, containing what he calls an account of the principal transactions or events of his life. Among these *memorabilia*, relating, chiefly, to his movements from one place to another, is observed nothing respecting the birth or education of his son William, which he did not foresee would be the only circumstances, or events of any importance, in his life, that might possibly rescue his name from oblivion. Our author, William Emerson, it appears, received his education principally from his father;—reading, writing and arithmetic, and a little Latin, perhaps, as far as *Corderii*, or Beza's *Latin Testament*; but that he afterwards received some assistance from a young gentleman, then Curate of Harworth, in the learned languages, and who, at that time, boarded in his father's house. It does not appear, however, that he made any considerable progress, or was much attached to his books whilst a boy, or exhibited any symptoms of those superior faculties, which he afterwards exerted with so much energy. Indeed, so careless and inattentive to learning was he, at this period, that he has been heard to say, that till he was nearly twenty years of age, his principle and favourite employment, for one season of the year, was that of seeking birds' nests. But his attachments to childish amusements having passed away, he began to be sensible to the charms and beauties of science. He first went to Newcastle, and afterwards to York, where he applied himself with considerable attention and diligence to the study of the mathematics, under the direction of schoolmasters; and of whom he used to speak with much respect, in the latter part of his life. He used to say, too, that his father was a tolerable mathematician; and with-

out his books and instructions, perhaps, his own genius (most eminently fitted for mathematical disquisitions) would never have been unfolded.

After his return from school at York, he resided principally at Hurworth, where he continued to pursue his studies and amusements, at intervals, until the time of his marriage. In what year of his life this happened is not exactly known, but it was about the thirty-second or thirty-third; and from this period we must date the commencement of his mathematical labours; or, perhaps, the communication of them to the public. What he had done in this line was merely an occasional application for his own amusement, or for the exercise and improvement of his leisure hours. But one of those accidents, which, as Dr. Johnson observes, in the Life of Cowley, "produce that particular designation of mind and propensity for some certain science, commonly called genius," took place upon this occasion, and added a powerful stimulus to his native thirst for knowledge and for fame. His wife was a niece of a Dr. Johnson, rector of Hurworth, vicar of Mansfield, in the county of York, and a prebendary of Durham, a man very eminent in his time for his skill in surgery, and who by a very extensive and successful practice in this profession, together with the emoluments arising from his livings, had accumulated a very considerable fortune. Dr. Johnson had promised to give his niece, who lived with him, five hundred pounds for her marriage portion. Some time after the marriage, Mr. Emerson took an opportunity to mention this matter to the Doctor, and to remind him of his promise. The Doctor did not recollect, or did not choose to recollect, any thing of it, but treated our young mathematician with some contempt, as a person of no consequence, and beneath his notice. The pecuniary disappointment, Emerson (who had as independent a spirit as any man, and whose patrimony, though not large, was equal to all his wants) would easily have surmounted, but this contemptuous treatment stung him to the very soul. He immediately went home, packed up his wife's clothes, and sent them off to the Doctor, saying, he would scorn to be beholden to such a fellow for a single rag;

and swearing, at the same time, that he would be revenged, and prove himself to be the better man of the two. His plan of revenge was truly noble and laudable. He was resolved to demonstrate to his uncourteous uncle, and to the world, that he was not to be rated as an insignificant or ignorant person; and that the contempt and indignity with which he had been treated, were much misplaced, and very unmerited; and, in order to demonstrate this, he determined to labour till he became one of the first mathematicians of the age.

He had received from nature a strong and vigorous mind, and had acquired a just relish for the beauties of mathematical science, and an ardent love of truth; he was, at the same time, stimulated with an eager desire of distinguishing himself from the illiterate crowd of mortals; the effects of his labour, influenced by such motives, and directed by such abilities, could not, therefore, be but great. He made himself a perfect master of the whole circle of the mathematics; and, after having carefully planned and digested, revised and completed the work to his own satisfaction, he published, in the forty-second year of his age, his book of *Fluxions*; and at his first appearance in the world as an author, stepped forth like a giant in all his might, and justly claimed a place amongst mathematicians of the very first rank. By the strictly scientific manner in which he established the principles, and demonstrated the truth, of the method of Fluxions in this work, he added another firm and durable support to the noble edifice of the Newtonian Philosophy, which, by some less accurate and penetrating observers, was supposed to have received a violent and dangerous concussion from the metaphysical artillery of the *Analyst*, and the cavils and objections advanced against the truth of the fluxionary method.

Having thus secured his mathematical fame upon a firm and solid basis, he continued, from time to time, to favour and instruct the public with other most valuable publications upon the several branches of the mathematics. These appeared in the order in which they stand arranged below, with the year of their publication, and the date of the author's life at the time.

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1763 ..	62	Arithmetic, Geometry, 8vo.
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Several of the above works have gone through new editions, which have been improved and augmented by the Author, and were mostly published by F. Wingrave, in the Strand, bookseller.

The above works, many of them allowed to be the best extant upon the subject of which they treat, will still remain a lasting monument of Mr. Emerson's genius, penetration, and industry, to the latest times; and render any further eulogium of their Author, as a man of science, totally unnecessary.

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Printed by D. S. Maurice, Fenchurch Street.

THE EDITOR'S PREFACE.

In commencing this series of popular and standard works on the different branches of science that are more expressly applicable to the purposes of the Mechanic, it will naturally be expected that something further than a reprint will be given, from the advanced state of science, and the many improvements of late years effected, both by the writings of men of genius, and the practical example of those whose occupations are more directed to what is of real benefit to mankind—the improvement of our manufactures,—than to speculations of the learned. Many obstacles have presented themselves, as regards the form of language, and the adaptation of style, to meet the wishes of all readers. Men of science are, in general, too fond of technological obscurity, and what is written in a familiar and plain way is apt to draw down their censure; but, as the present work is more particularly intended for the perusal of the Artisan, I have endeavoured to gratify both the Student and the practical Mechanic, by a careful

revision of the works themselves; and where I have thought any thing was wanting in clearness, or any subject that required familiar language to make it generally understood, I have preferred the method of supplying their defects by an Appendix, rather than that of altering the text of the original. How far I have been successful, will not be for me to decide; but I can, with safety, say, that my endeavours have been directed to the elucidation of subjects which appeared not sufficiently explained; and to place, in as clear a light as possible, every Proposition that presented itself. Some new matter has, also, been added, in order to render the work as complete as possible.

It will, I trust, be thought not too much if I add, that, independent of the great care that has been taken to correct any typographical errors, the corrections of the engravings and diagrams have been much improved from those of the original works. Indeed, our present specimen, both of typography and perspective drawing in the plates, will, at least, shield us from censure, if not gain some meed of praise, for the pains that have been taken to make the work worthy of a place in the library of the man of science as well as the mechanic. This first volume of the series thus presents itself, not as aiming at originality, but as an attempt to perpetuate the writings of a man who, for general mathematical knowledge, stands, I may say, unrivalled; and to bring into more general notice the theory of that art by which our wants, as well as our luxuries, are gratified; and by which the prosperity of the nation, and the comfort and happiness of individuals, is insured.

Should the present work be thought worthy of patronage, it is my intention to continue the series, by a careful selection from other authors of acknowledged merit, who have written on the other branches of science, and to include all those anywise connected with the business of the Mechanic, such as Geometry, Perspective, Algebra, &c.; and I shall endeavour, in the same manner, to illustrate them by an Appendix, using as familiar language as the subject will admit of, and noticing such new improvements as may have been added since the original publication; thus endeavouring to blend, as it were, together the various discoveries of the moderns with the original works of those who have preceded them.

I shall here take my leave; trusting that no material error will be found: and, as I cannot claim any merit in the production, I hope I shall not incur censure, but that the indulgent reader will peruse this with the same spirit in which I have written, that of endeavouring to apply to some useful purpose the talents which are entrusted to us.

G. A. S.

MEMOIRS OF MR. EMERSON;

WITH SOME ACCOUNT OF HIS WRITINGS.

In giving some account of the Life and Writings of this eccentric man, though excellent mathematician, it is but justice to own, I am more indebted to what has been mentioned by others, than to any information I have been able to collect in the vicinity of his place of usual residence, as it is now between forty and fifty years since his decease; I shall, therefore, avail myself of the information given by the Rev. W. Bowe of Scorton, near Catterick, Yorkshire, in his account of the Life of William Emerson, and endeavour to supply some omissions, and venture some remarks on his writings, not given by him.

Mr. Bowe commences with remarking, that, for the last three or four years of our Author's life, he was on terms of intimacy with him, and, therefore, had many opportunities of hearing from his own mouth accounts of circumstances which had taken place at former periods of his life, as well as gaining information from those who knew him many years: he commences his narrative by remarking, that Mr. Dudley Emerson, of Hurworth, near Darlington, in the county of Durham, had two sons, William, the elder, and Dudley, who died whilst he was young; William, who afterwards lived to become so eminent a mathematician, was born at Hurworth, in the year 1701, and appears, by the parish register, to have been baptized there on the 10th of June in that year.

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PRINCIPLES

OF

MECHANICS:

EXPLAINING AND DEMONSTRATING

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THE LAWS OF GRAVITY,
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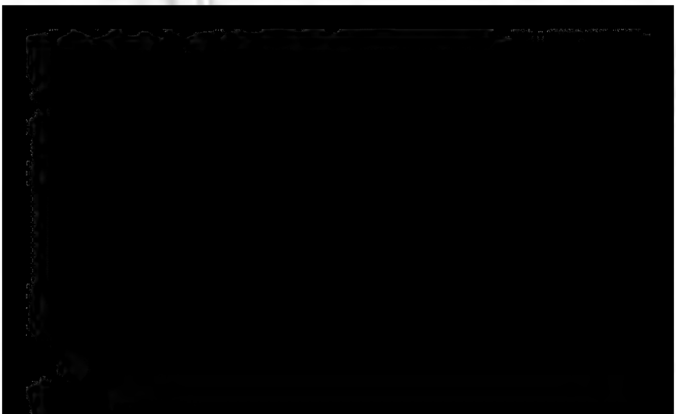

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"In a vacant leaf of an old prayer-book, in which Dudley Emerson, the father of our Author, had registered his marriage, and the births of several of his children, it is written, William Emerson was born on Wednesday, May 14th, at one o'clock in the morning and forty minutes, and baptized June 10th, 1701."

His father, Dudley, who was possessed of but a small estate, at that time, taught a school, and seems to have thought himself of some little consequence in the world, for there is a paper written by himself, containing what he calls an account of the principal transactions or events of his life. Among these *memorabilia*, relating, chiefly, to his movements from one place to another, is observed nothing respecting the birth or education of his son William, which he did not foresee would be the only circumstances, or events of any importance, in his life, that might possibly rescue his name from oblivion. Our author, William Emerson, it appears, received his education principally from his father;—reading, writing and arithmetic, and a little Latin, perhaps, as far as *Corderii*, or Beza's *Latin Testament*; but that he afterwards received some assistance from a young gentleman, then Curate of Harworth, in the learned languages, and who, at that time, boarded in his father's house. It does not appear, however, that he made any considerable progress, or was much attached to his books whilst a boy, or exhibited any symptoms of those superior faculties, which he afterwards exerted with so much energy. Indeed, so careless and inattentive to learning was he, at this period, that he has been heard to say, that till he was nearly twenty years of age, his principle and favourite employment, for one season of the year, was that of seeking birds' nests. But his attachments to childish amusements having passed away, he began to be sensible to the charms and beauties of science. He first went to Newcastle, and afterwards to York, where he applied himself with considerable attention and diligence to the study of the mathematics, under the direction of schoolmasters; and of whom he used to speak with much respect, in the latter part of his life. He used to say, too, that his father was a tolerable mathematician; and with-

out his books and instructions, perhaps, his own genius (most eminently fitted for mathematical disquisitions) would never have been unfolded.

After his return from school at York, he resided principally at Hurworth, where he continued to pursue his studies and amusements, at intervals, until the time of his marriage. In what year of his life this happened is not exactly known, but it was about the thirty-second or thirty-third; and from this period we must date the commencement of his mathematical labours; or, perhaps, the communication of them to the public. What he had done in this line was merely an occasional application for his own amusement, or for the exercise and improvement of his leisure hours. But one of those accidents, which, as Dr. Johnson observes, in the Life of Cowley, "produce that particular designation of mind and propensity for some certain science, commonly called genius," took place upon this occasion, and added a powerful stimulus to his native thirst for knowledge and for fame. His wife was a niece of a Dr. Johnson, rector of Hurworth, vicar of Mansfield, in the county of York, and a prebendary of Durham, a man very eminent in his time for his skill in surgery, and who by a very extensive and successful practice in this profession, together with the emoluments arising from his livings, had accumulated a very considerable fortune. Dr. Johnson had promised to give his niece, who lived with him, five hundred pounds for her marriage portion. Some time after the marriage, Mr. Emerson took an opportunity to mention this matter to the Doctor, and to remind him of his promise. The Doctor did not recollect, or did not choose to recollect, any thing of it, but treated our young mathematician with some contempt, as a person of no consequence, and beneath his notice. The pecuniary disappointment, Emerson (who had as independent a spirit as any man, and whose patrimony, though not large, was equal to all his wants) would easily have surmounted, but this contemptuous treatment stung him to the very soul. He immediately went home, packed up his wife's clothes, and sent them off to the Doctor, saying, he would scorn to be beholden to such a fellow for a single rag;

and swearing, at the same time, that he would be revenged, and prove himself to be the better man of the two. His plan of revenge was truly noble and laudable. He was resolved to demonstrate to his uncourteous uncle, and to the world, that he was not to be rated as an insignificant or ignorant person; and that the contempt and indignity with which he had been treated, were much misplaced, and very unmerited; and, in order to demonstrate this, he determined to labour till he became one of the first mathematicians of the age.

He had received from nature a strong and vigorous mind, and had acquired a just relish for the beauties of mathematical science, and an ardent love of truth; he was, at the same time, stimulated with an eager desire of distinguishing himself from the illiterate crowd of mortals; the effects of his labour, influenced by such motives, and directed by such abilities, could not, therefore, be but great. He made himself a perfect master of the whole circle of the mathematics; and, after having carefully planned and digested, revised and completed the work to his own satisfaction, he published, in the forty-second year of his age, his book of *Fluxions*; and at his first appearance in the world as an author, stepped forth like a giant in all his might, and justly claimed a place amongst mathematicians of the very first rank. By the strictly scientific manner in which he established the principles, and demonstrated the truth, of the method of Fluxions in this work, he added another firm and durable support to the noble edifice of the Newtonian Philosophy, which, by some less accurate and penetrating observers, was supposed to have received a violent and dangerous concussion from the metaphysical artillery of the *Analyst*, and the cavils and objections advanced against the truth of the fluxionary method.

Having thus secured his mathematical fame upon a firm and solid basis, he continued, from time to time, to favour and instruct the public with other most valuable publications upon the several branches of the mathematics. These appeared in the order in which they stand arranged below, with the year of their publication, and the date of the author's life at the time.

Years.	Ætat.	
1743 ..	42	Fluxions, 8vo.
1749 ..	48	Projections of the Sphere, and Elements of Trigonometry, 8vo.
1754 ..	53	Mechanics, 4to.
1755 ..	54	Navigation, 12mo.
1763 ..	62	Arithmetic, Geometry, 8vo. Method of Increment, 4to.
1764 ..	63	Algebra, 8vo.
1767 ..	66	Arithmetic of Infinites and Conic Sections, 8vo.
1768 ..	67	Elements of Optics and Perspective, 8vo.
1769 ..	68	Astronomy, Mechanics, Centripetal and Centrifugal Forces, 8vo.
1770 ..	69	Mathematical Principles of Geography, Navigation, and Dialling; Comment on the Principia, with the Defence of Newton; Tracts, 8vo.
1776 ..	75	Miscellanies, 8vo., which was his last work.

Several of the above works have gone through new editions, which have been improved and augmented by the Author, and were mostly published by F. Wingrave, in the Strand, bookseller.

The above works, many of them allowed to be the best extant upon the subject of which they treat, will still remain a lasting monument of Mr. Emerson's genius, penetration, and industry, to the latest times; and render any further eulogium of their Author, as a man of science, totally unnecessary.

His first publication, however, did not meet with immediate encouragement; so that it is probable the rest would never have appeared, or, at least, not in the Author's life-time, had he not, about the year 1763, been recommended by his great admirer, and friend, the late Edward Montague, Esq. to Mr. John Nourse, bookseller, in London, who was himself an eminent mathematician, and well skilled in the Newtonian Philosophy, having had an

University education, and been an early associate with the learned Doctors Pemberton and Wilson; the one, the companion of Newton, and the Editor of the best edition of the *Principia*; the other, of Mr. Robins's *Mathematical Tracts.* Mr. Nourse was so highly sensible of Mr. Emerson's superior abilities, that he engaged him, on very liberal terms, to furnish a regular Course of the Mathematics for the use of Young Students. Mr. Emerson made a journey to London, in the summer of the year 1763, to settle and fulfil this agreement. Even in London, he could not be idle: besides correcting the sheets for the press, (for he always made the revisal himself, and to "trust no eyes but his own" was his favourite maxim,) he took lodgings at a watchmaker's, near Smithfield, that he might improve himself in that branch of knowledge, during his stay there.

Besides the above regular works, published in Mr. Emerson's own name, he wrote several other fugitive pieces, in the *Ladies' Diaries,* and other periodical and miscellaneous works. In the *Ladies' Diaries,* he proposed and answered several new questions under the signature *Merones*; an anagram of his own name, containing all the letters of it transposed: the questions resolved by him, were as follow, viz. prize 1736, questions 193, 195, 197; prize 1737, questions 205, 206, 207, 209, 210, 215, 217, 221, 223; prize 1741, questions 226, 229; prize 1742, questions 238, 240; and he proposed the following new questions;—No. 193, 206, and 220; see the *Diarian Miscellany,* which is a republication of all the useful parts of the *Ladies' Diaries,* by Dr. Hutton. In the *Diaries,* &c. Mr. Emerson had, also, some warm controversies with Wadson, (Mr. Dawson of Sedgbergh,) and other eminent mathematicians.

Mr. Emerson also took a part in the *Miscellanea Curiosa Mathematica,* a work published in quarterly numbers by Mr. Francis Holliday, (his friend and correspondent,) from the year 1745 to 1755, in 4to. In this work he resolves many questions, as before in the Diaries; sometimes under the signature *Merones,* and sometimes under the still more whimsical one of *Philofluentimechanalgegeomastrolongo*; and, probably, under several

others. We shall now take a view of Mr. Emerson in his private life, as a man, and as a member of society.

Mr. Emerson was, in person, something below the common size, but firm, compact, and well made, very active and strong. He had a good, open, expressive countenance, with a ruddy complexion, a keen and penetrating eye, and an ardour and eagerness of look that was very expressive of the texture of his mind. His dress was very simple and plain, or what, by the generality of people, perhaps, would have been called grotesque and shabby. A very few hats served him through the whole course of his life, and when he purchased one, (or, indeed, any other article of dress), it was a matter of perfect indifference to him, whether the form and fashion of it was that of the day, or of half a century before. One of these hats, of immense superficies, had, in length of time, lost its elasticity, and the brim of it began to droop in such a manner, as to prevent him being able to view the objects before him in a direct line. This was not to be endured by an optician; he, therefore, took a pair of shears, and cut it round close to the body of the hat, leaving a little to the front, which he dexterously rounded into the resemblance of a jockey's cap. His wigs were made of brown or dirty flaxen coloured hair, which, at first, appeared bushy and tortuous behind, but which grew pendulous through age, till, at length, it became quite straight, having, probably, never undergone the operation of the comb; and, either through the original mal-conformation of the wig, or from a custom he had of frequently inserting his hand behind it, his hindhead and wig never coming into very close contact. His coat, or, more properly, jacket, or waistcoat with sleeves to it, which he constantly wore without any other waistcoat, was of a drab colour. His linen came not from Holland or Hibernia, but was spun and bleached by his wife, and woven at Hurworth, being calculated more for warmth and duration, than for shew. He had a singular custom of frequently wearing, especially in cold weather, his shirt with the wrong side before, and buttoned behind the neck. But this was not an affectation of singularity, (for Emerson had no singularity

about him, though his customs and manners were singular:) he had a reason for it;—he seldom buttoned more than two or three of the buttons of his waistcoat, one or two at the bottom, and sometimes one at the top; leaving all the rest open. In wind, rain, or snow, therefore, he must have found the aperture at the breast inconvenient, if his shirt had been put on in the usual manner. His breeches had an antique appearance, the lappet before, not being supported by two buttons, placed in a line parallel to the horizon, but by buttons descending in a line perpendicular to it. In cold weather, he used to wear, when he grew old, what he called shin-covers. Now these shin-covers were made of old sacking, tied with a string above the knee, and depending before the shins down to the shoe; they were useful in preserving his legs from being burnt, when he sat too near the fire (which old people are apt to do); and if they had their use, he was not solicitous about the figure or appearance they might make.

This singularity of dress and figure, together with his character for profound learning, and knowledge more than human, caused him to be considered, by ignorant and illiterate people in the neighbourhood, as a *wise* or *cunning man*, or *conjuror*; many of them are still persuaded that he was such, and will tell you wonderful stories of the feats he performed, and particularly how, by virtue of a magic spell, he pinned a fellow to the top of his pear or cherry-tree, who had got up with a design to steal his fruit, and compelled him to sit there a whole Sunday forenoon, in full view of the congregation going to, and returning from, church. That he did compel a man to sit for some time in the tree, I believe was a fact; not, however, by virtue of any magic spell, but by standing at the bottom of the tree, with a hatchet in his hand, and swearing that if he came down he would hag (*i. e.* hew) his legs off. This opinion of his skill in the *black* art, was of service in defending his property from such depredation, and therefore it would have been impolitic to discourage it: but he was apt to lose his patience very much when he was applied to for the recovery of stolen goods, or to investigate the secrets of

futurity. A woman came one day to him, to inquire about her husband who had gone six years before to the West Indies or America, and had not been heard of since. She requested, therefore, to be informed, whether he was dead or living, as a man in her neighbourhood had made proposals of marriage to her. It was with much difficulty the supposed prophet repressed the rising furor till the conclusion of the tale; when, hastily rising from the tripod, or three-footed stool, on which he usually sat, in terms more energetic than ever issued from the shrine at Delphi, he gave this plain and unequivocal response: "D—n thee for a b—h! thy husband's gone to hell, and thou may go after him." The woman went away, well pleased and satisfied with the answer she had received, thinking she might now listen to the proposals of her lover with a safe conscience. Another damsel, with a similar errand, met with a milder reception. Her mistress had lost some caps, or linen, and she wanted to know whether her fellow-servant (of whom she entertained suspicions) had purloined them or not. "Thou's a canny young lass," replied the smiling conjuror, "but thou's over late o' coming; I can do nought for thee." The poor girl went away, grieved that she had not made her application sooner, supposing he meant that the mysterious moment of divination was past.

He was by some people looked upon as an atheist, but he was as much an atheist as he was a magician. He firmly believed in the being of God; he did not *believe* it, as he sometimes said,—he *knew* it; he was *certain* of it, to a demonstration. But it must be acknowledged, that he did not always speak of revealed religion, the Church of England, or the clergy, in terms of respect. It has often been observed and lamented, that minds merely mathematical are apt to tend towards scepticism and irreligion. The man who is always accustomed to demonstrative proofs, and wholly engaged in a science which admits of them at every step, will not so readily acquiesce in a series of probabilities, where investigations of another kind are presented to him; and, perhaps, will not have patience to examine circumstances deeply enough, to ascertain on which side there is a prepon-

derance of evidence amounting nearly to a demonstration. Besides, Emerson's resentment of Dr. Johnson's treatment of him, and which I have mentioned before, might produce a bias on his mind unfavourable to religion. A man of his natural impetuosity of temper would be too apt to associate the idea of the *profession* with that of the *professor*; and because he had quarrelled with the *priest*, would also quarrel with his *doctrine*. Under the influence of this prepossession, he set himself to work to examine the Scriptures of the Old and New Testament, and collected two small quarto volumes of what he conceived to be contradictory passages, and arranged them, like hostile troops confronting each other, on the opposite pages of his book.

His diet was as simple and plain as his dress, and his meals gave little interruption to his studies, employments, or amusements. During his days of close application, he seldom sat down to eat, but would take a piece of cold pie or meat of any kind in his hand, and, retiring with it to his place of study, could satisfy his appetite for knowledge and food at the same time. He catered for himself, and pretty constantly made his own market. When his stock of groceries or other necessaries grew low, on the Monday morning he took his wallet, which he slung obliquely across his shoulders, and set forward for the market at Darlington, three miles distant from Hurworth, whither he always walked on foot, for he seldom or never kept a horse, and had an aversion to riding. "He would frequently lead the horse, when he had one from market, by the halter, bearing the wallet stuffed with the provisions he had bought at market." After having provided all the necessary articles, he did not always make directly home again: but, if he found good fair ale, and company to his mind, he would sit himself down contentedly in some public house, for the remainder of the day, and frequently during the night too; sometimes he did not reach home till late on Tuesday or even Wednesday. He remained talking or disputing on various topics—mechanics, politics, or religion,—just as his company might be, varying the scene sometimes with a beef-steak,

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MDCCCXXV.

But we meet with no considerable inventions in the mechanical way, for a long series of ages; or, if there had been any, the accounts of them are now lost, through the length of time; for we have nothing upon record for two or three thousand years forward. But, afterwards, we find an account of everal machines that were in use. For we read in Genesis, that ships were as old, even on the Mediterranean, as the days of Jacob. We likewise read that the Philistines brought thirty thousand chariots into the field against Saul; so that chariots were in use 1070 years before Christ. And about the same time architecture was brought into Europe. And 1030 years before Christ, Ammon built long and tall ships with sails, on the Red Sea and the Mediterranean. And, about ninety years after, the ship Argo was built; which was the first Greek vessel that ventured to pass through the sea, by help of sails, without sight of land, being guided only by the stars. Dædalus also, who lived 980 years before Christ, made sails for ships, and invented several sorts of tools, for carpenters and joiners to work with. He also made several moving statues, which could walk or run of themselves. And, about 800 years before Christ, we find in 2 Chron. xv. that Uzziah made in Jerusalem, engines, invented by cunning men, to be on the towers and upon the bulwarks, to shoot arrows and great stones withal. Corn-mills were early invented; for we read in Deuteronomy, that it was not lawful for any man to take the nether or the upper mill-stone to pledge; yet water was not applied to mills before the year of Christ 600, nor wind-mills used before the year 1200. Likewise, 580 years before Christ, we read in Jeremiah xviii. of the potter's wheel. Architas was the first that applied mathematics to mechanics, but left no mechanical writings behind him: he made a wooden pigeon that could fly about. Archimedes, who lived about 200 years before Christ, was a most subtle geometer and mechanic. He made engines that drew up the ships of Marcellus at the siege of Syracuse; and others that would cast a stone of a prodigious weight to a great distance, or else several lesser stones, as also darts and arrows; but there have been many fabulous reports concerning these engines. He also made a sphere which showed the motions of the sun, moon, and planets. And Posidonius, afterwards, made another which shewed the same thing. In these days, the liberal arts flourished, and learning met with proper encouragement; but, afterwards, they became neg-

lected for a long time. Aristotle, who lived about two hundred and ninety years before Christ, was one of the first that writ any methodical discourse of mechanics. But, at this time, the art was contained in a very little compass, there being scarce any thing more known about it, than the six mechanical powers. In this state, it continued till the sixteenth century, and then clock-work was invented; and, about 1650, were the first clocks made. At this time, several of the most eminent mathematicians began to consider mechanics; and, by their study and industry, have prodigiously enlarged its bounds, and made it a most comprehensive science. It extends through heaven and earth; the whole universe, and every part of it, is its subject. Not one particle of matter but what comes under its laws. For what else is there in the visible world, but matter and motion? and the properties and affections of both these, are the subject of mechanics.

To the art of mechanics is owing all sorts of instruments to work with, all engines of war, ships, bridges, mills, curious roofs and arches, stately theatres, columns, pendent galleries, and all other grand works in building. Also clocks, watches, jacks, chariots, carts and carriages, and even the wheel-barrow. Architecture, navigation, husbandry, and military affairs, owe their invention and use to this art; and whatever hath artificial motion by air, water, wind, or cords; as all manner of musical instruments, water-works, &c. This is a science of such importance, that, without it, we could hardly eat our bread, or lie dry in our beds.

By mechanics, we come to understand the motions of the parts of an animal body; the use of the nerves, muscles, bones, joints, and vessels: all which have been made so plain, as proves an animal body to be nothing but a mechanical engine. But this part of mechanics, called anatomy, is a subject of itself. Upon mechanics are also founded the motions of all the celestial bodies, their periods, times, and revolutions. Without mechanics, a general cannot go to war, nor besiege a town, or fortify a place; and the meanest artificer must work mechanically, or not work at all; so that all persons, whatever, are indebted to this art, from the king down to the cobbler.

Upon mechanics is also founded the Newtonian, or only true philosophy in the world. For all the difficulty of philosophy consists in this; from some of the principal phænomena of motions to investigate the forces of nature. And then, from these forces to

revision of the works themselves; and where I have thought any thing was wanting in clearness, or any subject that required familiar language to make it generally understood, I have preferred the method of supplying their defects by an Appendix, rather than that of altering the text of the original. How far I have been successful, will not be for me to decide; but I can, with safety, say, that my endeavours have been directed to the elucidation of subjects which appeared not sufficiently explained; and to place, in as clear a light as possible, every Proposition that presented itself. Some new matter has, also, been added, in order to render the work as complete as possible.

It will, I trust, be thought not too much if I add, that, independent of the great care that has been taken to correct any typographical errors, the corrections of the engravings and diagrams have been much improved from those of the original works. Indeed, our present specimen, both of typography and perspective drawing in the plates, will, at least, shield us from censure, if not gain some meed of praise, for the pains that have been taken to make the work worthy of a place in the library of the man of science as well as the mechanic. This first volume of the series thus presents itself, not as aiming at originality, but as an attempt to perpetuate the writings of a man who, for general mathematical knowledge, stands, I may say, unrivalled; and to bring into more general notice the theory of that art by which our wants, as well as our luxuries, are gratified; and by which the prosperity of the nation, and the comfort and happiness of individuals, is insured.

made. For never a philosopher before Newton ever took the method that he did. For whilst their systems are nothing but hypotheses, conceits, fictions, conjectures, and romances, invented at pleasure, and without any foundation in the nature of things, he, on the contrary, and by himself alone, set out upon a quite different footing. For he admits nothing but what he gains from experiments, and accurate observations. And from this foundation, whatever is further advanced, is deduced by strict mathematical reasoning. And where this thread does not carry him, he stops, and proceeds no further; not pretending to be wise above what is written in nature; being rather content with a little true knowledge, than, by assuming to know every thing, run the hazard of error. Contrary to all this, these scheming philosophers, being men of strong imaginations and weak judgments, will run on, ad infinitum, and build one fiction upon another, till their Babel, thus erected, proves to be nothing but a heap of endless confusion and contradiction. And then it is no wonder, if the whole airy fabric tumbles down, and sinks into ruin. And yet it seems, such romantic systems of philosophy will please some people as well as the strictest truth, or most regular system. As if philosophy, like religion, was to depend on the fashion of the country, or on the fancies and caprice of weak people. But, surely, this is nothing but rambling in the dark, and saying that the nature of things depends upon no steady principles at all. But, in truth, the business of true philosophy is to derive the nature of things from causes truly existent; and to inquire after those laws on which the Creator choosed to found the world; not those by which he might have done the same, had he so pleased. It is reasonable to suppose, that, from several causes, something differing from each other, the same effect may arise. But the true cause will always be that from which it truly and actually does arise; the others have no place in true philosophy. And this can be known no way, but by observations and experiments. Hence, it evidently follows, that the Newtonian philosophy, being thus built upon this solid foundation, must stand firm and unshaken; and being once proved to be true, it must eternally remain true, until the utter subversion of all the laws of nature. It is, therefore, a mere joke to talk of a new philosophy. The foundation is now firmly laid: the Newtonian philosophy may, indeed, be improved, and further

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Should the present work be thought worthy of patronage, it is my intention to continue the series, by a careful selection from other authors of acknowledged merit, who have written on the other branches of science, and to include all those anywise connected with the business of the Mechanic, such as Geometry, Perspective, Algebra, &c.; and I shall endeavour, in the same manner, to illustrate them by an Appendix, using as familiar language as the subject will admit of, and noticing such new improvements as may have been added since the original publication; thus endeavouring to blend, as it were, together the various discoveries of the moderns with the original works of those who have preceded them.

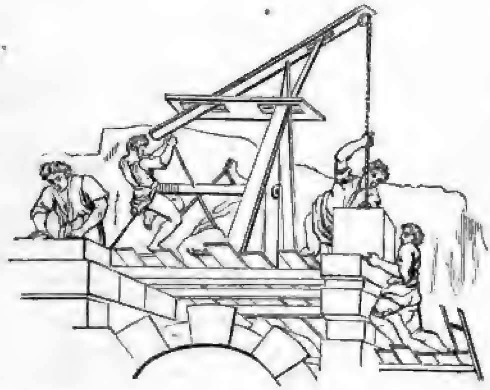
I shall here take my leave; trusting that no material error will be found: and, as I cannot claim any merit in the production, I hope I shall not incur censure, but that the indulgent reader will peruse this with the same spirit in which I have written, that of endeavouring to apply to some useful purpose the talents which are entrusted to us.

G. A. S.

will be to despise their dull criticisms, and laugh at their ignorance.

> Men' moveat cimex Pantilius? aut crucier, quod
> Vellicit absentem Demetrius? aut quod ineptus
> Fannus Hermogenis lœdat Conviva Tigelli?
> Plotius, & Varius, Mæcenas, Virgiliusque,
> Valgius, —— probet hæc ————
>
> Hor.

WILLIAM EMERSON.



MECHANICS.

DEFINITIONS.

1. *Mechanics* is a science, which teaches the proportion of the forces, motions, velocities, and, in general, the actions of bodies upon one another.

2. *Body* is the mass or quantity of matter. If a body yields to a stroke and recovers its former figure again, it is called an *elastic body*: if not, it is *inelastic*.

3. *Density* of a body is the proportion of the quantity of matter contained in it, to the quantity of matter in another body of the same bigness. Thus, the density is said to be double or triple, when the quantity of matter contained in the same space is double or triple.

4. *Force* is a power exerted on a body to move it. If it act but for a moment, it is called the force of *percussion* or *impulse*. If it act constantly, it is called an *accelerative force*. If constantly and equally, it is called a *uniform accelerative force*.

5. *Velocity* is an affection of motion, by which a body passes over a certain space in a given time. The velocity is said to be greater or less, according as the body passes over a greater or less space in the same time.

6. *Motion* is a continual and successive change of place. If a body moves through equal spaces in equal times, it is called *equable motion*. If its velocity continually increases, it is called *accelerated motion;* if it decreases, it is *retarded motion*. If it increases or decreases uniformly, it is *equably accelerated* or *retarded*. Likewise, if its motion be considered in regard to some other body at rest, it is called *absolute motion*. But if its motion be considered with respect to other bodies also in motion, then it is *relative motion*.

7. *Direction of motion* is the way the body tends, or the right line it moves in.

8. *Quantity of motion* is the motion a body has, both in gard to its velocity, and quantity of matter. This is called *momentum* of a body, by some mechanical writers.

9. *Vis inertiæ* is that innate force of matter by which it resi any change, and endeavours to preserve its present state of moti or rest.

10. *Gravity* is that force wherewith a body endeavours to scend towards the centre of the earth. This is called *absol gravity*, when the body tends downwards in free space: and *re tive gravity* is the force it endeavours to descend with in a flu

11. *Specific gravity* of bodies is the greater or less weight bodies of the same magnitude; or the proportion between th weights. The specific gravity is said to be double or triple, wh the weight of the same bulk of matter is double or triple.

12. *Centre of gravity* of a body is a certain point in it, up which the body being freely suspended, it would rest in any po tion.

13. *Centre of motion* of a body is a fixed point about whi the body is moved. And the *axis of motion* is the fixed axis moves about.

14. *Weight and power*, when opposed to one another, sign the body to be removed, and the body that moves it. That bo which communicates the motion is called the *power;* and th which receives it, the *weight*.

15. *Equilibrium* is when two or more forces acting against o another, none of them overcome the others, but destroy one a other's effects, and remain at rest.

16. *A fluid* is a body whose parts yield to any impress force; and by yielding are easily moved among themselves.

17. *Hydrostatics* is a science that treats of the properties fluids.

18. *Hydraulics* is the art of raising or conveying water by

contrary to this is *strength*, which is the resistance any beam is able to make against a force endeavouring to break it.

24. *Friction* is the resistance arising from the parts of machines, or of any bodies rubbing against one another.

POSTULATA.

1. That a small part of the surface of the earth, or the horizon, may be looked upon as a plane. Though this is not strictly true, yet it differs insensibly in so small a space as we have any occasion to consider it.

2. Heavy bodies descend in lines parallel to one another, and perpendicular to the horizon: and they always tend perpendicular to the horizon by their weight. For this is true as to sense, because the lines of their direction meet only at the centre of the earth, taken as a perfect sphere.

3. The weight of any body is the same in all places at or near the surface of the earth. For the difference is insensible at any heights to which we can ascend. Though, in strictness, the force of gravity decreases in ascending, from the earth's surface, in the reciprocal ratio of the squares of the heights from the earth's centre.

4. We are to suppose all planes perfectly even and regular, all bodies perfectly smooth and homogeneous, and moving without friction or resistance; lines perfectly straight, and inflexible, without weight or thickness; cords extremely pliable, &c. For though bodies are defective in all these, and the parts or matter, whereof engines are made, subject to many imperfections, yet we must set aside all these irregularities, till the theory is established; and afterwards make such allowance as is proper.

AXIOMS.

1. Every body perseveres in its present state, whether of rest, or moving uniformly in a right line, till it is compelled to change that state by some external force.

2. The alteration of motion, or the motion generated or destroyed in any body, is proportional to the force applied, and is made in the direction of that right line in which the force acts.

3. The action and re-action between two bodies are equal, and in contrary directions.

4. The motion of the whole body is made up of the sum of the motions of all the parts.

5. The weights of all bodies in the same place, are proportional to the quantities of matter they contain, without any regard to their bulk, figure, or kind. For twice the matter will be twice as heavy, and thrice the matter thrice as heavy; and so on.

6. The vis inertiæ of all bodies is proportional to the quantity of matter.

7. Every body will descend to the lowest place it can get to.

8. Whatever sustains a heavy body, bears all the weight of i

9. Two equal forces acting against one another in contra directions, destroy one another's effects.

10. If a body is acted on with two forces in contrary dir tions, it is the same thing as if it were only acted on with t difference of these forces, in direction of the greater.

11. If a body is kept in equilibrio, the contrary forces, in a one line of direction, are equal, and destroy one another.

12. Whatever quantity of motion any force generates in a giv time, the same quantity of motion will an equal force destroy the same time, acting in a contrary direction.

13. Any active force will sooner or more easily overcome lesser resistance than a greater.

14. If a weight be drawn or pushed by any power, it push or draws all points of the line of direction equally. And it is t same thing, whatever point of that line the force is applied to.

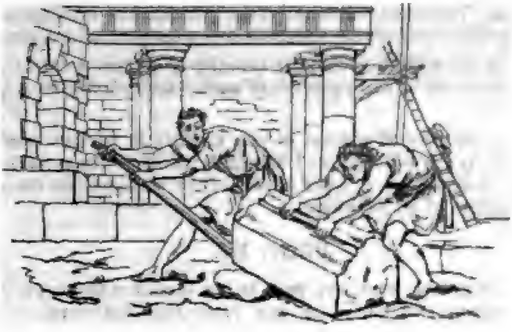
15. If two bodies be moving the same way, in any right li their relative motion will be the same, as if one body stood st and the other approached, or receded from it with the differen of their motions; or with the sum of their motions, if they mo contrary ways.

16. If a body is drawn or urged by a rope, the direction of th force is the same as the direction of that part of the rope next a joining to the body.

17. If any force is applied to move or sustain a body, by mea of a rope, all the intermediate parts of the rope are equa distended, and that in contrary directions.

18. If a running rope go freely over several pullies, all t parts of it are equally stretched.

19. If any forces be applied against one end of a free lever beam, the other end will thrust or act with a force, in direction



SECTION FIRST.

THE GENERAL LAWS OF MOTION.

PROPOSITION I.

THE QUANTITIES OF MATTER IN ALL BODIES ARE IN THE COMPLICATE RATIO OF THEIR MAGNITUDES AND DENSITIES.

FOR by Definition 3, if the magnitudes be equal, the matter will be as the densities. And if the densities be equal, the matter will be as the magnitudes. Therefore, the matter is universally in the compound ratio of both.

Corollary 1.—*The quantities of matter in similar bodies, are as the densities and cubes of their like dimensions, in a sphere.* The magnitude is as the cube of the diameter.

Cor. 2. *The quantities of matter are as the magnitudes and specific gravities.* For the specific gravities are as the densities, by Axiom 5.

PROP. II.

THE QUANTITIES OF MOTION, IN ALL MOVING BODIES WHATEVER, ARE IN THE COMPLICATE RATIO OF THE QUANTITIES OF MATTER AND THE VELOCITIES.

For if the velocities are equal, it is manifest (by Axiom 4,) that the quantities of motion will be as the quantities of matter. And if the quantities of matter are equal, the motions will be as the

velocities. Therefore, universally, the quantities of motion are the compound ratio of the velocities and quantities of matter.

Cor. In any sort of motion, the quantity of motion is as the s of all the products of every particle of matter multiplied by its resp tive velocity.

For the quantity of motion of any particle is as that parti multiplied by its velocity; and as each particle of the same bo moves with equal velocity, the motion of the whole will be the s of the motions of all the parts.

PROP. III.

In all uniform motions, the space described is in t complicate ratio of the time and velocity.

For it is evident, if the velocity be given, the space describ by any body, will be as the time of its moving. And, if the ti be given, the space described will be greater or less, according the velocity is greater or less; that is, the space will be as the locity. Therefore, if neither be given, the space will be in compound ratio of both the time and velocity.

Cor. The time is as the space directly, and velocity reciprocally.

PROP. IV.

The motion generated by any momentary force, is the force that generates it.

For if a certain quantity of force generates any motion, a doul quantity of force will generate double the motion; and a trip force, triple the motion; and so on.

Cor. The space described is as the force and time directly, a quantity of matter reciprocally.

all suppositions. And thence all the laws and proportions belonging to uniform motion, may be readily and universally resolved: expunging such as are not concerned in the question; and rejecting all those that are given or constant; and also those that are in both terms of the proportion. Thus we shall get in general,

$$f \propto m \propto bv \propto \frac{bs}{t}$$

$$m \propto f \propto bv \propto \frac{bs}{t}$$

$$s \propto tv \propto \frac{tm}{b} \propto \frac{tf}{b}$$

$$v \propto \frac{m}{b} \propto \frac{s}{t} \propto \frac{f}{b}$$

$$t \propto \frac{s}{v} \propto \frac{sb}{m} \propto \frac{sb}{f}$$

$$b \propto \frac{m}{v} \propto \frac{f}{v} \propto \frac{mt}{s} \propto \frac{ft}{s}.$$

PROP. V.

IN ANY MOTION, GENERATED BY A UNIFORMLY ACCELERATING FORCE, THE MOTION GENERATED IN ANY TIME IS IN THE COMPLICATE RATIO OF THE FORCE AND TIME.

For in any given time, the motion generated will be proportional to the force that generates it, this being its natural and genuine effect. And since in all the several parts of time, the force is the same, and has the same efficacy, therefore the motion generated will also be as the time: whence universally, the motion generated is in the compound ratio of both the force and the time of acting.

Cor. 1. *This Proposition is equally true in respect to the motion lost or destroyed in a moving body, by a force acting in a contrary direction.* By Axiom 12.

Cor. 2. *If the space through which a body is moved by any force, be divided into an indefinite number of small equal parts; and if, in each part, the accelerative force acts differently upon the body, according to any certain law; and if there be taken the product of the accelerating force in each part multiplied by the time of passing through it; then I say,*

As any uniform accelerative force × by the time of acting (FT):
Is to the motion generated in that time (M) ::

the lines AB, AC. Then since the force acting in the directi AC parallel to BD, by Axiom 2, will not alter the velocity wards the line BD; the body therefore will arrive at BD in same time, whether the force in the direction AC be impressed not: therefore, at the end of the time, it will be found somewh in BD. By the same argument, it will be found somewhere in line CD; therefore it will be found in D, the point of intersectio and by Axiom 1, it will move in a right line from A to D.

Otherwise:

Suppose the line AC to move parallel to itself into the pl BD, whilst A moves from A to C, then since this line and body are both equally moved towards BD, it is plain the bo must be always in the moveable line AC. Therefore, when A comes to the position *bg*, let the body be arrived at *d*; th since both the line AC moves uniformly along AB, and the bo A along AC; therefore it will be as A*b* : *bd* : : AB : BD, theref A*d*D is a right line.

Case II. (*Fig.* 1. *Pl.* I.)

Let the body be carried through AB, AC, by an accelerat force. Then, by Prop. VI. the space described will be as the ti and velocity, and therefore the velocity will be as the space direc and time reciprocally. Also, by Cor. 3, Prop. V., when the fo and the body is given, the velocity is as the time. Whence time will be as the space directly and time reciprocally, and space as the square of the time. That is, the same body acted by the same force will describe spaces which are as the squa of the times. Now let the time of describing AB or AC be and let the line AC move along with the body, always parallel itself, and in the time *t*, let it arrive at *bg*, and the body

Cor. 1. (*Fig.* 1. *Pl.* I.) *The forces, in the directions* AB, AC, AD, *are respectively proportional to the lines,* AB, AC, AD.

For by Cor. Prop. IV. the time and the quantity of matter being given; the force is directly as the space described. And in accelerated motion, the same is true, by Prop. V. Cor. 3. and Prop. VI.

Cor. 2. *The two oblique forces,* AB, AC, *are equivalent to the single direct force* AD, *which may be compounded of these two, by drawing the diagonal of the parallelogram* AD.

Cor. 3. *Any single direct force* AD, *may be resolved into the two oblique forces whose quantities and directions are* AB, AC, *having the same effect, by describing any parallelogram whose diagonal is* AD.

Cor. 4. *A body being agitated by two forces at once, will pass through the same point, as it would do if the two forces were to act separately and successively. And if any new motion be impressed on a body already in motion, it does not alter its motion in lines parallel to its former direction.*

Cor. 5. (Fig. 2. Pl. I.) *If two forces, as* AB, AC, *act in the directions* AB, AC, *respectively, draw* AR *to the middle of the right line* BC, *and* 2 AR *is the force compounded out of these, and* AR *its direction.*

PROP. VIII. (*Fig.* 3. *Pl.* I.)

LET THERE BE THREE FORCES, A, B, C, OF THE SAME KIND, ACTING AGAINST ONE ANOTHER, AT THE POINT D, AND WHOSE DIRECTIONS ARE ALL IN ONE PLANE; AND IF THEY KEEP ONE ANOTHER IN EQUILIBRIO, THESE FORCES WILL BE TO EACH OTHER RESPECTIVELY, AS THE THREE SIDES OF A TRIANGLE DRAWN PARALLEL TO THEIR LINES OF DIRECTION, DI, CI, CD.

Let DC represent the force C, and produce AD, BD, and complete the parallelogram DICH: And by the last Prop. the force DC is equivalent to the two forces DH, DI; put, therefore, the forces DH, DI instead of DC, and all the forces will still be in equilibrio. Therefore, by Ax. II., DI is equal to its opposite force A, and DH or CI equal to its opposite force B. Therefore, the three forces A, B, C are respectively as DI, CI, CD.

Cor. 1. *Hence the forces,* A, B, C *are respectively as the three sides of a triangle, drawn perpendicular to their lines of direction, or in any given angle to them, on the same side.* For such a triangle will be similar to the former triangle.

Cor. 2. *The three forces* ABC *will be to each other as the sines of the angles through which their respective lines of direction do pass, when produced:*

For DI : CI :: S.DCI or CDB : S.CDI or CDA.
And CI : CD :: S.CDI or CDA : S.CHD or HDI or ADI

Cor. 3. *If there be ever so many forces acting against any pc in one plane, and keep one another in equilibrio, they may be all duced to the action of three, or even of two equal and opposite one*

For if HD, ID be two forces, they are equivalent to the sin force DC. and, in like manner, A and B may be reduced to a s gle force.

Cor. 4. *And if ever so many forces in different planes, act against one point, keep one another in equilibrio, they may be reduced to the actions of several forces in one plane, and, consequen to two equal and opposite ones.*

For if the four forces, A, B, H, I, act against the point D, a H, I, be out of the plane ABD, let DC be the common section the planes ADB, HDI; then the forces H, I, are reduced to force C, in the plane ADB.

Scholium.—This Prop. holds true of all forces whatever, wh ther of impulse or percussion, thrusting, pulling, pressing, whether instantaneous or continual, provided they be all of t same kind.

Hence, if three forces act in one plane, their proportions a had; and if one force be given, the rest may be found. And four forces act, and two be given, the other two may be foun but if only one be given, the rest cannot be found; for in t three forces, A, B, C, the force C may be divided into otl two, an infinite number of ways, by drawing any parallelogra DICH about the diagonal DC; and, in general, if there be a number of forces acting at D, and all be given but two, the two may be found; otherwise not, though the positions of the all be given.

by the pressure of B against it by the force EB; which, if the surface be perfectly smooth and void of tenacity, will be nothing. The force DB, therefore, having no effect, the remaining force EB will be the only one whereby the body B acts against the surface DB, and that in direction EB, perpendicular to it.

Cor. 1. *If a given body* B *strike another body* C *obliquely, at any angle* ABD, *the magnitude of the stroke will be directly as the velocity and the sine of the angle of incidence* ABD; *and the body* C *receives that stroke in the direction* EB *perpendicular to the surface* DB.

For if the angle ABD be given, the stroke will be greater in proportion to the velocity; and if the velocity AB be given, the stroke will be as AD, or S ∠ ABD; or the magnitude of the stroke is as the velocity wherewith the body approaches the plane.

Cor. 2. (Fig. 5. Pl. I.) *If a perfectly elastic body* A *impinges on a hard or elastic body* CB *at* B, *it will be reflected from it, so that the angle of reflection will be equal to the angle of incidence.*

For the motion at B parallel to the surface is not at all changed by the stroke; and, because the bodies are elastic, they recover their figure in the same time they lose it by the stroke; therefore, the velocity in direction BE is the same after as before the stroke. Let AE, BE, represent the velocities before the stroke, and ED (= AE) and BE the respective velocities after the stroke; then in the two similar and equal triangles, AEB, BED, ∠ ABE is equal to ∠ EBD.

But since no bodies in nature are perfectly elastic, they are something longer of regaining their figure; and, therefore, the angle DBF will be something more acute than the ∠ ABG.

Cor. 3. *If one given body impinges upon another given body, the magnitude of the stroke will be as the relative velocity between the bodies.*

For the magnitude of the stroke is as the line BE, (*Fig.* 4. *Pl.* I.) or the velocity wherewith the bodies approach each other; that is, as the relative velocity.

Cor. 4. *And in any bodies whatever, if a body in motion strike against another, the magnitude of the stroke will be as the motion lost by the striking body.*

For the motion impressed on the body that receives the stroke is equal to the magnitude of the stroke; and the same motion by Ax. 3, is equal to the motion lost in the striking body.

Cor. 5. *A non-elastic body striking another non-elastic body, only loses half as much motion as if the bodies were perfectly elastic.*

For the non-elastic bodies only stop; but elastic bodies recede with the same velocity they meet with.

Cor. 6. *Hence, also, it follows, that if one body acts upon another, by striking, pressing, &c. the other re-acts upon this in the direction of a line perpendicular to the surface whereon they act.* By Ax. 3.

SCHOLIUM.—Though the momentum, or quantity of motion, in a moving body, is a quite distinct thing from the force that generates it, yet, when it strikes another body and puts it into motion, it may, with respect to that other body, be considered as a certain quantity of force proportional to the motion it generates in the other body.

Also, although the motion generated by the impulse of another body is considered as generated in an instant, upon account of the very small time it is performed in, yet, in mathematical strictness, it is absolutely impossible that any motion can be generated in an instant, by impulse or any sort of finite force whatever. For when we consider that the parts of the body which yield to the stroke, are forced into a new position, there will be required some time for the yielding parts to be moved through a certain space into this new position. Now, during this time, the two bodies are acting upon each other with a certain accelerative force, which, in that time, generates that motion, which is the effect of their natural impulse. So that it is plain that this is an effect produced in time; and the less the time, the greater the force; and if the time be infinitely small, the force ought to be infinitely great, which is impossible. But by reason that this effect is produced in so small a time as to be utterly imperceptible, so that it cannot be brought to any calculation, upon this account the time is entirely set aside, and the whole effect imputed to the force only, which is therefore supposed to act but for a moment.

The quantity of motion in bodies has been proved to be as the velocity and quantity of matter. But the momentum or quantity of motion may be the same in different bodies, and yet may have [redacted]

pieces; and great bodies with small velocity, to shake or move the whole.

PROP. X.

THE SUM OF THE MOTIONS OF ANY TWO BODIES IN ANY ONE LINE OF DIRECTION, TOWARDS THE SAME PART, CANNOT BE CHANGED BY ANY ACTION OF THE BODIES UPON EACH OTHER, WHATEVER FORCES THESE ACTIONS ARE CAUSED BY, OR THE BODIES EXERT AMONG THEMSELVES.

Here I esteem progressive motions, or motions towards the same part, affirmative; and regressive ones, negative.

CASE I.

Let two bodies move the same way, and strike one another directly. Now, since action and re-action (by Ax. 3.) are equal and contrary, and this action and re-action is the very force by which the new motions are generated in the bodies, therefore,(by Ax. 2.) there will be produced equal changes towards contrary parts. And, therefore, whatever quantity of motion is gained by the preceding body will be lost by the following one; and, consequently, their sum is the same as before.

And if the bodies do not strike each other, but are supposed to act any other way, as by pressure, attraction, repulsion, &c., yet still, since action and re-action are equal and contrary, there will be induced an equal change in the motion of the bodies, and in contrary directions; so that the sum of the motions will still remain the same.

CASE II.

Suppose the bodies to strike each other obliquely; then, since (by the last Prop.) they act upon each other in a direction perpendicular to the surface in which they strike, the action and re-action in that direction being equal and contrary, the sum of the motions, the same way, in that line of direction, must remain the same as before. And since the bodies do not act upon each other in a direction parallel to the striking surface, therefore there is induced no change of motion in that direction. And therefore, universally, the sum of the motions will remain the same, considered in any one line of direction whatever. And if the bodies act upon one another by any other forces whatever, still (by Ax. 2, and 3,) the changes of motion will be equal and contrary, and their sum the same as before.

Cor. 1. *The sum of the motions of any system or number of bodies, in any one line of direction, taken the same way, remains always the same, whatever forces these bodies exert upon each other; esteeming contrary motions to be negative, and, therefore,*

Cor. 2. *The sum of the motions of all the bodies in the world, es mated in one and the same line of direction, and always the same w is eternally and invariably the same; esteeming these motions af mative which are progressive, or directed the same way; and the gressive motions negative. And, therefore, in this sense, motion c neither be increased nor diminished. But,*

Cor. 3. *If you reckon the motions in all directions to be affirm tive, then the quantity of motion may be increased or decreased an finite number of ways. As, suppose two equal non-elastic bodies meet one another with equal velocities, they will both stop and lose their motions.*

For let M be the motion of each, then, before meeting the su of their motions is M+M; and after their meeting, it is 0. B in the sense of this Prop. M—M is the sum of the motions befo they meet, because they move contrary ways, which is 0; and it the same after they meet. And thus a man may put several bodi into motion with his hands, which had no motion before; a that in as many several directions as he will.

PROP. XI.

The motions of bodies included in a given space, a the same among themselves; whether that space is rest, or moves uniformly forward in a right line.

For if a body be moving in any right line, and there be a force equally impressed, both upon the body and the right line any direction; and, in consequence of this, they both move ui formly with the same velocity; now, as there is no force to ca the body out of that line, it must still continue in it as before; a as there is no force to alter the motion of the body in the rig line, it will (by Ax. 1.) still continue to move in it as before. F the same reason, the motions of any number of bodies moving

such as pressure, gravity, &c.: concerning this *vis viva* they talk so obscurely, that it is hard to know what they mean by it. But they measure its quantity by the number of springs which a moving body can bend to the same degree of tension, or break; whether it be a longer or shorter time in bending them. So that the *vis viva* is the total effect of a body in motion, acting till its motion be all spent. And, according to this, they find that the force (or *vis viva*) to overcome any number of springs, will always be as the body multiplied by the square of the velocity.

Suppose any number of equal and similar springs placed at equal distances in a right line, and a body be moved in the same right line against these springs; then the number of springs which that body will break before it stop, will be as the square of its velocity; whatever be the law of the resistance of any spring in the several parts of its tension; for, from the foregoing Prop. it appears, that the swifter the body moves, so much the less time has any spring to act against it to destroy its motion; and, therefore, the motion destroyed by one spring will be as the time of its acting; and by several springs, as the whole time of their acting; and, consequently, the resistance is uniform. And since the resistance is uniform, the velocity lost will be as the time, that is as the space directly and velocity reciprocally; whence the space, and therefore, the number of springs, is as the square of the velocity. And upon this account they measure the force of a body in motion, by the square of the velocity. So at last the *vis viva* seems to be the total space passed over, by a body meeting with a given resistance, which space is always as the square of the velocity. And this comes to the same thing as the force and time together, in the common mechanics.

Now it seems to be a necessary property of the *vis viva*, that the resistance is uniform; but there are infinite cases where this does not happen; and, in such cases, this law of the *vis viva* must fail. And since it fails in so many cases, and is so obscure itself, it ought to be weeded out, and not to pass for a principle in mechanics.

Likewise, if bodies in motion impinge on one another, the conservation of the *vis viva* can only take place when the bodies are perfectly elastic. But as there are no bodies to be found in nature which are so; this law will never hold good in the motion of bodies after impulse, but, in this respect, it must eternally fail.

This notion of the *vis viva* was first introduced by M. Leibnitz, who believed that every particle of matter was endued with a living soul.

SECTION SECOND.

THE LAWS OF GRAVITY, THE DESCENT OI HEAVY BODIES, AND THE MOTION OF PRO JECTILES, IN FREE SPACE.

PROP. XII.

THE SAME QUANTITY OF FORCE IS REQUISITE TO KEEP A BOI IN ANY UNIFORM MOTION, DIRECTLY UPWARDS, AS IS R QUIRED TO KEEP IT SUSPENDED, OR AT REST.

AND IF A BODY DESCENDS UNIFORMLY, THE SAME FORCE TH. IS SUFFICIENT TO HINDER ITS ACCELERATION IN DESCENDIN IS EQUAL TO THE WEIGHT OF IT.

For the force of gravity will act equally on the body in a state whether of motion or rest; therefore, if a body is project directly upwards or downwards, with any degree of velocity, would for ever retain that velocity if it were not for the force gravity that draws it down, (by Ax. 1.) If, therefore, a force equ to its gravity were applied directly upwards; then (by Ax. these two forces destroy each other's effects; and it is the sar thing as if the body was acted on by no force at all; and, ther

Cor. 1. *All bodies falling by their own weight, gain equal velocities in equal times.*

Cor. 2. *Whatever velocity a falling body gains in any time, if it be thrown directly upwards, it will lose as much in an equal time; by* Ax. 12. *And, therefore,*

Cor. 3. *If a body be projected upwards with the velocity it acquired by falling in any time; it will, in the same time, lose all its motion. Hence, also,*

Cor. 4. *Bodies thrown upwards lose equal velocities in equal times.*

PROP. XIV.

THE SPACES DESCRIBED BY FALLING BODIES ARE AS THE SQUARES OF THE TIMES OF THEIR FALLING FROM REST.

For by Postul. 3. gravity is a uniformly accelerating force; therefore, (by Prop. VI.) the space described is as the time and velocity. But by the last Prop. the time is as the velocity; and, therefore, the space described is as the square of the time.

Cor. 1. *The spaces described by falling bodies are also as the squares of the velocities; or the velocity is as the square root of the height fallen.*

Cor. 2. *Taking any equal parts of time; then the spaces described by a falling body, in each successive part of time, will be as the odd numbers,* 1, 3, 5, 7, 9, 11, *&c.*

For, in the times 1, 2, 3, 4, &c., the spaces described will be as their squares 1, 4, 9, 16, &c. And, therefore, in the differences of the times, or in these equal parts of time, the spaces described will be as the differences of the squares, or as 1, 3, 5, 7, &c.

Cor. 3. *A body moving with the velocity acquired by falling through any space, will describe twice that space in the time of its fall.* By Cor. 1. Prop. VI.

Cor. 4. *If a body be projected upwards with the velocity it acquired in falling, it will, in the same time, ascend to the same height it fell from; and describe equal spaces in equal times, both in rising and falling, but in an inverse order, and will have the same velocity at every point of the line described.*

For by Cor. 2. of the last Prop. equal velocities will be gained or lost in equal times, (reckoning from the last moment of the descent.) Therefore, since, at the several correspondent points of time, the velocities will be equal, the spaces described in any given time will be equal, and the wholes equal.

Cor. 5. *If bodies be projected upwards with any velocities, the*

heights of their ascent will be as the squares of the velocities, or as t *squares of the times of their ascending.*

For, in descending bodies, the spaces descended are as t squares of the last velocities, by Cor. 1. And by Cor. 4. t spaces ascended will be equal to those descended.

Cor. 6. *If a body is projected upwards with any velocity, with* t *same velocity undiminished, it would describe twice the space of whole ascent, in the same time.* By Cor. 3. and 4.

Cor. 7. *Hence, also, all bodies from equal altitudes descend to* t *surface of the earth in equal times.*

SCHOLIUM.—It is known by experiments, that a heavy bo falls $16\frac{1}{12}$ feet in a second of time, and acquires a velocity whi will carry it over $32\frac{1}{6}$ feet in a second; which being known, t spaces described in any other times, and the velocities acquire will be known by the foregoing propositions; and the contrar These propositions are exactly true, where there is no resistan to hinder the motion; but because bodies are a little resisted l the air, descended bodies will be a little longer in falling; and body projected upwards, will be something longer in descendi than in ascending, and falls with a less velocity; and, consequentl a body projected upwards with the velocity it falls with, will n ascend quite to the same height; but these errors are so small, tha in most cases, they may safely be neglected.

If the force by which a body is accelerated in falling was d rectly as the height fallen from; it may be computed (by Cor. Prop. V.) that the velocity acquired will also be as the heigh or the space described directly as the velocity. And, therefore, bodies were projected upwards, they would in this case ascend heights, which are as the velocities with which they are projecte This being compared with Cor. 5. of the last Prop. it is easy conclude that bodies projected upwards, and acted upon by

PROP. XV. (*Figures 6 and 7. Pl.* I.)

A BODY BE PROJECTED EITHER PARALLEL TO THE HORIZON, OR IN ANY OBLIQUE DIRECTION, IT WILL, BY ITS MOTION, DESCRIBE A PARABOLA.

Let AD be the direction of the motion, AFG the curve described; and let AB, BC, CD, &c. be all equal; draw AM, BF, CG, DH, &c. perpendicular to the horizon; and complete the parallelograms, AF, AG, AH, &c., then, by Ax. 1. if the body were without gravity, it would move on in the line AD, and describe the lines AB, BC, CD, &c. in equal times. Now, since gravity acts in lines perpendicular to the horizon, it does not affect the motion in direction AD, but generates a motion in direction AM. So that the body, instead of being at B, C, D, &c., will, at the same points of time be at F, G, H, &c. But in the time of describing AB, AC, AD, the body, by the force of gravity, will descend through the spaces BF, CG, DH; which are as the squares of the times they are described in, (by Prop. XIV.) that is, as the squares of the lines, AB, AC, AD. But AB, AC, AD, are equal to KF, LG, MH; and BF, CG, DH, equal to AK, AL, AM. Therefore, the parts of the axis of the curve, AK, AL, AM, &c. are respectively as the squares of the ordinates KF^2, LG^2, MH^2, &c. And therefore, by the conic sections, the curve AFG is a parabola.

Cor. 1. *The line of direction* AD *is a tangent to the curve in* A. *And the latus rectum to the point* A *is* $\frac{KF^2}{AK}$ *or* $\frac{AB^2}{BF}$. *And in the oblique projection,* (*Fig.* 7. *Pl.* I.) *the parameter is* $\frac{AO^2}{BF}$, *supposing* AOP *perpendicular to* AM, *and* G *the vertex.* For then $\frac{AO^2}{BF} = \frac{AP^2}{CG = GP}$.

Cor. 2. *If the horizontal velocities of projectiles be the same, whatever their elevations be, they will describe the same parabola.*

For if AB be the velocity and direction of the projectile, then AO is the horizontal velocity. When the body comes to the vertex G, its motion is then parallel to the horizon, which parallel motion remains the same as before, that is, equal to AO. Therefore it describes the same parabola, as a body projected from G with the velocity AO parallel to the horizon.

Cor. 3. *The velocity of a projectile in any point of the curve, is as the secant of the angle of its direction above the horizon.*

For AO the horizontal velocity is the same at all points of the curve; and the velocity AB at A, in the curve, is the secant of the angle of elevation OAB.

Cor. 4. *The velocity at any point of the curve is the same that is*

acquired by falling through $\frac{1}{4}$ the parameter belonging to that poi or, which is the same, through $\frac{1}{4}$ of the principal latus rectum + abscissa to that point.

For let IA be the space fallen through to acquire the velocity any point as A; then the space AD described in the same ti with that velocity in direction AV, will be 2AI (by Cor. 3. Pr XIV.) but in the same time, by the same force of gravity, the bo will descend through an equal space DH, therefore AD or MH 2DH or 2AM, but the parameter $= \frac{MH^2}{AM} = \frac{4AM^2}{AM}$ or 4A Therefore AM or AI $= \frac{1}{4}$ parameter.

PROP. XVI. (*Fig.* 7. *Pl.* I.)

THE HORIZONTAL DISTANCES OF PROJECTIONS, MADE WITH A VELOCITIES, AND AT ANY ELEVATIONS, ARE AS THE SINES THE DOUBLED ANGLES OF ELEVATION, AND THE SQUARES THE VELOCITIES CONJUNCTLY.

Let $v =$ *velocity of the projectile; measured by the spac passes through in time* 1″.
$f =$ *descent of a body by gravity in the same time.*
$x =$ AE *the horizontal distance, or amplitude.*
$s =$ *sine*, $c =$ *cosine* } *of the elevation* VAE.
$A =$ *sine*, $B =$ *vers. sine* } *of twice the elevation.*

Then by trigonometry, $2sc = A$, and $2ss = B$, when the radi is 1. And in the right angled triangle AEV.

$$c : x :: (\text{rad.})\ 1 : \frac{x}{c} = AV, \text{ and}$$

$$c : x :: s : \frac{sx}{c} = VE.$$

sines of elevation, and the squares of the velocities; or, as the versed sines of the doubled angles of elevation, and the squares of the velocities.

For if G be the vertex, GP $=$ CP $= \frac{1}{4}$ VE, and VE $= \frac{sx}{c} = \frac{vvss}{f}$. Therefore GP $= \frac{vvss}{4f} = \frac{vvB}{8f}$. Therefore GP is as $vvss$ or as vvB.

Cor. 2. *The times of flight of projectiles are as the velocities, and the sines of elevation.*

For the time $= \frac{x}{cv} = \frac{vs}{f}$

Cor. 3. *The greatest random or horizontal projection, is at the elevation of* 45 *degrees; and the horizontal distances are equal, at elevations equally distant above or below* 45°.

SCHOLIUM.—Let h be the height of the perpendicular projection with the velocity v; then will $h = \frac{vv}{4f}$. Whence

Horizontal distance $= \frac{vvsc}{f} = \frac{vvA}{2f} = 4sch = 2Ah$.

Altitude of the projection $= \frac{vvss}{4f} = \frac{vvB}{8f} = ssh = \frac{1}{2}Bh$.

Time of flight $= \frac{vs}{f} = 2s\sqrt{\frac{h}{f}}$.

PROP. XVII. (*Fig.* 8. *Pl.* I.)

THE DISTANCES OF PROJECTIONS MADE ON ANY INCLINED PLANES, ARE IN THE COMPLICATE RATIO OF THE SINES OF THE ANGLES WHICH THE LINES OF DIRECTION MAKE WITH THE PLANE AND ZENITH, AND THE SQUARES OF THE VELOCITIES, DIRECTLY; AND THE COSINES SQUARED OF THE PLANE'S ELEVATION RECIPROCALLY.

Let AE be the inclined plane, AV the direction of the projectile, SA, CP, VE, perpendicular to the horizon; AGE the path of the projectile, and let $v =$ velocity of the projectile in A, measured by the space it describes in the times 1. $f =$ space described by a descending body in the same time. $s =$ sine of VAE. $c =$ sine of VAS. $z =$ sine of SAE. $x =$ AE the oblique distance or random.

Then by plane trigonometry.

$c : x$ (AE) $:: z :$ AV $= \frac{xz}{c}$, and

$c : x :: s :$ VE $= \frac{sx}{c}$.

And by Prop. III.

Space v : time 1 :: (AV) $\frac{xz}{c}$: $\frac{xz}{cv}$ = time of describing Al

And by Prop. XIV.

Space f : time 1 :: (VE) $\frac{sx}{c}$: $\frac{sx}{fc}$ = square of the time of scending through VE.

But the times of describing AV, VE, being equal, we h $\frac{sx}{fc} = \frac{xxzz}{ccvv}$, or $\frac{s}{f} = \frac{xzz}{cvv}$; whence $x = \frac{scvv}{fzz}$, and f being a gi quantity, x or AE is as $\frac{scvv}{zz}$.

Cor. 1. *The heights above the planes are as the squares of velocities, the squares of the sines of elevation above the pl directly; and the squares of the cosines of the plane's elevat reciprocally.*

For if AP = PE, then G is the vertex of the parabola, respect of the plane AE. And GP = $\frac{1}{4}$ VE = $\frac{sx}{4c} = \frac{ssvv}{4fzz}$.

Cor. 2. *The times of flight are as the velocities and sines elevation above the plane, and the cosines of the plane's elevat reciprocally.*

For the time is = $\frac{zx}{cv} = \frac{sv}{fz}$.

Cor. 3. *Hence, also, the altitude is as the square of the time flight.*

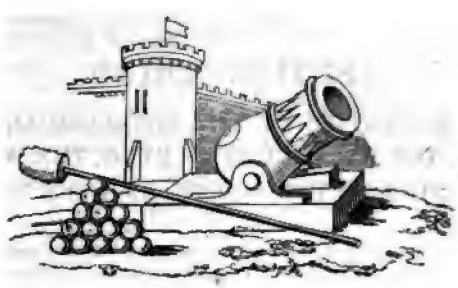
For the altitude is $\frac{ssvv}{zz}$, and the time as $\frac{sv}{z}$.

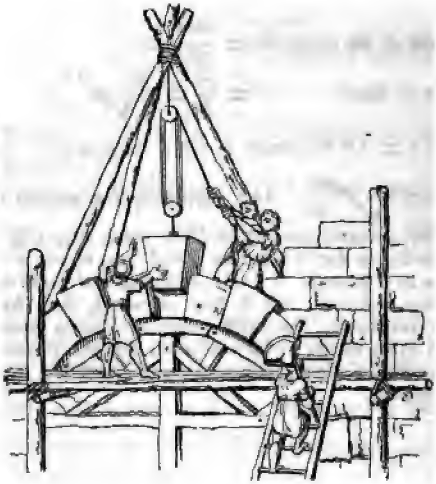
Cor. 4. *The greatest projection upon an inclined plane is w*

Height of the projection $= \frac{ssvv}{4fzz} = \frac{ssh}{zz}$.

Time of flight $= \frac{sv}{fz} = \frac{2s}{z}\sqrt{\frac{h}{f}}$.

And if $d =$ AE the length of the projection, then $\frac{scvv}{fzz} = d$, whence $v = z\sqrt{\frac{df}{sc}}$. And supposing the utmost random of one of our greatest guns to be 5864 paces, then $v = 194$ paces $= 324$ yards, so that a ball shot out of her, moves at the rate of 324 yards in a second. All this supposes, that there is no resistance of the medium; but it may be noted, that, by reason of the air's resistance, the upper randoms, being more resisted, scarce go so far as the under randoms; and the greatest random upon a horizontal plane, is, therefore, at something less elevation than 45 degrees.





SECTION THIRD.

THE PROPERTIES OF THE MECHANICAL PO' ERS; THE BALANCE, THE LEVER, THE WHEI THE PULLEY, THE SCREW, AND THE WEDG

Aa, Bb, described by these bodies, will be equal; consequently, the velocities and quantities of matter of A, B, being equal, their momenta, or motions, will be equal; and, because ACB is a right line, they move in a contrary direction; and, therefore, by Ax. 9, these bodies cannot, of themselves, raise one the other, but must remain in equilibrio.

Cor. Hence, equal forces, A, B, applied at equal distances from the centre of motion C, will have the same effect in turning the balance.

PROP. XIX. (*Figures* 10. 19. 20. *and* 12. *Plates* I. *and* II.)

IN ANY STRAIGHT LEVER, IF THE POWER P BE TO THE WEIGHT W AS THE DISTANCE OF THE WEIGHT FROM THE FULCRUM C TO THE DISTANCE OF THE POWER FROM THE FULCRUM, THE POWER AND WEIGHT (ACTING PERPENDICULARLY ON LEVER) WILL BE IN EQUILIBRIO.

A lever is any inflexible beam, staff, or bar, whether of metal or wood, &c., that can any way be applied to move bodies. There are four kinds of levers:

1. A lever of the first kind is, that where the fulcrum is between the weight and the power, (*fig.* 10. *Pl.* I.)
2. A lever of the second kind is, where the weight is between the fulcrum and the power, (*fig.* 19. *Pl.* II.)
3. The lever of the third kind is, where the power P is between the weight and the fulcrum, (*fig.* 20. *Pl.* II.)
4. The fourth kind is the bended lever, (*fig.* 12. *Pl.* I.)

CASE I. (*Fig.* 10. *Pl.* I.)

In the lever of the first kind WCP, instead of the power P, apply a weight P to act at the end of CP. And let the lever WCP be moved into the position *a*C*b*. Then will the arches W*a*, P*b* be as the radii CW, CP; that is, as the velocities of the weight and power. Whence, since P : W :: CW ; CP, therefore P : W :: velocity of W : velocity of P : therefore P $\times$ velocity of P $=$ W $\times$ velocity of W. Consequently the momenta or motions of P and W are equal. And since they act in contrary directions, therefore by Ax. 9. neither of them can move the other, but they will remain in equilibrio.

CASE II. (*Figures* 19 *and* 20. *Pl.* II.)

The levers of the second and third kind may be reduced to the first, thus; make C*p* $=$ CP, and instead of the power P, apply a weight equal to it at *p*. Then by Case I., the weight W and power *p* will keep one another in equilibrio; and (by Cor. Prop. XVIII.) the weight *p* and power P will have the same effect in turning the lever about its centre, therefore the power P and weight W will be in equilibrio.

Cor. 1. (Figures 10, 11, 12, 13, 14, 15, 16.) ***In any sort*** *lever, whether straight or bended, and whether moveable* ***about a sing*** *point* C, *or an axis* AB; *or whether the lever be fixed to* ***the axis a*** *both together moveable about two centres* A, B; *or* ***whatever fo*** *the levers have; if* AB *be a right line, and from the ends* P, W, ***th*** *be drawn lines to the centre* C, *or perpendiculars to the* ***axis*** Al *and if the power and weight act perpendicular to* ***these lines, and*** *always reciprocally as these distances drawn to the centre* C ***or a*** AB; *then they will be in equilibrio.*

Cor. 2. (Figures 17 and 18.) *In any sort of lever* WCP, ***and*** *whatever directions the power and weight act on it; if* ***their quantit*** *be reciprocally as the perpendiculars to their several lines of* ***directi*** *let fall from the centre of motion, they will be in* ***equilibrio. Or th*** *will be in equilibrio, when the weight multiplied by* ***its distance, and*** S. *angle of its direction, is equal to the power multiplied by* ***its distan*** *and* S.∠ *of its direction.* $W \times WC \times S.DWC = P \times PC \times S.EP$

For the power and weight will be in equilibrio if they be su posed to act at E and D; and (by Ax. 14.) it is the same thi whether they act at E and D, or at P and W. Also by trigon metry, $WC \times S.W = DC$, and $PC \times S.P = CE$.

Cor. 3. *Hence universally, if any force be applied to a lever, effect, in moving the lever, will be as that force multiplied by the d tance of its line of direction from the centre of motion. Or the eff is as the force* × *by its distance from the centre, and by the sine the angle of its direction,* $P \times PC \times S.P$.

Cor. 4. *If two bodies be in equilibrio on the lever, each weight reciprocally as its distance from the centre.*

Cor. 5. *In the straight lever when the weight and power are equilibrio, and act perpendicularly on the lever, or in parallel dir tions; then of these three the power, weight, and pressure upon t*

For the force of each weight to move the lever is as the weight multiplied by the distance (by Cor. 3. last Prop.); and the sum of the products is as the whole forces; which if they be equal, the forces on both sides are equal, and the lever remains at rest.

PROP. XXI. (*Fig.* 22. *Pl.* II.)

If a bended lever WCP be kept in equilibrio by two powers, acting in the directions PB, WA perpendicular to the ends of the lever CP, CW; and if the lines of direction be produced till they meet in A, and AC be drawn, and CB parallel to WA;—I say the power P, the weight or power W, and the force acting against the fulcrum C, will be respectively as AB, BC, AC; and in these very directions.

Draw CB, CF parallel to WA, PA; then the angle WFC = WAP = CBP, and the right angled triangles WCF and BCP are similar; whence CF : CB :: CW : CP :: (by Cor. 2. Prop. XIX.) power P : power W. Now since (by Ax. 14.) it is the same thing to what points of the lines of direction PB, WF, the forces P, W be applied; let us suppose them both to act at the point of intersection A; then since the point A is acted on by two forces which are as CF and CB, or as AB and AF; and both these are equivalent to the single force AC (by Cor. 2. Prop. 7.) Therefore the fulcrum C is acted on by the force AC, and in that direction, by Ax. 11.

Cor. 1. *Hence the power* P, *the weight* W, *and the pressure the fulcrum* C *sustains; are respectively as* WC, PC, *and* PW. *That is, any one is as the distance of the other two.*

For since the angles at P, W are right; CA is the diameter of a circle passing through the points A, P, C, W; therefore the angle WPC = WAC = ACB, and the angle CWP = CAP; therefore the triangles ABC, WCP are similar; and AB : BC :: WC : CP, and BC : AC :: CP : PW. Therefore, &c.

Cor. 2. *In any lever* WCP, *the lines of direction of the powers* PW, WF, *and of the pressure on the fulcrum* C, *all tend to one point* A.

For if not, the lever would not remain in equilibrio.

PROP. XXII. (*Fig.* 23. *Pl.* II.)

If AB, CD, be two levers moveable about A and C, and some force acts upon the end B of the lever AB, in a given direction BF; whilst the lever AB acts upon CD at F: if BE be drawn perpendicular to CB, and AE parallel to BF: and if these levers keep one another in equilibrio:—then I say, the force in direction BF,

FORCE AGAINST DC IN DIRECTION EB, AND THE PRESSU AGAINST THE CENTRE A, ARE RESPECTIVELY AS AE, BE, AE

For since (by Prop. IX.) the lever AB acts upon BC at B, the direction EB perpendicular to BC; and the lever CD re-a in direction BE; and (by Ax. 19.) the point A is acted on in rection BA. Therefore the point B is acted on with three forc BF the force applied at B, and BE the re-action of the lever C and AB the re-action of the centre A; and AE is parallel to B therefore (by Prop. VIII.) these forces are as AE, BE, a AB.

Cor. If two forces BF, BE, *acting perpendicular to the le* AB, DC, *keep these levers in equilibrio. The force* BE, *force* I *and pressure at* A, *are respectively as radius*, *Cos*. ABD, S.ABD.

For then EAB is a right-angled triangle; and these forces as BE, AE, and AB; that is, as radius, S.ABE, and S.AE that is, as radius, Cos. ABD, and S.ABD.

PROP. XXIII. (*Fig.* 24. *Pl.* II.)

IF AB, BC BE TWO LEVERS MOVEABLE ABOUT THE CENTRES A A C; IF THE CIRCLES KBM AND DBE BE DESCRIBED WITH RADII BC, BA; AND UPON THE CIRCLE BM AS A BASE, W THE GENERATING CIRCLE DBE, THE EPICYCLOID BNE BE SCRIBED; AND THE LEVER CB AND EPICYCLOID BNE BE JO ED TOGETHER IN THAT VERY POSITION, SO AS TO MAKE BUT CONTINUED LEVER CBNE. AND IF THESE LEVERS CB AND AB MOVE ABOUT THE CENTRES C, A; SO THAT THE EN OF THE LEVER AD BE ALWAYS IN THE CURVE OF THE EPI CLOID DK; I SAY, THAT TWO EQUAL AND CONTRARY FORCES D AND K, ACTING PERPENDICULAR TO THE RADII DA, I WILL ALWAYS KEEP THESE LEVERS IN EQUILIBRIO.

For let the levers AB, CBNE come into the position

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Cor. 3. (Fig. 25. Pl. II.) *After a like manner if* BE *be an epicycloid described within the circle* BRM, *by the generating circle* BD; *and the lever* CBE *be compounded of the right line* CB *and the epicycloid* BE; *then the levers* CBE *and* AB, *by equal forces acting at* B, *will keep one another in equilibrio in any position, as* CKF *and* AD.

For when AB is come to AD, and CBE to CKDF; then the arch BK = arch BD. Whence the weight or forces acting at the distances CB, AB, have equal velocities; and, therefore, will sustain one another.

Cor. 4. (Fig. 26. Pl. II.) *If* BC *be infinite, or (which is the same thing) if* BK *be a right line perpendicular to* AB; *then* BE *or* KDF *will be the common cycloid. Therefore, whilst the point* D *moves uniformly about the centre* A, *the point* K *will move uniformly along the right line* BK, *and with equal velocities and forces: the point* D *in the mean time acting upon the cycloidal tooth* KD. *And any equal opposite forces will sustain one another.*

In like manner, (Fig. 27. Pl. II.) *if* BA *be infinite, or* BD *a right line perpendicular to* BC; *then* BE *or* DK *will be an epicycloid generated by the tangent* DB *revolving on the circle* BK. *And the velocities of* K *in the right line* BK, *and of* D *in the right line* BD, *will be always equal: and equal forces will be sustained at* B, *in all positions of the lever* CKD.

Cor. 5. (Fig. 1. Pl. III.) *If the figure of the tooth ef, at the end of the lever* AB, *be given; and the epicycloid* BE *be described as before. And if the levers* AD, CB, *be made to revolve about their centres* A, C; *so that the point* B *always move in the epicycloid* BE *or* KD. *And if the tract of the extreme points of the tooth be marked out upon the plane of the cycloidal tooth, as fgggge or fnnnne. And if the part fgDdKf be cut away, if the tooth be to act on the concave side of the epicycloid; or fne* DdKf, *if on the convex side. Then if the levers revolve so that the tooth move along the curve gg or nn; the points* D *and* K *of the levers* AB, CD, *will move with equal velocities, in the arches* BD, BK, *as before.* For the fixed point B in the tooth will still describe the epicycloid.

Cor. 6. (Fig. 2. Pl. III.) *If the two epicycloids* BE, BO, *be described upon* BM, BL, *with the generating circles* BD, BK; *and the levers* AB, CB, *revolve about the centres* A, C; *so that the point* B *or* D *of the lever* AB *move along the epicycloid* BE *or* KDS. *Then the point* B *or* K, *of the lever* CB, *will at the same time move along the epicycloid* BO *or* DKT; *and the points* D, K, *will describe the equal arches* BD, BK. *And, therefore, it is the same thing on which lever the cycloidal tooth be placed, or whether on one or both.*

For the epicycloid DKT generated on DB, will pass through K, if BD = BK. Also the epicycloid KDS, generated upon KB, will pass through D, when BK = BD.

Scholium.—(*Figures* 24 *and* 25. *Pl.* II.) The levers AB, CB, are supposed only to act upon one another, below the line AC; for was the action supposed to be continued above the line AC, the point B would no longer act on the same, but on a different epicycloid; and the equality of motion would hold no longer.

PROP. XXIV. (*Fig.* 3. *Pl.* III.)

In the wheel and axle, if the power P be to the weight W, as the diameter of the axle EF where the weight acts, to the diameter of the wheel AB, where the power acts; then the power and weight will be in equilibrio, and the contrary.

For let AB be the wheel, CD the axle; and suppose the wheel and axle to turn once round; then it is plain the power P will have descended a space equal to the circumference of the wheel; and the weight W will have risen a height equal to the circumference of the axis. Therefore, velocity of P, to velocity of W :: as circumference of the wheel, to circumference of the axis :: or as diameter of the wheel to diameter of the axis :: that is (by supposition) as W to P. Therefore the motions of P and W are equal; and have equal forces to move each other; and therefore (by Ax. 9.) will remain in equilibrio.

This Prop. will appear otherwise. For the wheel and axle may be reduced to a lever of the first kind: for the fulcrum will be in the middle of the axis CD. Therefore, drawing lines from the middle of the axis to the power and weight, parallel to the horizon; and the radius of the wheel will be the distance of the power, and the radius of the axle the distance of the weight. And, as their radii are reciprocally as the weight and power, therefore (by Prop. XIX.) they will be in equilibrio. And thus, the wheel and axle is no more but a perpetual lever.

Cor. 1. *If the rope have any sensible thickness; then if the*

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Cor. 4. *And it is the same thing, if instead of a wheel there be only spokes fixed in the axis, whose length is equal to the radius of the wheel: and any other equal force be applied for a power, instead of the weight* P.

Cor. 5. *The force of the weight is increased when one or more spires of the rope is folded about the axle.* For that, in effect, augments the diameter of the axle.

Cor. 6. *It matters not how low the weight hangs. For whilst the axle remains the same, the resistance of the weight remains the same, setting aside the weight of the rope.*

PROP. XXV. (*Fig.* 6. *Pl.* III.)

LET NBD, MBK BE TWO TOOTHED WHEELS IN THE SAME PLANE, AND IF THE TEETH OF THE WHEEL BM BE THE EPICYCLOIDS Be, *kd*, KD, DESCRIBED ON THE BASE KBM, WITH THE GENERATING CIRCLE BN, AND THESE TEETH ALL EQUIDISTANT; AND IF B, *d*, D, THE ENDS OF THE TEETH OF THE WHEEL NBD BE ALSO EQUIDISTANT, AND THESE DISTANCES B*d*, *d*D EQUAL TO B*k*, *k*K. THEN, I SAY, THE POINTS OF THE TEETH B. *d*, D, WILL ALL ACT TOGETHER, ON THE CYCLOIDAL TEETH BE, *kd*, KD, AS THE WHEELS TURN ROUND. AND ANY POINTS D, K, WILL MOVE THROUGH EQUAL ARCHES BD, BK IN EQUAL TIMES.

Draw the radii AD, A*d*, CK, and C*k*; then AD and CKD may be considered as two levers moving about A, C, and acting on one another in D: and the same of A*d*, C*kd*, acting at *d*. But by the motion of the wheels BD, BK, suppose D always to be in the epicycloid KD; then (Cor. 1. Prop. XXIII.) will BD = BK, and since D*d* = K*k*, therefore B*d* will be = B*k*, and consequently (by Cor. 1, Prop. XXIII.) the point *d* will be in the epicycloid *kd*. And thus, if there be never so many teeth B, *d*, D, &c. they will always be in the curves of the epicycloids BE, *kd*, KD, &c. Therefore the working teeth either act all at once upon one another, or they act not at all. And as the velocities of any points are equal in the two wheels BD, BK, when only one tooth acts upon one, they will still be equal, if never so many act together.

Cor. 1. *Hence equal weights or forces applied to the circumferences of these two wheels, as at* B, *and acting one against the other, will keep these wheels in equilibrio. Likewise, it is the same thing whether the wheel* AB *drive the wheel* CB, *its teeth acting upon the concave side of the cycloids; or the wheel* CB *drive* AB, *the convex side of the cycloid acting against the teeth of* AB.

Cor. 2. (Fig. 7. Pl. III.) *Hence, instead of the points* B, *or the infinitely small teeth of the wheel* ABD; *if any sort of a tooth rs be*

placed at B; *and if the wheels be made to move about so that the gi point* B *may describe the epicycloid* BE *or* KD, *whilst the track the extreme points of the tooth is marked out as* KeD, K*f*D; *and space* KeD*f be cut away; and the same be done for all the other tee being equidistant and of the same form and bigness. Then if one these wheels is supposed to drive the other, by these teeth running the spaces* D*f*Ke; *I say, the circumferences of these wheels will m with equal velocities, and all the working teeth will act togeth* This is evident, because the points B, D, will, by this motion, scribe the epicycloids as before.

Cor. 3. (Fig. 8. Pl. III.) *If the epicycloid* BV *be described on base* KBH, *with the generating circle* BD; *and a portion of the epi cloid be placed at equal distances* B, L, K, *for teeth; then the teeth the wheel* A *acting against the cycloidal teeth, will make the mot equal in the two wheels. Where we may take as great a portion the cycloid as we will; and the sides* BO, LI, *which act not, may be any figure, not to hinder the motion of the teeth of* A. *And it is same thing what part of the tooth* LO, *the tooth* G *acts against.*

Cor. 4. *But the teeth ought not to act upon one another bef they arrive at the line* AC, *which joins their centres. And thou the side* BO *of the tooth may be of any form; yet it is better make them both sides alike, which will serve to make the wheels tu backwards. Also a part, as pqr, may be cut away on the back every tooth, to make way for those of* A. *And the more teeth th work together, the better; at least one tooth should always begin fore the other hath done working: the teeth ought to be disposed such manner as not to trouble or hinder one another, before they gin to work; and that there be a convenient length, depth, a thickness given them, that they may more easily disengage themselv as well as for strength.*

PROP. XXVI. (*Fig*. 1. *Pl*. IV.)

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For the cords supply the place of teeth.

Cor. 2. (Fig. 1. Pl. 4.) *In any combination of wheels with teeth, if the power* P *be to the weight* W; *as the diameter of the axle* F *where the weight acts, multiplied into the product of the teeth in each pinion or spindle, is to the diameter of the wheel* A, *where the power acts, multiplied by the product of the teeth in each of the wheels (that the pinions act against); the weight and power will be in equilibrio.*

For the number of teeth in each wheel and pinion that act against one another are as the circumferences or as the diameters of that wheel and pinion.

Cor. 3. *And hence also, if the power be to the weight, in a ratio compounded of the diameter of the axle* F, *where the weight acts to the diameter of the wheel A, where the power acts, and the ratio of the number of teeth in the first axle,* (B), *reckoning from the power; to the number of teeth in the second wheel* (C), *and of the number of teeth in the second axle* (D), *to the number in the third wheel* (E); *and so on till the last; then they will be in equilibrio.*

Cor. 4. *In a combination of wheels, the number of revolutions of the wheel* F *where the weight acts, to the number of revolutions of the wheel* A *where the power acts, in the same time; is as the product of the teeth in the pinions, to the product of the teeth in the wheels which act in them; or as the product of the diameters of the pinions, to the product of the diameters of the wheels.*

Scholium.—Wheels with oblique teeth come under the same rules; but as they are related to the screw, we refer you thither for a farther account thereof.

In wheels whose teeth work together, they should not encounter before they come to the line joining their centres; because the rubbing is greater on that side; but being past the line, the teeth slip easily along one another, in making their escape, so that the friction is very inconsiderable.

PROP. XXVII. (*Figures* 4 *and* 5. *Pl.* IV.)

If a power sustain a weight by means of a rope going over a fixed pulley; then the power is equal to the weight. But if the pulley be moveable together with the weight, and the other end of the rope fixed; then the power will be but half the weight.

For suppose a horizontal line AB drawn through the centre of the pulley C; then that line will represent a lever, and (in *fig.* 4.) where the pulley is fixed, the centre C being kept immoveable, represents the fulcrum; whilst the weight acts at B, and the power at A. And because BC $=$ CA, therefore (by Prop. XIX.) the power P is equal to the weight W.

And (in *fig.* 5.) the fixed point B is the fulcrum, and t weight acts at C, and the power at A; and since BC is half A therefore (by Prop. XIX.) the power at P is half the weight W.

Cor. (Fig. 6. Pl. IV.) *Hence all fixed pullies are equivalent levers of the first kind. And they add no new force to the power,* l *only serve to change the direction, and facilitate the motion of* t *rope: but a moveable pulley doubles the force. And if a rope go* o *several pullies,* A, B, C, *whose blocks are all fixed; the power neither increased nor diminished.*

PROP. XXVIII. (*Fig.* 7. *Pl.* IV.)

IN A COMBINATION OF PULLIES ALL DRAWN BY ONE RUNNI ROPE; IF THE POWER P BE TO THE WEIGHT W; AS 1 TO T NUMBER OF PARTS OF THE ROPE AT THE MOVEABLE BLOCK THEY WILL BE IN EQUILIBRIO.

For (by Ax. 18.) all the parts of the rope *m, o, n, r, s, t, v,* t equally stretched; and the weight W is sustained by the numl of ropes that act against the moveable block; and the rope *v* the power P acts with the force of one or unit. Therefore t power is to the weight, as 1 to the number of ropes pulling at t moveable block A.

Cor. Hence the power is to the force by which the immovea block B *is drawn; as* 1 *to the number of ropes acting against* t *immoveable block.*

PROP. XXIX. (*Fig.* 2. *Pl.* IV.)

IN THE SCREW, IF THE POWER P BE TO THE WEIGHT W, AS T HEIGHT OF ONE THREAD (RECKONED ACCORDING TO THE LENG OF THE SCREW) TO THE CIRCUMFERENCE DESCRIBED BY O REVOLUTION OF THE POWER; THEN THEY WILL BE IN EQUI

For, in one revolution of P, the wheel DC with the weight W, has moved the distance of one tooth.

Cor. 2. *And universally, if there be several worms or spiral leaves, upon the axis* AB, *and the weight* G *hangs upon the axle* EF. *Then if the power* P, *is to the weight* G :: *as the radius of the axle* EF × *number of worms in* AB, *to* AP × *number of teeth in* CD. *Then the power and weight are in equilibrio.* For by *Cor.* 3. Prop. XXVI., *if* n *be the number of worms, then* P : G :: n × ½ EF : AP × *teeth in* CD.

Cor. 3. (Fig. 1. Pl. V.) *And by reason of the obliquity of the teeth, the force acting perpendicular to the teeth, the lateral force perpendicular to the wheel, and the direct force in the plane of the wheel; will be respectively, as radius, the sine, and cosine of the obliquity of the teeth.*

For let GD be the side of a tooth acted on: GE parallel to the axis of the wheel, and DE perpendicular to it, or in the plane of the wheel. Now, if GD represent the force acting perpendicular to the tooth. Then DE, GE, will be the forces acting in the directions GE, DE, (by *Cor.* 1. Prop. VIII.) but if GD be radius, DE is the sine of the obliquity, and GE the cosine.

Cor. 4. *In the common screw the less the distances of the threads are, and the longer the handle is, the easier any given weight is moved.*

Cor. 5. *What is here demonstrated, will hold equally true, if the wheel* CD *act upon another wheel with oblique teeth, instead of the worm* AB.

Scholium.—The force of the screw resembles the force that drives a body up an inclined plane; the force acting parallel to the base of the plane.

All things here laid down relating to the perpetual screw, do suppose that the axis of the worm spindle lies in the plane of the wheel it works in, and that their axles are perpendicular to each other; but if they are in oblique position, and the teeth of one or both also oblique, they cannot work without loss of power; a part being lost proportional to the obliquity.

If any worm spindle contains one leaf or worm, then a spindle of twice the diameter will require two worms, and one of thrice the diameter, three worms, &c., to work in the same wheel; and the power is best estimated by the rise or fall of a tooth of CD (*fig.* 1.) for a revolution of the power P.

PROP. XXX. (*Fig.* 3. *Pl.* V.)

Let EFG be the back or base of a wedge in form of an isoceles triangle; then if the power acting perpendicular to the back FG, is to the force or resistance acting

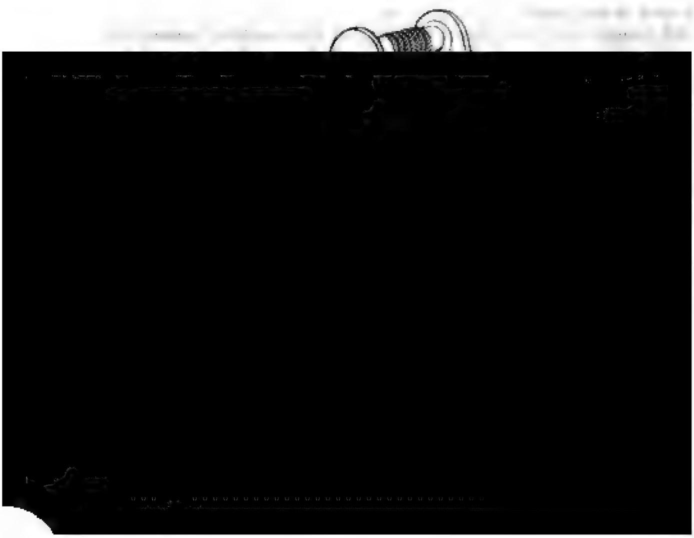
AGAINST EITHER SIDE, IN A DIRECTION PERPENDICULAR THAT SIDE; AS THE BACK OF THE WEDGE FG, TO EITHER OF T SIDES EF, EG: THEN THE WEDGE IS IN EQUILIBRIO, C WHICH IS THE SAME THING, THE POWER IS TO THE WHOLE R SISTANCE AGAINST BOTH SIDES, AS THE BACK FG, to THE SU OF THE SIDES EF, EG.

For draw the axis ED perpendicular to the base FG; and C. CB, perpendicular to the sides EF, EG; then DC is the directi of the power. And (by Prop. IX.) the impediment to be r moved, acts against the wedge in the directions AC, BC; an therefore, (by *Cor.* 1. Prop. VIII.) the power, and the actions the impediment, are as FG, FE, EG respectively, when they a in equilibrio.

Cor. 1. *The power acting perpendicular to the base, is to the fo acting against either side, in a direction parallel to the base* FG, *perpendicular to the axis* DE; *as the base* FG, *to the height* EI *when the wedge is in equilibrio, or the power is to the whole for against both sides (in direction parallel to* FG) *as the back* FG, *twice the height* DE.

For the force EG may be divided into the two ED, DG, (l *Cor.* 3. Prop. VII.) Then since (by this Prop.) EG is the for acting in direction CB; ED will be the force acting in directi DG.

Cor. 2. *The sharper the wedge, or the more acute its angle, t easier it will divide any thing or overcome any resistance.*



SECTION FOURTH.

THE DESCENT OF BODIES UPON INCLINED PLANES, AND IN CURVE SURFACES. ALSO, THE MOTION OF PENDULUMS.

PROP. XXXI. (*Fig.* 4. *Pl.* V.)

IF A HEAVY BODY W, BE SUSTAINED UPON AN INCLINED PLANE AC, BY A POWER ACTING IN A DIRECTION PARALLEL TO THAT PLANE; THEN

THE WEIGHT OF THE BODY,	THE LENGTH AC,
THE POWER THAT SUSTAINS IT,	THE HEIGHT CB,
AND ITS PRESSURE AGAINST THE PLANE,	AND THE BASE AB,
ARE, RESPECTIVELY, AS	OF THE PLANE.

Draw BD perpendicular to AC; then as the force of grav tends perpendicular to the horizon, or parallel to CB; and the rection of the power is parallel to DC; and the pressure agai the plane is (by Prop. IX.) parallel to DB; and, therefore, th quantities are respectively as the three lines CB, CD, BD, (Prop. VIII.) that is, by similar triangles, as AC, CB, and AB.

Cor. 1. *The weight, power, and pressure on the plane, are resp tively, as radius, the sine, and cosine of the plane's elevation.*

For the sides of a triangle are as the sines of the oppo angles.

Cor. 2. *The relative weight of a body, to make it run down an clined plane, is as the height directly, and length reciprocally, tha* $\frac{BC}{AC}$ *; or it is as the sine of the plane's elevation.*

Cor. 3. (Fig. 5. Pl. V.) *If a cylinder be sustained upon an incli plane, by a power drawing one end of a rope parallel to the pl whilst the other end is fixed; this power is to the weight of the linder, as half the height to the length of the plane.*

For half the relative weight of the cylinder is sustained by other end of the rope, which is fixed.

Scholium.—(*Fig.* 4. *Pl.* V.) If it be required to find the p tion of the plane AC, whose height BC is given, so that the gi weight W may be raised through the length of the plane AC, in least time possible, by any given power P, acting in the direct DC; make $AC = \frac{2W}{P} \times BC$, and you have your desire.

PROP. XXXII. (*Fig.* 4. *Pl.* V.)

are respectively, as radius, the sine, and cosine of the plane's elevation.

PROP. XXXIII. (*Figure* 6. *and* 7. *Pl.* V.)

IF A HEAVY BODY W BE SUSTAINED UPON AN INCLINED PLANE AC, BY A POWER P ACTING IN ANY GIVEN DIRECTION WP; AND IF BED BE LET FALL PERPENDICULAR ON WP; THEN

THE POWER P, } DB,
THE WEIGHT OF THE BODY W, } AB,
THE PRESSURE OF THE PLANE, } AD.
WILL BE RESPECTIVELY, AS

For, since BD is perpendicular to the direction of the power, AB to the direction of gravity, and AD to the direction of the pressure on the plane; therefore (by Cor. 1. Prop. VIII.) these forces will be respectively as BD, AB, AD, when they are in equilibrio.

Cor. 1. *The power, weight, and pressure against the plane, are respectively as the sine of the plane's elevation, cosine of the angle of traction* CWP, *and the cosine of the angle of direction of the power above the horizon.*

The angle of traction is the angle that the direction of the power makes with the plane. And in the triangle ABD, the sides are as the sines of the opposite angles, where $\angle$ D = complement of DWP.

Cor. 2. *Hence, whether the line of direction of the power be elevated above or depressed below the plane; if the angles of traction be equal, equal powers will sustain the weight; but the pressure is greater when the line of direction runs below the plane.*

Cor. 3. *The power* P *is least when the line of direction is parallel to the plane; and infinite, when perpendicular to it; and equal to the weight, when perpendicular to the horizon.*

Cor. 4. (Fig. 8. Pl. V.) *If a weight upon an inclined plane be in equilibrio with another hanging freely, their perpendicular velocities will be reciprocally as their quantities of matter.*

For, let the weight at W be made to descend to A, and draw Wr perpendicular to AE, and Wt, Dv to AB; then the weight P will have ascended a height = Ar, which is its perpendicular ascent; and Wt is the perpendicular descent of W. The figures Arwt and AEDv are similar, as are also the triangles AEB, DvB; whence Wt : Ar :: Dv : AE :: DB : AB = (by this Prop.) P : W.

Cor. 5. *And therefore if any two bodies be in equilibrio upon two inclined planes, their perpendicular velocities will be reciprocally as their quantities of matter.*

PROP. XXXIV. (*Fig.* 9. *Pl.* V.)

THE SPACE WHICH A BODY (DESCENDING FROM REST) DESCRI UPON AN INCLINED PLANE, IS TO THE SPACE WHICH A BO FALLING PERPENDICULARLY, DESCRIBES IN THE SAME TI AS THE HEIGHT OF THE PLANE CB, TO ITS LENGTH AC.

The force wherewith a body endeavours to descend upon inclined plane, is equal to the power that sustains it; and (Prop. XXXI) that power is to the weight of the body as CB CA; therefore the body is urged upon the plane, by an u formly accelerating force, which is to the force of gravity as (to CA. But (by Prop. V.) the motion generated in the same tin and in the same body, is as the force, that is (since the body given) the velocity is as the force. And (by Prop. III.) the spa uniformly described with the last velocities will be as these ve cities; and (by Cor. 1. Prop. VI.) these spaces are double t spaces described by the accelerating forces; therefore the spa described on the plane and in the perpendicular, are as the l velocities, or as the forces, that is, as CB to CA.

Cor. 1. *Hence, if* BD *be let fall perpendicular to* AC, *then in t time a body falls through the height* CB, *another body, descendi along the inclined plane, will run through the space* CD.

For these spaces are as CA to CB, that is, as CB to CD, by milar triangles.

Cor. 2. *The velocity acquired upon an inclined plane, is to velocity acquired in the same time by falling perpendicularly, as* CB CA, *or as* CD *to* CB.

Cor. 3. *The space described by a body moving down any plane a given time, is as the sine of the plane's elevation.*

For if CB be given, CD is as the sine of CBD or CAB.

Cor. 4. *The spaces described by a body descending on any gi*

For it will be uniformly retarded in ascending; and, in all points, will have the same velocity in ascending as descending.

Scholium.—Since the force by which bodies descend down an inclined plane, is a uniformly accelerative force, therefore whatever is demonstrated of falling bodies in Sect. II. holds equally true, in regard to the motion of bodies upon an inclined plane; substituting the relative weight upon the plane, instead of the absolute weight of the body.

Hence, therefore, a body projected on an inclined plane, will describe a parabola. And if the velocity of projection upon the plane, be to the velocity of a projectile in the air, as the relative gravity on the plane, to the absolute gravity, and both projected at the same obliquity, the same parabola will be described in both cases.

PROP. XXXVI. (*Fig.* 9. *Pl.* V.)

A body acquires the same velocity in descending down an inclined plane CD, as by falling perpendicularly through the height of that plane CE.

For draw DB perpendicular to CD, and the bodies will descend through CD, or CB in the same time; then (by Cor. 2. Prop. XXXIV.) the velocity in D : the velocity in B :: CE CD, and (Cor. 1. Prop. XIV.) the velocity in B : the velocity in E :: $\sqrt{CB} : \sqrt{CE}$ :: CD : CE. Therefore the velocity in D : the velocity in E :: CE : CE, and, therefore, the velocities in D and E are equal.

Cor. 1. *A body acquires the same velocity in falling from any height, whether it falls perpendicularly, or down an inclined plane of equal height.*

Cor. 2. *Hence the velocities acquired by heavy bodies falling from the same height, to the same horizontal right line, on any planes whatever, are equal among themselves.*

Cor. 3. *If the velocities be equal at any two equal altitudes* D, E; *they will be equal at any other two equal altitudes* A, B: *and acquire equal increases of velocity, in passing through* EB, *and* DA *of equal perpendicular heights.*

Cor. 4. *The velocities acquired by descending down any planes whatever, are as the square roots of the heights.*

PROP. XXXVII. (*Fig.* 12. *Pl.* V.)

In a circle whose diameter CB is perpendicular to the horizon, a body will descend through any chord CD or DB, in the same time as it will descend perpendicularly through the diameter CB.

For the angle at D is right; therefore, (by Cor. 1. Pr XXXIV.) the time of descending through CD will be equal to time of descending perpendicularly through CB. Draw CE rallel to DB, then will CE be equal to DB; and a body will scend through the chords CE, DB in the same time. But time of descending through CE is the same as falling through diameter; therefore the time of descending through any ch CD, DB, is the same as falling through the diameter CB.

Cor. 1. *The times of descending through all the chords of a cir drawn from either point* C *or* B, *are equal among themselves.*

Cor. 2. *The velocity acquired by descending through any chord* C *or* DB, *is as the length of the chord.*

For draw DF perpendicular to CB, then $CD = \sqrt{CB \times C}$ and $DB = \sqrt{CB \times BF}$; and (by Prop. XXXVI.) a body quires the same velocity in descending through CD, as in fall through CF, but this (by Cor. 1. Prop. XIV.) is as $\sqrt{CF}$, tha as CD. Also a body acquires the same velocity through DB FB, and that is as $\sqrt{BF}$, or as DB.

Cor. 3. *But a body will descend sooner through the small arc a circle, than through its chord* TB.

For if BG, TG be two tangents, then the relative gravity T in the arch and chord, will (by Cor. 1. Prop. XXXI.) be as sines of the angles TGO, TBO, or as BT and TG, or BG, t is, nearly as two to one when the arch is very small. And accelerative force in the circle being double to that in chord, therefore the velocity will be greater in the arch, and time of description shorter, though their lengths are nearly same.

PROP. XXXVIII. (*Fig.* 10. *Pl.* V.)

[redacted]

pondent points as I and D, then (by Cor. 3. Prop. XXXVI.) they will be equal in K and E, after the descent through IK; and being equal in K and E, they will also be equal in G and F, after the descent through KG, and so on. Therefore, since the motion begins in A, they will acquire equal velocities in descending through the first plane, and, likewise, through the 2d, 3d, 4th, &c. And, therefore, the velocities will be equal in all correspondent points I and D, K and E, G and F, &c., and at B and C.

Cor. 1. *Therefore, if a body be suspended by a string, and, by oscillating describes any curve* AB; *or, if it is any way forced to move in any polished and perfectly smooth surface* AB, *whilst another body ascends or descends in a right line; then, if their velocities be equal at any one equal altitude, they will be equal at all other equal altitudes.*

For the same thing is effected by the string of the pendulous body, as by the smooth surface of a polished body.

Cor. 2. *Hence a body oscillating in any curve line whatever, acquires the same velocity in the curve, as if it had fallen perpendicularly from the same height. And, therefore, the velocity in any point of the curve, is as the square root of the height descended.*

Cor. 3. *And a body, after its descent through any curve, will ascend to the same height in a similar and equal curve, or even in any curve whatever. And the velocities will be equal at all equal altitudes. And the ascent and descent will be in the same time, if the curves are the same.*

For the forces that generated the motion in descending, will equally destroy it in ascending, and therefore they will lose equal velocities by ascending equal heights. And if the curves are similar and equal, every particle of the curve will be described with the same velocity, and, therefore, in the same time, whether ascending or descending.

Cor. 4. *This Prop. is equally true, whether the curve* AKB *be in one plane perpendicular to the horizon, or in several planes* IK, KG, *&c., winding about in nature of a spiral.*

PROP. XXXIX. (*Fig.* 11. *Pl.* V.)

THE TIMES OF DESCENT THROUGH TWO SIMILAR PARTS OF SIMILAR CURVES, ARE IN THE SUBDUPLICATE RATIO OF THEIR LENGTHS, *ab*, AB.

Divide both curves into an equal number of infinitely small parts, similar to each other; and let *bc*, BC, be two of them, similarly posited; and draw *rb*, RB, perpendicular to *ah*, AH. By Prop. III. the space described is as the time and velocity,

and the time of describing any space is, as the space directly a velocity reciprocally. By Cor. 2. Prop. XXXVIII. the velo ties in b and B are as $\sqrt{rb}$ and $\sqrt{RB}$, that is because *arb* a ARB are similar, that is, as $\sqrt{ab}$ and $\sqrt{AB}$. Therefore, the ti of describing bc : to time of describing BC $::\ \frac{bc}{\sqrt{ab}} : \frac{BC}{\sqrt{AB}}$ $\frac{ab}{\sqrt{ab}} : \frac{AB}{\sqrt{AB}} :: \sqrt{ab} : \sqrt{AB} :: \sqrt{ad} : \sqrt{AD}$, because t curves are similarly divided. Whence, by composition, t whole time of describing ab : whole time of describing AB is in the same given ratio of $\sqrt{ab} : \sqrt{AB}$, or $\sqrt{ad} : \sqrt{AD}$.

Cor. 1. *Hence, if two pendulums describe similar arches, the ti of their vibrations are as the square roots of their lengths, or lengths as the squares of the times of vibration.*

For, let hd, Hd, be the lengths of the pendulums; then, becau the figures are similar, it is ad : AD :: hd : HD.

Cor. 2. *If a pendulum vibrates in a circle, the velocity in lowest point is as the chord of the arch it described in descending.*

For (by Cor. 2. Prop. XXXVIII.) it acquires the same ve city in the arch as in the chord; and (by Cor. 2. Prop. XXXVI the velocity in the chord is as the chord.

Cor. 3. *The lengths of pendulums vibrating in similar arches, reciprocally proportional to the squares of the number of their vib tions, in a given time.*

PROP. XL. (*Fig.* 13. *Pl.* V.)

IF A PENDULUM VIBRATES IN A CYCLOID, THE TIME OF ONE V BRATION IS TO THE TIME OF A BODY'S FALLING PERPENDIC

plane ED, since this is a motion uniformly accelerated, therefore, (by Cor. 1. Prop. VI.) it would, in the time of its fall, describe 2ED, with the velocity acquired in D. And since, (by Cor. Prop. III.) the times are as the spaces directly, and velocities reciprocally; and (by Cor. 2. Prop. XXXVIII) the velocities are as the square roots of the heights; therefore it will be, as time of describing ED : time in Cc :: $\frac{2ED}{\sqrt{MD}} : \frac{Cc}{\sqrt{MN}} :: \frac{2MD}{\sqrt{MD}}$ or $2\sqrt{MD}$ $\frac{Nn}{\sqrt{MN}}$ :: $2\sqrt{MD \times MN}$: Nn; by similar triangles.

Again, when the velocity is given, the time is as the space described. Therefore it will be, as time in Cc : time in Bb :: Cc : Bb or Gg :: CD : GD or $\sqrt{CD \times DE}$ :: $\sqrt{CD} : \sqrt{DE}$ :: $\sqrt{DN} : \sqrt{DM}$; by similar triangles. Therefore, ex equo, time in ED : time in Bb :: $2\sqrt{MD \times MN \times DN}$: $Nn\sqrt{DM}$ :: $2\sqrt{MN \times DN}$ or 2NL : Nn :: MD : Ll. Therefore, by composition it is, as time in ED : time in the arch Hb :: MD : arch Ml. And as the time in ED : whole time in HD :: MD : semicircumference MLD.

And since the time of descending through HD is equal to the time of descending through Dh: and (by Prop. XXXVII.) the time of descending through ED is equal to the time in the diameter FD. And 2FD is = DV, the length of the pendulum (being the radius of curvature in D); therefore, as the time of falling through half the length of the pendulum FD : time in HDh, or time of one vibration :: diameter MD : circumference 2MLD.

Cor. 1. *Hence all vibrations, great and small, are performed in equal times. And, in unequal arches the velocities generated, and the parts described, and those to be described, in the same time, will always be as the whole arches; and in any arch* HD, *the accelerative force at any point* B, *will be as the length* BD *from the bottom.*

For, the descent through HD is always the same, wherever the point H is taken. Also, (by the nature of the cycloid) the tangent at B is parallel to GD, and BD = 2 DG. And (by Cor. 2. Prop. XXXI.) the relative weight on GD (which is the accelerating force along GD, or the tangent at B) is as $\frac{ND}{GD}$, that is as $\frac{GD}{FD}$, or $\frac{2GD}{FD}$, or as $\frac{BD}{FD}$, or as BD, because FD is given. Whence, the accelerating force being always as the distance from the bottom, therefore, in any two arches, the velocities generated every moment, and the parts continually de-

scribed, will be as these forces; that is, as the whole arches. A
consequently, the spaces described, and the velocities generat
in any time, will be as the whole arches; and, therefore, the p
to be described will also be as the wholes.

Cor. 2. *The time of descent in* HB, *to the time of descent* HD, *is as the arch* ML, *to the semi-circumference* MLD.

Cor. 3. *The velocity of the pendulum in any point* B, *is* $\sqrt{DH^2-DB^2}$, *or* $\sqrt{HB \times BDh}$.

For (by Cor. 1. Prop. XIV.) the square of the velocity in B as MN, that is, as MD—ND, or $\frac{ED^2-GD^2}{DF}$ or $DH^2 - DB$ because DH $=$ 2DE, and DB $=$ 2GD, by the nature of the cloid; and DF is given.

Cor. 4 (Fig. 1. Pl. VI.) *If the length of the pendulum* VD *be m double the axis* FD; *and* ARV, *ar*V, *be two semi-cycloids equal* AHD, *and so placed, that the vertex (as* D) *be at* A *and* a. *T the pendulum* VH *vibrating between the cycloidal cheeks* ARV, *ar the point* H *will describe the cycloid* AHD*ha*; *and the time of its bration will be* 3.1416 × *time of falling through* FD, *half length of the pendulum.*

All this follows from the nature of the cycloid.

Cor. 5. *Hence, also, it appears from experiments on pendulu that, at the surface of the earth, a heavy body will descend throug space of* $16\frac{1}{12}$ *English feet nearly, in one second of time.*

For it is found, by observations upon clocks, that a pendulu 39, 13 inches long, vibrates once in a second; therefore $\frac{1}{3.14}$

$=$ time of a body's falling through FD or $\frac{39.13}{}$ inches. A

(by Cor. 3. Prop. V.) if the matter be given, the velocity generated in descending bodies is as the force and time; and (by Prop. VI.) the space descended is as the velocity and time, that is as the force and square of the time. Therefore, half the length of the pendulum is as the force, and square of the time of descending half its length; whence, the length is as the force and square of the time of vibration.

Cor. 7. *From the motion of pendulums it also follows, that, in any one place, the quantity of matter in any body is proportional to its weight.*

For it is certain, from experience, that pendulums of equal length, whatever quantities of matter they contain, vibrate in the same time. Therefore they will descend through half the length of the pendulum in the same time; and, consequently, would acquire equal velocities in the same time. Therefore (by Prop. V.) the velocity and time being given, the quantity of matter is as the force of gravity.

Cor. 8. *Hence it also follows, that there are vacuities or empty spaces in bodies.*

For since (by Cor. 7.) the quantity of matter is as the weight of the body, if it were true that there is an absolute *plenum*, all bodies of the same bulk must be of equal weight; which is contrary to all experience.

PROP. XLI. (*Fig.* 2. *Pl.* VI.)

If a pendulum AT oscillates in a circle TRQ, and, in the mean time, be acted on in the several points T, by a force tending perpendicular to the horizon, which is to the uniform force of gravity, as the arch TR, is to the sine TN; the times of all vibrations will be equal, whether great or less.

For, from any point T draw TZ perpendicular to the horizon, and TY a tangent to the circle in T; and let AT express the uniform force of gravity, TZ the variable force at T; draw ZY perpendicular to TY; then the force TZ will be resolved into the two TY, YZ. Of which YZ, acting in direction AT, does not at all change the motion of the body. But the force TY directly accelerates its motion in the circle TR. The triangles ATN, ZTY are similar, and TZ : TA :: TY : TN; but (by supposition) TZ : TA :: arch TR : TN; therefore TY = arch TR : that is, the force TY is as the arch to be described TR. Therefore, if AT, A*t*, be let fall together from the points T, *t*; the velocities generated in equal times, will be as the forces TY, *ty*; that is, as the arches TR, *t*R, to be described. But the parts described at the beginning of the motion, are as the velocities; that is, as the wholes to be described at the beginning;

and, therefore, the parts which remain to be described, the subsequent accelerations proportional to these parts, also as the wholes, &c. Therefore the velocities generated, the parts described with these velocities, and the parts to be- scribed, are always as the wholes. And, therefore, the parts to described, being every where as the velocities they are descril with, will be described in equal times and vanish together; is, the two bodies oscillating will arrive at the perpendicular together.

Cor. 1. *Hence that the vibrations in a circle may be isocronal; force* TZ *must be* $= \frac{TR}{TN} \times$ *gravity.*

Cor. 2. *Hence, if a pendulum vibrates by the force of grav only, the times of vibration, in very small different arches, will be v nearly equal.*

For, in small arches, the ratio of the arch to the cord is nea a ratio of equality.

Cor. 3. *But the time of vibration in larger arches, is greater ti the time in less arches of a circle.*

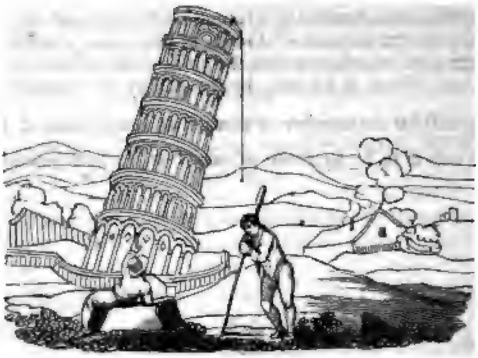
For the gravity at T being less than the isocronal force, body will be longer in describing that arch.

Cor. 4. *Hence, also, if a pendulum vibrates in the small arch o circle, the time of one vibration is to the time of a body's falli through twice the length of the pendulum, as half the circumferei of a circle to the diameter.*

For AR is the radius of curvature of a cycloid, whose axis is AR. Therefore the circle and cycloid coincide at R, and small arches of both will be described in the same time; that as expressed by Prop. XL., only here we take twice the length the pendulum and half the circumference which comes to t

And, if a pendulum vibrates seconds in an arch $2a$, and c be the cord of a, or of half the whole arch. Then $3\frac{1}{3} \times \overline{CC - cc}$ will be the seconds lost in twenty-four hours, by vibrating in the arch, the chord of whose half is C.

Also, if the bob of such a pendulum can be screwed up or down, and you put $n =$ number of threads of the screw contained in an inch, $y =$ time in minutes that the clock gains or loses in twenty-four hours. Then it follows, by the theory of pendulums, that $\frac{2}{37} ny$ will be the number of threads or revolutions of the nut, that the bob is to be let down or raised up, to beat seconds.



SECTION FIFTH.

OF THE CENTRE OF GRAVITY AND ITS PROPERTIES.

PROP. XLII. (*Fig.* 3. *Pl.* VI.)

[redacted]

the line, RC, moveable about R; then by Ax. 8. the body AB, together with the line RC, will endeavour to descend from the position RC towards D. Also, for the same reason, the body and the line CS will endeavour to descend from the position CS towards D; but as these two motions oppose each other, the body will be sustained by the points R, S, and therefore it will stand. And the same is true of every two opposite points R, S.

CASE II. (*Fig.* 5. *Pl.* VI.)

But if CD fall without the base, then the line RC and the body at C will endeavour to descend towards D; also the body C and line CS will endeavour to descend towards D likewise; and as this motion does not oppose the other, there will be nothing to support the body; therefore it must necessarily fall towards D.

Cor. 1. *Hence it follows, that if the centre of gravity of a body be supported, the whole body is supported. And the centre of gravity of the body must be esteemed the place of the body. And if it be sustained by any lever or beam, its place is at the point where the beam is cut by a line drawn from the centre of gravity perpendicular to the horizon.*

Cor. 2. *All the gravity of a body, or the force it endeavours to descend with, is collected into the centre of gravity; and therefore whatever sustains the centre of gravity, sustains the whole weight. And the descent of a body must be estimated by the descent of its centre of gravity.*

Cor. 3. *Hence, also, the larger the base is, upon which a body stands, and the further within it the centre of gravity lies, the firmer the body will stand, and the more difficult to be removed. On the contrary, the less the base, or the less the centre of gravity falls within it; so much the easier it is to be moved out of its place.*

Cor. 4. (Fig. 4. Pl. VI.) *If a body be laid upon a plane* GF, *and one end* F *gradually raised up, the body will slide down the plane, if the perpendicular* CD *fall within the base; but if it fall without, it will roll down.*

PROP. XLIII. (*Fig.* 6. *Pl.* VI.)

THE COMMON CENTRE OF GRAVITY C OF TWO BODIES A, B, IS IN THE RIGHT LINE JOINING THEIR CENTRES OF GRAVITY; AND THE DISTANCE OF EITHER BODY FROM THE COMMON CENTRE OF GRAVITY, IS RECIPROCALLY AS THE QUANTITY OF MATTER IN IT.

Let A, B be the centres of gravity of A and B, and suppose AB to be an inflexible right line, or lever; and C the fulcrum.

Then, if C be the centre of gravity of the bodies A, B, those bodies (by Def., 12.) will be in equilibrio. And consequently (by Cor. 4. Prop. XIX.) AC : CB :: B : A.

Cor. 1. *If there be never so many bodies, the common centre of gravity of them all, is in the right line drawn from the centre of gravity of any one, to the common centre of gravity of all the rest; and it divides this line into two parts, reciprocally as that body to the sum of all the rest of the bodies.*

For let D be another body, and let B and A be placed in C, then will C : D :: DE : CE. And so on for more bodies.

Cor. 2. (Fig. 7. Pl. VI.) *If several bodies* A, B, D, E, F, *be in equilibrio upon a straight lever* AF, *then the fulcrum* C *is at the common centre of gravity of all these bodies.*

PROP. XLIV. (*Fig.* 8. *Pl.* VI.)

IF THERE BE SEVERAL BODIES, A, B, D, E, F; AND IF ANY PLANE PQ BE DRAWN PERPENDICULAR TO THE HORIZON; THE SUM OF THE PRODUCTS OF EACH BODY MULTIPLIED BY ITS DISTANCE FROM THAT PLANE, IF THEY ARE ALL ON ONE SIDE; OR THEIR DIFFERENCE, IF ON CONTRARY SIDES; IS EQUAL TO THE SUM OF ALL THE BODIES MULTIPLIED BY THE DISTANCE OF THEIR COMMON CENTRE OF GRAVITY FROM THAT PLANE.

Draw lines perpendicular and parallel to the plane PQ, as in the figure, and let C be the centre of gravity. Then, (by Cor. 3. Prop. XIX.,) the force of all the bodies to move the plane PQ about R, will be $mD \times D + oE \times E + rF \times F - Ak \times A - Bl \times B$. That is, $\overline{dC+RC} \times D + \overline{eC+RC} \times E + \overline{RC-fC} \times F - \overline{aC-RC} \times A - \overline{bC-RC} \times B$, or $dC \times D + eC \times E - fC \times F - aC \times A - bC \times B + RC \times \overline{D+E+F+A+B}$. But because C is the centre of gravity of the bodies, therefore, (by Prop. XX.,) $dC \times D + eC \times E = fC \times F + aC \times A + bC \times B$; therefore we have $mD \times D + oE \times E + rF \times F - Ak \times A - Bl \times B = RC \times \overline{A+B+D+E+F}$.

Cor. 1. *This Prop. is equally true for any plane whatever.*

For suppose the plane and the bodies to be put into any oblique position, all the distances will remain the same as before.

Cor. 2. *If any plane be drawn through the common centre of gravity* C, *of any number of bodies* A, B, D, *&c. and each body be multiplied by the distance of its centre of gravity from that plane; the sum of the products on each side are equal:* $A \times aC + B \times bC + F \times fC = D \times dC + E \times eC$.

For the distance of a body must be estimated by the distance of its centre of gravity.

Cor. 3. *Hence, also, the sum (or difference) of the products of each particle of a body multiplied by its distance from any plane whatever, is equal to the whole body multiplied by the distance of its centre of gravity from that plane; and if the plane pass through the centre of gravity, the sums of the products on each side are equal.*

Cor. 4. *The sum of the forces of a system of bodies is the very same, as if all the bodies were collected into their common centre of gravity, and exerted their several forces there.*

For the sum of all the forces are $mD \times D + oE \times F$, &c., or $RC \times \overline{A + B + D + E + F}$.

Cor. 5. *And the same is true of any forces whatever, with regard to the centre of gravity of those forces; and, therefore, if several forces act in parallel directions, the sum of all these forces will be equivalent to one single force; and their common centre of gravity, the place where it acts.*

Cor. 6. (Fig. 4. Pl. VIII.) *If a circle be described about the centre of gravity* G, *of a system of bodies* A, B, C; *and any point* S *be taken at pleasure in the circumference; then* $SA^2 \times A + SB^2 \times B \times SC^2 \times C$, *is a given quantity; and the same holds true for the surface of a sphere, and the bodies not all in one plane.*

For, draw SG, on which let fall the perpendiculars Aa, Bb, Cc; then, (by Eucl. II. 12, 13.) $SA^2 \times A + SB^2 \times B + SC^2 \times C = \overline{SG^2 + GA^2 + 2SG \times Ga} \times A + \overline{SG^2 + GB^2 - 2SG \times Gb} \times B + \overline{SG^2 + GC^2 + 2SG \times Gc} \times C$. But, (by Cor 2.) $Ga \times A - Gb \times B + Gc \times C = 0$, and all the rest are given quantities.

PROP. XLV. (*Fig.* 9. *Pl.* VI.)

If there be several forces in one plane, acting against one another in the point C, whose quantities and directions are CA, CB, CD, CE, CF; and if they keep one another in equilibrio; I say, C is the common centre of gravity of all the points, A, B, C, D, E; and any one of them as EC being produced, will pass through the centre of gravity G of all the rest.

Since all the forces are in equilibrio, the sum of the forces acting against EC will destroy its effects, and act against it in the same line of direction.

Upon EC let fall the perpendiculars Aa, Bb, Dd, Ff; then any force AC is divided into two Aa, aC. Now, as the point C is in equilibrio, all the perpendicular forces Aa, Bb, on one side, are

equal to all those Dd, Ff, on the other, by Ax. 11; and if th body 1 be supposed to be suspended at A, B, D, E, F; then, sin Aa × 1 + Bb × 1 = Dd × 1 + Ff × 1; the centre of gr vity of the bodies A, B, C, D, (and, also, of all the bodies) is the line EC. Again, it follows from the equilibrium of the force that EC + aC = Cb + Cd + Cf, by Ax. 11; and, therefore, the body 1 be suspended at the points E, a, d, b, f; C is th centre of gravity, that is, C is the centre of gravity of E, A, D, F.

Cor. 1. *If* G *be the centre of gravity of* A, B, D, F; *then* EC CG × *number of points* A, B, D, F.

For EC = Cb + Cd + Cf — Ca = CG × $\overline{A + B + D +}$ or 4CG, by Prop. XLIV.

Cor. 2. *The sum of all the perpendiculars on one side,* Aa, Bb *sum* Dd, Ff, *on the other side of* EC; *and the sum of their d tances* CE, Ca, *on one side* = *sum* Cd, Cb, Cf, *on the other s of* C.

PROP. XLVI. (*Fig.* 10. *Pl.* VI.)

If a body be acted on by several forces A, B, C, D, E, the parallel directions Aa, Bb, &c. and kept in equi brio; and if any plane RN be drawn from any poi R; the sum of the forces on each side are equal, A + = B + C + E; and the sum of the products on t other side, Ra × A + Rd × D = Rb × B + Rc × C — × E; and the contrary; where any product lying t contrary way from R, must be taken negative.

For, suppose RN to be the plane, acted on by these force then (by Cor. 5. Prop. XLIV.) the effect of the forces A and

the body at p, q, &c., and the perpendiculars Aa, Bb, *&c., are drawn; then, I say,* 1. *the sum of the perpendicular forces on each side are equal,* $Aa + Dd = Bb + Cc + Ec$. 2. *The sums of the contrary forces in direction of the line* RN *are equal,* $pa + yb = rc + sd + te$. 3. *The sums of the rectangles on each side, from any point* R, *are equal,* $Rp \times Aa + Rs \times Dd = Rq \times Bb + Rr \times Cc + Rt \times Ee$. *But where the points lie the contrary way from* R, *the rectangles must be negative; and when all these are equal, the body is at rest.*

For, since it is the same thing whether any force A act, at A, at F, or at p, we will suppose it to act at p; then if the oblique force pA be divided into the two pa, aA; and the same for the rest; then the sum of all the forces pa must be equal to the sum of all the contrary forces cr, by Ax. 11. The rest follows from this Prop.

Cor. 2. *And if a body be kept in equilibrio by several forces acting at different points, and in different directions, either in the same plane, or in different planes; it will still be in equilibrio by the same forces, acting from any one point, and in directions respectively parallel to the former.*

For, in the same plane, the forces parallel and perpendicular to RN, will remain the same as before. And when the directions of any of the forces are out of this plane, all these extraneous forces may be reduced to others, one part acting in the plane, the other perpendicular to it; and both these remain the same in quantity as before. And since the forces acting in the plane, kept one another in equilibrio at first, they will do the same still. And as the parts perpendicular to this plane, also kept one another in equilibrio at first, they will do the same when applied to their common centre of gravity, or to any other point.

Cor. 3. *If several forces acting after any manner keep a body unmoved; and any plane whatever be drawn; and the vagrant forces be all reduced to that plane; then all the perpendicular forces on one side, are equal to those on the other; and their centres of gravity fall in the same point. When this does not happen in all planes, the body will be moved some way or other.*

PROP. XLVII. (*Fig.* 1. *Pl.* VII.)

To find the centre of gravity of a system of bodies, A,B,C.

Draw any plane ST, and from the centres of gravity of all the bodies, draw perpendiculars to this plane, Aa, Bb, Cc; then (by Cor. 3. Prop. XIX.) the forces of A, B, C, at the distances Aa, Bb, Cc, from the plane, will be $A \times Aa$, $B \times Bb$, $C \times Cc$. Let G be the centre of gravity, then the sum of the forces $A \times Aa +$

$B \times Bb + C \times Cc$ must be $= \overline{A + B + C} \times Gg$, the pow of all the bodies situated in G (by Prop. XLIV.) whence t distance of the centre of gravity from the plane, that is Gg $\frac{Aa \times A + Bb \times B + Cc \times C}{A + B + C}$: where, if any of the bod be situate on the other side of the plane, the correspondent re angles will be negative.

And if the distance be in like manner found from the pla TV, perpendicular to ST, the point G will be determined making the parallelogram TG with the respective distances fr those planes.

Cor. 1. *Let b be any body, p any particle in it, d its dista from a given plane; then the distance of its centre of gravity fr that plane is* $= \frac{\textit{Sum of all the } dp^{*}}{b}$.

Cor. 2. *To find the centre of gravity of an irregular plane gure. Suspend it by the string* AEB, *at* E; *and draw the plu line* ECF. *Then suspend it by another point of the string as and draw another plumb line through* E, *to intersect* CF; *and point of intersection is the centre of gravity.*

Cor. 3. *To find the centre of gravity of a flexible body, lay upon a board whose centre of gravity is known; lay the centre of g vity of the board upon the edge of a prism; and lay the body up it, and remove it back or forwards, till it be in equilibrio upon board.*

Scholium.—The centres of gravity of several planes and sol have been determined to be as follows:

1. If two lines be drawn from two angles of a *triangle*, to middle of the opposite sides, the point of intersection is the cen

7. In the *cone and pyramid*, the distance of the centre of gravity from the vertex is $\frac{3}{4}$ of the axis.

8. In a *paraboloid*, the distance of the centre of gravity from the vertex is $\frac{2}{3}$ of the axis.

9. For the *segment of a sphere*, let $r =$ radius, $x =$ height of the segment; then the distance of the centre of gravity from the vertex is $\frac{8r-3x}{12r-4x}x$.

PROP. XLVIII.

IF TWO OR MORE BODIES MOVE UNIFORMLY IN ANY GIVEN DIRECTIONS, THEIR COMMON CENTRE OF GRAVITY WILL EITHER BE AT REST, OR MOVE UNIFORMLY IN A RIGHT LINE.

CASE I.

Let one body stand still, and the other move directly to or from it in a right line; then, since the centre of gravity divides the distance, in a given ratio, and the distance increases uniformly, therefore that centre moves uniformly. Now, suppose the other body likewise to move in the same right line, and any quantity of space to move along with it; then, since the body is relatively at rest in this space, the centre of gravity, in regard to that space, moves uniformly; to which, adding or subtracting the uniform motion of that space, the centre of gravity will still move uniformly.

CASE II. (*Fig.* 4. *Pl.* VII.)

Let the bodies move in one plane, in the directions DE, AB; produce their lines of direction till they meet in D, and when one body is in D and E, let the other be in A and B respectively. Let H be their centre of gravity, when in D and A; and K, when in E and B; and draw HK, and make BP $=$ AD, and draw EP, and KL parallel to AB; then DE is to AB or DP, in the given ratio of the motion of the bodies; and since the $\angle$ EDP is given, therefore all the angles of the triangle EDP are given, and DP will be to PE in a given ratio. But by similar triangles PE is to PL in the given ratio of BE to BK, by the property of the centre of gravity, therefore DP is to PL in a given ratio. And all the angles in the triangle DPL are given, and therefore the angle PDL. Therefore the point L is always in the line DL given in position. And by the nature of the centre of gravity, DA : DH :: EB : EK :: PB or DA : LK. Therefore DH $=$ LK, whence DHKL is a parallelogram, and HK parallel to DL, and therefore the angle BHK is given; and the centre of gravity K is always in the right line HK given by position. And because all the angles of the triangles DPL, and DLE are given; therefore the lines DP, DE, DL, that is, AB, DE, HK are in a given ratio; and, consequently, the point K moves uniformly along the right line

HK. And the demonstration is in the same manner, if one of the bodies B moves from B towards A.

CASE III. (*Fig.* 5. *Pl.* VII.)

Let the paths of the bodies AB, DE, be in differen planes. Through the path AB draw a plane B*de* paral lel to the path DE, and through DE draw the plane D*de*E per pendicular to B*de*, produce AB to *d*, and let D*d*, E*e* be perpendi cular to *de*. Then the planes D*d*A, E*e*B, will be perpendicular t the plane *ed*B. Let one body be in A and B, when the other i in D and E respectively. Now, if the body at D were to mov in *de*, then, (by Case 2.,) the centre of gravity would move uni formly along some right line HK; through HK erect the plan HK*kh* perpendicular to HBK. Then, by similar triangles, an the nature of the centre of gravity, A*h* : *h*D :: (AH : HD :: BK Ke ::) B*k* : *k*E. And *hk* is the path of the centre of gravity o the bodies moving in AB, DE. Likewise, D*d* : H*h* :: A*d* : Al :: *b*E : BK :: *e*E or D*d* : K*k*; therefore, H*h* = K*k*, and *hk* i equal and parallel to H*k*, therefore the centre of gravity of th bodies (moving in AB, DE) moves uniformly through the rigl line *hk*.

CASE IV.

The common centre of gravity of two bodies, and a third either at rest, or moves uniformly in a right line; for these tw may be put into the place of their centre of gravity, which befo moved uniformly; and then the centre of gravity of the three wi move uniformly. Likewise the common centre of gravity of thr bodies and a fourth, will move uniformly in a right line, and s on.

PROP. XLIX.

upon each other, is at rest; and the actions of all the bodies being the sum of the actions of every two, it is evident the centre of gravity of all the bodies remains the same, as if they did not act at all upon one another; and, therefore, is at rest in this space, or moves uniformly forward along with it.

Cor. 1. *Hence if a body be projected into free space, if it have any circular motion, this motion will be performed uniformly about an axis passing through the centre of gravity.*

For if every particle of the body retained the distinct motion first impressed on it; the common centre of gravity of the whole would move in a right line, by the last Prop. And since the cohesion of the parts of the body retains the particles in one mass, therefore (by this Prop.) the motion of the centre of gravity is not altered, which it would be if the axis of circular motion did not pass through the centre of gravity, but through some other point.

Cor. 2. *And if a body be hurled into the air, its centre of gravity will either move in a right line, or describe a parabola; whilst that body revolves about an axis passing through the centre of gravity, if it have any circular motion.*

PROP. L. (*Fig.* 6. *Pl.* VII.)

THE SUM OF THE MOTIONS OF SEVERAL BODIES IN ANY GIVEN DIRECTION, IS THE SAME AS THE MOTION OF ALL THE BODIES IN THE SAME DIRECTION, MOVED WITH THE VELOCITY OF THEIR COMMON CENTRE OF GRAVITY.

Let the bodies, A, B, move round the centre of gravity C at rest, to the places a, b; draw BCA, bCa. Then, since A : B :: BC : AC :: bc : ac; therefore the triangles ACa, BCb, are similar, and $\angle$ bBc = $\angle$ CAa, therefore Bb is parallel to Aa, and the bodies move in contrary directions. Also, since Aa : Bb :: AC : CB :: B : A, or Aa × A = Bb × B. Therefore the motions of A, B, in contrary directions are equal, or their motion the same way is 0. Now let the space and bodies moving in it, be moved in any direction with any velocity v; it is manifest, the motion of each body in that direction will be greater than before, by the quantity of matter × velocity. Therefore, the sum of the motions is now vA + vB or $v \times \overline{A + B}$, that is, equal to the sum of the bodies × velocity of the centre of gravity.

After the same manner, the motion of three bodies is the same as the motion of two of them, moved with the velocity of their common centre of gravity, together with the motion of the third; that is (by what has been shown) equal to the sum of all the three, moved with the velocity of the centre of gravity of all the three and so for more bodies.

Cor. The centre of gravity of a body must be taken for the pla of the body. And the motion of any body, or of any system of bodi must be estimated by the motion of the centre of gravity.

PROP. LI.

IF TWO WEIGHTS ON ANY MACHINE KEEP ONE ANOTHER IN EQU LIBRIO, IF THEY BE ANY HOW RAISED OR MOVED BY HELP THE MACHINE, THE CENTRE OF GRAVITY OF THE WEIGHT A POWER WILL ALWAYS BE IN THE SAME HORIZONTAL RIG LINE.

For in the lever, the centre of gravity is at the fulcrum, an therefore, it neither ascends nor descends. In the wheel a axle, and in the pulley or any combination of pulleys, the weig and power approach or recede from each other, by spaces whi which are reciprocally as the bodies; and, therefore, their cent of gravity is at rest. And upon any inclined plane, the perpe dicular velocities of the power and weight (by Cor. 4. Pr XXXIII.) are reciprocally as their quantities, and the distance the centre of gravity from each, being in the same ratio, is a at rest. And, universally, in any combination of these, or a machine whatever, where the equilibrium continues, the asce and descent of the power and weight being reciprocally as th quantities, the centre of gravity neither ascends nor descends.

PROP. LII. (*Fig.* 8. *Pl.* VII.)

IF A HEAVY BODY AB BE SUSPENDED BY TWO ROPES AC, B A RIGHT LINE PERPENDICULAR TO THE HORIZON PASSI THROUGH THE INTERSECTION F, OF THE ROPES, WILL A PASS THROUGH THE CENTRE OF GRAVITY G, OF THE BODY.

[redacted]

Cor. 3. *If the centre of gravity fall not in the line* FG, *the body will not rest till it fall in that line.*

PROP. LIII. (Fig. 7. Pl. VII.)

IF ANY BODY WATEVER, AS BC, OR ANY BEAM LOADED WITH A WEIGT, BE SUPPORTED BY TWO PLANES AB, CD, AT C AND B; AND FROM THE POINTS C, B, THE LINES CF, BF, BE DRAWN PERPENDICULAR TO THESE PLANES; AND FROM THE INTERSECTION F, THE LINE FH BE DRAWN PERPENDICULAR TO THE HORIZON, IT WILL PASS THROUGH THE CENTRE OF GRAVITY G, OF THE BODY.

For since the body is sustained by the planes at B, C, and these planes re-act against the body in the perpendicular directions BF, CF; therefore it is the same thing as if the body was sustained by the two ropes BF, CF; and, consequently (by Prop. last), FH will pass through G, the centre of gravity of the whole weight.

Cor. 1. *If* EG *be drawn parallel to* CF, *then the whole weight, the pressure upon the planes* CD, AB, *are respectively as* FG, EG, EF; *and in these very directions; or, as the sines of the angles* BFC, BFG, *and* CFG.

Cor. 2. *If the line* FG *drawn (from the intersection of the perpendiculars* FC, FB) *perpendicular to the horizon, does not pass through the centre of gravity, the body will not be sustained, but will move till the centre of gravity fall in that line.*

Cor. 3. *Hence, if the position of one plane* C D *be given, and the position of the body* CB, *and its centre of gravity* G; *the position of the other plane* AB *may be found, by which the body will be supported, by drawing* CF *perpendicular to* CD, *and* GF *perpendicular to the horizon; and from* F *drawing* FB; *then* BA *perpendicular to it is the other plane.*

SCHOLIUM.—Some people have objected against the truth of the two last Propositions, as well as some others, though demonstrably proved. But this arises only from their own ignorance of the principles. They that have a mind may see this very Proposition demonstrated five or six different ways in Prop. XXIX. of the small Treatise of Mechanics, published in Vol. VII. of the Cyclomathesis.

PROP. LIV. (*Fig.* 10. *Pl.* VII.)

IF A HEAVY BODY AD, WHOSE CENTRE OF GRAVITY IS G, BE SUSTAINED BY THREE FORCES A, B, C, IN ONE PLANE, ACTING IN DIRECTIONS AH, BI, CD. AND IF FGP BE DRAWN PERPENDICULAR TO THE HORIZON, AND CD PRODUCED TO CUT IT IN P; AND IF AH, BI PRODUCED, INTERSECT IN O; THEN IF

OP BE DRAWN, AND IF EP AND OF BE DRAWN PARALLEL T AO AND PC; THEN I SAY THE WEIGHT OF THE BODY, TH THREE FORCES A, B, C, ARE, RESPECTIVELY, AS FP, EP, E(OF.

Because the line OP is unmoved, the point O is sustained b three forces in directions OP, OA, OB; which, therefore, are the lines OP, EP, OE. Also, the point P is sustained by thr forces in the directions PO, PC, GP; which, therefore, are as t lines OP, OF, FP: of which that in direction FP is the weig of the body, at G the centre of gravity. And the forces at and P, in the direction OP and PO, are equal and contrary.

Cor. Hence, if any other force instead of the weight act at G, *direction* GP; *then the forces at* P, A, B, C, *will be respectively* FP, EP, EO, OF.

Scholium.—If one of the forces be given, all the rest may found, if they act two and two at different points O, P. But, five forces act in one plane, two of them must be given.

PROP. LV. (*Fig.* 9. *Pl.* VII.)

If EBDF BE ANY PRISMATIC SOLID ERECTED UPON A PLANE AD AND IF IT BE CUT BY ANY PLANE AGH; I SAY, THE SURFAC OR SOLID GBDH, CUT OFF BY THIS PLANE, IS RESPECTIVEL EQUAL TO THE SURFACE OR SOLID EBDF, WHOSE ALTITUDE I CI, THE LINE PASSING THROUGH THE CENTRE OF GRAVITY O THE BASE, AND PARALLEL TO THE AXIS OF THE SOLID.

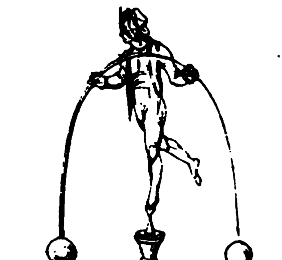
I shall not demonstrate this geometrically by measuring, bu mechanically by weighing them. Suppose the periphery, or th base, BD, to be divided into an infinite number of equal part by planes perpendicular to the horizon, and parallel to the axi of the solid, and to one another; and imagine AD to be a leve and let each particle be placed on AD where its plane cuts it

surface or solid whose base is the line or figure given, and height equal to the arch described by the centre of gravity.

(Fig. 1. Pl. VIII.) Let BD*db* be the figure generated. On the base BCD erect the surface or solid BDFE, and let C be the centre of gravity. Since the arches B*b*, C*c*, D*d*, are as the radii, AB, AC, AD, that is, as BG, CI, DH: therefore, if CI = C*c*, then will all the lines BG, CI, DH, &c. = all the arches B*b*, C*c*, D*d*, &c.; that is, the surface or solid BD*db* = BDHG, that is (by this Prop.) = BDFE.

Cor. 2. *Also, if a curve revolves about any right line drawn through its centre of gravity; the surfaces generated (either by a partial or total revolution) on opposite sides of the line, will be equal.*

For (by Cor. 2. Prop. XLIV.) each part of the curve multiplied by the distance of its centre of gravity from this line, must be equal on both sides. And, by Cor. 2., each surface generated, is equal to the curve multiplied by the arch described at that distance; and these arches (being similar) are as these distances. Whence, each surface is as the curve multiplied by the distance of its centre of gravity; and, therefore, they are equal.



SECTION SIXTH.

OF THE CENTRES OF PERCUSSION, OSCILLATION, AND GYRATION.

PROP. LVI. (*Fig.* 2. *Pl.* VIII.)

LET THERE BE ANY SYSTEM OF BODIES A, B, C, CONSIDERED WITHOUT WEIGHT, AND MOVEABLE ABOUT AN AXIS PASSING THROUGH S; AND IF ANY FORCE f CAN GENERATE THE ABSOLUTE MOTION m IN A GIVEN TIME; IF THE SAME FORCE ACT AT P, PERPENDICULAR TO PS; THE MOTION GENERATED IN THE SYSTEM, IN THE SAME TIME, REVOLVING ABOUT THE AXIS AT S, WILL BE $\frac{A \times SA + B \times SB + C \times SC}{A \times SA^2 + B \times SB^2 + C \times SC^2} \times SP \times m$.

For, suppose PS perpendicular to the axis at S, and to the line of direction PQ. And SA, SB, SC, perpendicular to the axis at S. And, suppose the force f divided into the parts p, q, r, acting separately at P, to move A, B, C. Then (by Cor. 3. Prop. XIX.) the bodies A, B, C, will be acted on respectively with the forces $\frac{SP}{SA}p$, $\frac{SP}{SB}q$, $\frac{SP}{SC}r$.

Since the angular motion of the whole system is the same, the velocities of A, B, C, are as SA, SB, SC; and their motions as A × SA, B × SB, C × SC; and these motions are as their generating forces $\frac{SP}{SA}p$, $\frac{SP}{SB}q$, $\frac{SP}{SC}r$. Whence p, q, r, are as $\frac{A \times SA^2}{SP}$, $\frac{B \times SB^2}{SP}$, $\frac{C \times SC^2}{SP}$: put the sum of these $= s$, and since $p + q + r = f$. Therefore, $s : f :: \frac{A \times SA^2}{SP} : \frac{f \times A \times SA^2}{s \times SP} = p$. And $\frac{SP}{SA}p = \frac{f \times A \times SA}{s}$ = force acting at A. Then $f : m :: \frac{f \times A \times SA}{s} : \frac{m \times A \times SA}{s} =$ motion of A. After the same manner $\frac{m \times B \times SB}{s}$, $\frac{m \times C \times SC}{s}$, are the motions of B and C. Therefore, the whole motion generated in the system is, $\frac{A \times SA + B \times SB + C \times SC}{S} \times m$.

Cor. 1. *If you make* $SO = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{A \times SA + B \times SB + C \times SC}$, *then if all the bodies be placed in* O, *the motion generated in the system will be the same as before, as to the quantity of motion, or the sum of all the absolute motions; but the angular velocity will be different.*

For the motion generated in these two cases, will be $\frac{\overline{A \times SA + B \times SB + C \times SC}}{A \times SA^2 + B \times SB^2 + C \times SC^2} \times SP \times m$, and $\frac{\overline{A+B+C} \times SO}{\overline{A+B+C} \times SO^2} \times SP \times m$; and if these be supposed to be equal, there comes out $SO = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{A \times SA + B \times SB + C \times SC}$.

Cor. 2. *The angular velocity of any system* A, B, C, *generated in a given time, by any force* f, *acting at* P, *perpendicular to* PS, *is as* $\frac{SP \times f}{A \times SA^2 + B \times SB^2 + C \times SC^2}$.

For the angular velocity of the whole system is the same as of one of the bodies A. But the absolute motion of A is $= \frac{m \times A + SA}{s}$, and the absolute velocity of $A = \frac{m \times SA}{s}$; but the angular velocity is as the absolute velocity directly, and the radius or distance reciprocally; therefore the angular vel. of A, and,

consequently, of the whole system, is as $\frac{m}{s}$ or $\frac{m \;\; \times}{A \times SA^2, \&c}$

that is, (because m is as the force f,) as $\frac{f \times SP}{A \times SA^2 + B \times SB}$

Cor. 3. *Hence, there will be the same angular velocity rated in the system, and with the same force, as there wou in a single body placed at* P, *and whose quantity of mat* $\frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{SP^2}$.

For let P $=$ that body, then (by Cor. 2.) since f and Sl given, the angular velocities of the system and body P, w

to one another, as $\frac{1}{A \times SA^2 + B \times SB^2, \&c.}$ to $\frac{1}{P \times SP^2}$. Whic ing supposed equal, we shall have $P = \frac{A \times SA^2 + B \times SB^2 + C}{SP^2}$

Cor. 4. *The angular motion of any system, generated by form force, will be a motion uniformly accelerated.*

PROP. LVII. (*Fig.* 3. *Pl.* VIII.)

TO FIND THE CENTRE OF PERCUSSION OF A SYSTEM OF BODIE THE POINT, WHICH STRIKING AN IMMOVEABLE OBJECT SYSTEM SHALL INCLINE TO NEITHER SIDE, BUT REST AS IT IN EQUILIBRIO.

Through the centre of gravity G of the system, draw a perpendicular to the axis of motion in S. And if the bodi not all situated in that plane, draw lines perpendicular to it the bodies, and let A, B, C, be the places of these bodies i plane. Draw SGO, and let O be the centre of percussion.

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the distance of the centre of percussion, from the axis of motion. Where note, if any points f, g, h, fall on the contrary side of S, the correspondent rectangles must be negative, $-A \times Sf$, $-B \times Sg$, &c.

Cor. 1. *If* G *be the centre of gravity of a system of bodies* A, B, C, *the distance of the centre of percussion from the axis of motion, that is,* $SO = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2, \&c.}{SG \times \overline{A + B + C, \&c.}}$.

For, by Prop. XLIV., $A \times Sf + B \times Sg + C \times Sh = \overline{A + B + C} \times SG$.

Cor. 2. *The distance of the centre of percussion from the centre of gravity* G, *is* $GO = \frac{GA^2 \times A + GB^2 \times B + GC^2 \times C}{SG \times \overline{A + B + C}}$

For, $A \times SA^2 + B \times SB^2 + C \times SC^2 = A \times \overline{SG^2 + GA^2 - 2SG \times Gf} + B \times \overline{SG^2 + GB^2 + 2SG \times Gg} + C \times \overline{SG^2 + CG^2 + 2SG \times Gh}$, by Eucl. II. 12 and 13. But, (by Cor. 2. Prop. XLIV.) $-A \times Gf + B \times Gg + C \times Gh = 0$; therefore $A \times SA^2 + B \times SB^2 + C \times SC^2 = \overline{A + B + C} \times SG^2 + A \times GA^2 + B \times GB^2 + C \times GC^2$.
Whence (by Cor. 1.) SO or $SG + GO = \frac{SG^2 \times \overline{A + B + C}}{SG \times \overline{A + B + C}} + \frac{A \times GA^2 + B \times GB^2 + C \times GC^2}{SG \times \overline{A + B + C}}$.

Cor. 3. *Hence,* $SG \times GO =$ *the given quantity* $\frac{A \times GA^2 + B \times GB^2 + C \times GC^2}{A + B + C}$; *and, therefore,* GO *is reciprocally as* SG.

For each of the bodies A, B, C, and their distances from G, are given.

Cor. 4. *Hence, also, if* SG *be given,* GO *will be given also. And, therefore, if the plane of the motion remain the same, in respect to the bodies, and the distance* SG *remain the same, the distance of* O *from* G *will remain the same also.*

Cor. 5. *The percussion or quantity of the stroke at* O, *by the motion of the system, is the same as it would be at* G; *supposing all the bodies placed in* G, *and the angular velocity the same. For the sum of the motions of* A, B, C, *in the system, acting against* O, *is as* $A \times SA \times \frac{SA}{SO} + B \times SB \times \frac{SB}{SO} + C \times SC \times \frac{SC}{SO} = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{SO} = SG \times \overline{A + B + C}$ (by this Prop.) *but* $SG \times \overline{A + B + C}$, *denotes the*

motion of A+B+C, *acting against* G, *or an obstacle placed the*

Cor. 6. *In a body or system of bodies, oscillating about a centre if* V *be the velocity of* O, *the centre of percussion; the shock quantity of the stroke at any point* P, *against an obstacle the is* $\frac{SG}{SP} \times V \times$ *sum of the bodies, or of the whole system, and, the fore, are reciprocally as* SP. *For the velocity of* A *being denoted* SA, *its quantity of motion is* $A \times SA$. *But, by the property of lever* ASP, *its quantity of motion against* P *is*, $A \times SA \times \frac{SA}{SP}$, $\frac{A \times SA^2}{SP}$. *In like manner, the motions of* B, C, *against* P *are* $\frac{B \times B}{SP}$ $\frac{C \times SC^2}{SP}$, *therefore the sum of all, or the whole stock against* P,

$$\frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{SP},$$

and that is reciprocally as SP, *the rest being given quantities. But by this* Prop. $\frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{SP} = \frac{SG}{SP} \times SO \times \overline{A+B+C}$; *where* SO *denotes* t *velocity of* O, *the same as* V.

Cor. 7. *If* OT *be drawn perpendicular to* SO, *then* OT *will the locus of all the centres of percussion.*

For the direction of O is in the line OT; and, therefore, it the same thing which point of the line OT strikes an obstacle.

PROP. LVIII. (*Fig.* 6. *Pl.* VIII.)

[redacted]

Likewise the angular velocity which any particle p, situated in O, generates in the system, by its weight, is $\frac{Sr \times p}{p \times SO^2}$ or $\frac{Sr}{SO^2}$, or $\frac{Sg}{SG \times SO}$, because of the similar triangle SgG, SrO. But their vibrations, and every part of them, are performed alike; therefore their angular velocities must be every where equal; that is, $\frac{-Se \times A + Sn \times B + Sd \times C}{s} = \frac{Sg}{SG \times SO}$; whence by reduction $SO = \frac{Sg}{SG} \times \frac{s}{-Se \times A + Sn \times B + Sd \times C}$. But (by Prop. XLIV.) $-Se \times A + Sn \times B + Sd \times C = Sg \times \overline{A+B+C}$. Therefore the distance of the centre of oscillation from the axis of motion, $SO = \frac{s}{SG \times \overline{A+B+C}} = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2, \&c.}{SG \times A + B + C, \&c.}$ $= \frac{A \times SA^2 + B \times SB^2 + C \times SC^2, \&c.}{A \times Sa + B \times Sb + C \times Sc, \&c.}$ Where $A \times Sa$, $B \times Sb$, &c. must be negative, when a, b, &c. lie on the contrary side of S. And since all these quantities are the same at all elevations of the axis SO; therefore the point O is rightly found; and the system has such a point as is required. Likewise, it appears by Cor. 1. of the last Prop. that the centre of oscillation is the same with the centre of percussion.

Cor. 1. *If p be any particle of a body, d its distance from* S, *the axis of motion; G, O, the centres of gravity and oscillation; then the distance of the centre of oscillation of the body, from the axis of motion,* $SO = \frac{\text{sum of all the } p \times dd}{SG \times \text{body}}$.

Cor. 2. *If the bodies* A, B, C, *be large, and, therefore, the centre of oscillation of each, not in the centre of gravity; let d, e, f, be the respective distances of their centres of gravity, and p, q, r, of their centres of oscillation, from* S. *Then will the distance of the centre of oscillation from* S, *the axis of motion,* $SO = \frac{dpA + eqB \times frC}{A \times Sa + B \times Sb + C \times Sc} = \frac{dpA + eqB + frC}{SG \times \overline{A+B+C}}$.

For let a, b, c, be any particles in A, B, C; and x, y, z, their distances from S respectively. Then, by this Prop. $SO = \frac{\text{sum } x^2a + \text{sum } yyb + \text{sum } zzc}{SG \times \overline{A+B+C}}$. But $\frac{\text{sum } xxa}{dA} = p$, or sum xxa $= dpA$, and sum $yyb = ebB$, and sum $zzc = frC$; and $SG \times \overline{A+B+C} = Sa \times A + Sb \times B + Sc \times C$.

Cor. 3. *To find the centre of oscillation of an irregular body, su pend it at the given point, and hang up a single pendulum of such length, that making them both vibrate, they may keep time togeth Then the length of this pendulum is equal to the distance of the ce tre of suspension from the centre of oscillation of the body.*

Cor. 4. *What has been demonstrated by the last Prop. and C 1, 2, 3, 4, for the centre of percussion holds equally true for the ce tre of oscillation.*

Scholium.—I shall just take notice, that if the distance of t axis of suspension from the centre of gravity SG, be made equ to $\sqrt{\dfrac{GA^2 \times A + GB^2 \times B + GC^2 + C}{A + B + C.}}$; the body will osc late in the least time possible.

In very small bodies, or any bodies oscillating at a great d tance from the axis of motion, the centre of oscillation or percu sion is in or very near the centre of gravity. And the reas why the centre of oscillation or percussion is not always the centre of gravity, is because the body in vibrating made to turn about a centre. But, if it be so contrived always to move parallel to itself, without any circular motion, centres of gravity, of oscillation, and percussion will be same.

The distance of the centres of oscillation and percussion, fr the axis of motion, as calculated by Cor. 1, is as follows. Wh the axis of motion is at the vertex, and in the plane of figure.

1. In a right line, small parallelogram, and cylinder, ⅔ the a of the figure.

2. In a triangle, ¾ the axis

[redacted]

and let SA, SB, SC, be the nearest distances of the bodies A, B, C, from the axis SR; and let the force f act at P, in direction PQ perpendicular to PS; then (by Cor. 2. Prop. LVI.) the angular velocity generated in the system by the force f, will be as $\frac{SP \times f}{A \times SA^2 + B \times SB^2 + C \times SC^2}$, and in the system placed in O, it will be $\frac{SP \times f}{\overline{A + B + C} \times SO^2}$; and if these velocities be made equal we shall have $SO^2 = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{A + B + C}$. Whence the distance of the centre of gyration O from the axis of motion at S, that is $SO = \sqrt{\frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{A + B + C}}$.

Cor. 1. (Fig. 8. Pl. VIII.) *Let* b = *quantity of matter in any body* ABRCS, p *any particle,* $d = ap$, *its distance from the axis of rotation* SR : *then the square of the distance of the centre of gyration, from the axis of motion, that is,* $SO^2 = \frac{\text{sum of all the } ddp}{b}$.

Cor. 2. (Fig. 5. Pl. VIII.) *If any part of the system be supposed to be placed in the centre of gyration of that particular part; the centre of gyration of the whole system will continue the same as before.*

For, by this Prop. the same degree of force which moved this part of the system before, along with the rest, will move it now without any alteration; and, therefore, if each part of the system be collected into its proper centre of gyration, the centre of gyration of the whole will continue the same.

Cor. 3. (Fig. 5. Pl. VIII.) *If a circle be described from* G, *the centre of gravity of the system; and the axis of rotation be made to pass through any point* S, *in its periphery; the distance of the centre of gyration from that point will always be the same.*

For (by Cor. 6. Prop. XLIV.) the quantity $A \times SA^2 + B \times SB^2 + C \times SC^2$ will be given.

Cor. 4. *The distance of the centre of gyration from the axis of motion, is a mean proportional between the distances of the centres of gravity and percussion from that axis.*

It follows from this and the last Prop.

Cor. 5. *The momentum or quantity of motion of the whole system, acting against an obstacle at* O, *the centre of gyration, is the same as if all the bodies were placed in* O, *the angular velocity remaining the same. For the momenta or quantities of motion are as the forces.*

Scholium.—It is the same thing on whatever side of the ax of rotation SR, the point O or centre of gyration be taken, pr vided it be at its proper distance.

By a computation from Cor. 1. the distance of the centre gyration from the axis of rotation, in the following bodies, wi be,

1. In a *right line* or small *cylinder* (revolving about the en SO $=$ length $\times \sqrt{\frac{1}{3}}$.

2. The *plane* of a *circle*, or *cylinder*, (revolving about the axi SO $=$ radius $\times \sqrt{\frac{1}{2}}$.

3. The *periphery* of a *circle*, (about the diameter) SO $=$ radi $\times \sqrt{\frac{1}{2}}$.

4. The *plane* of a *circle*, (about the diameter) SO $=$ $\frac{1}{2}$ radius

5. The *surface* of a *sphere*, (about the diameter) SO $=$ radi $\times \sqrt{\frac{2}{3}}$.

6. A *globe*, (revolving about the diameter) SO $=$ radius $\times$

7. In a *cone*, (about the axis) SO $=$ radius $\times \sqrt{\frac{3}{10}}$.

If the periphery of a circle revolve about an axis in the cent perpendicular to its plane, it is the same thing as if all the mat was collected into any one point in that periphery. And t plane of a circle of double the matter of this periphery, and t same diameter, will, in an equal time, acquire the same angu velocity.

If the matter of any gyrating body were actually to be plac in its centre of gyration, it ought either to be disposed of in circumference of a circle, whose radius is SO, or else into t points O, diametrically opposite, equal and equi-distant from For, by this means, the centre of motion S, will be in the cen of gravity; and the body will revolve without any lateral force wards any side.

[redacted]

vity G would still move forward with the same velocity Gg; and the body, instead of revolving about S, would (by Cor. 1. Prop. XLIX.) revolve about G with the same angular motion as before. Therefore, if rGo be drawn parallel to Sgd, Gg will represent the velocity of G, and Oo the velocity of O about G. And, because Oo, Gg are very small similar arches, therefore their circumferences will be described in equal times; that is, in the time that O, or the body itself, makes one revolution about G, the point G will advance forward a space, equal to the circumference of a circle, whose radius is SG.

Now, this is the motion acquired by revolving about S. But (by Prop. LVII.) if a body so revolving, strikes an immoveable object at O, both the progressive and circular motion will be destroyed, and the body will be at rest. It is evident, on the contrary, that if a moving body strike the body at rest in the point O, with the same force, the same motion will be restored again; and is the same as above described.

Cor. 1. *At the beginning of the motion, and, also, after every revolution of the body, when the line* SGO *comes into its original position, so as to be perpendicular to the line of direction* OB, *the point* S *will be at rest for a moment.*

For, in this position, it will be (by this Prop.) as velocity of O about G : velocity of G :: OG : GS. And, by composition, velocity O about G + velocity, G that is absolute velocity, O : absolute velocity, G :: Og + GS or OS : GS. Therefore, since the absolute velocities of O and G are directly as their distances from S, it follows, that the point S is at rest.

Cor. 2. *Let a body* A $= \frac{SG}{SO} \times$ *body* EF; *and if* V *be the velocity which the body* A *would receive by the direct stroke of* B; *then I say, the absolute velocity of the body* EF (*or of its centre of gravity* G,) *which it receives by* B *impinging at* O, *will be* $\frac{SG}{SO}V$.

For let p be any particle of the body EF, and Sp its distance from S. Then, (by Cor. 3. Prop. LVI.) if a body $= \frac{\text{sum of all the } Sp^2 \times p}{SO^2}$ be placed in O, it will receive the same angular velocity, by the stroke, about S at rest, as the body EF when struck in O. But (by Cor. 1. Prop. LVIII.) sum $Sp^2 \times p = SO \times SG \times$ body EF; whence the body $\frac{SO \times SG \times \text{body} EF}{SO^2}$ or A, placed in O, receives the same angular velocity about S, as the point O of the body EF. But velocity of O or A : velocity G :: SO : SG. For, at the beginning of the motion, S is at rest, by Cor. 1.

Cor. 3. *The velocity lost in* B *by the stroke, will be* $\frac{body\ EF}{body\ B}\ \frac{SG}{SO}$ V.

For the sum of the motions of all the bodies, after the stroke, the same as the motion of B before it, by Prop. X.

SCHOLIUM.—The point S is, by some, called the *spontane centre of rotation;* because the body (or system of bodies) at t beginning of the motion, moves, as it were, of its own acco or without any compulsion, about the centre S at rest.

PROP. LXI. (*Fig.* 9. *Pl.* VIII.)

LET DE BE ANY BODY, C ITS CENTRE OF GRAVITY, AND IF FR THE CENTRE C, THE CIRCLE BFS BE DESCRIBED, AND IF ABO BFS AS AN AXIS, A CORD ASBFS, BE WOUND, AND THE E FIXED AT A; AND IF O BE THE CENTRE OF OSCILLATION, RESPECT TO THE CENTRE OF SUSPENSION S, THEN IF T BODY DESCEND BY A ROTATION ROUND THE AXIS BFS, BY U WINDING THE CORD ASBF, &c.; THEN, I SAY, THE SPACE D SCENDED BY THE WHIRLING BODY DE, IS TO THE SPACE D SCENDED IN THE SAME TIME BY A BODY FALLING FREELY, SC TO SO.

Through the point of contact S and the centre of gravity draw the horizontal line SCO. Then (by Prop. LVIII.) the a gular velocity of the body about the point of suspension S, at t beginning of the motion, will be the same as if the whole bo was placed in O. But, if a body was placed in O, its veloci generated at the beginning, will be the same as of a body f ling freely. Therefore, drawing Sco infinitely near SCO, and t small arches Oo, Cc, then the velo ity of O is to the velocity

point O be let go, the force acting at O will generate a motion about S, whilst the pressure at S, and, consequently, the tension of the cord, is neither increased nor decreased, but remains the same as before.

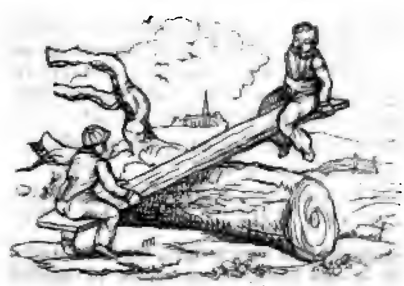
Cor. 2. *If a circular body, as* BFS, *runs down an inclined plane, whilst the thread* ASB *unfolds; or, if a round body roll down an inclined plane, and, by its friction, be hindered from sliding, the space it describes in any time, is to the space described by a body sliding down freely without friction, as* SC *to* SO.

For the forces that generate their motions are both decreased, in the same ratio, that is, as the absolute gravity to the relative gravity upon the plane; therefore, the spaces described will remain in the same ratio of SC to SO. And in the rolling body, the friction supplies the place of the cord, the same as if it had teeth.

Cor. 3. *This motion of the body* DE *by rotation, is a motion uniformly accelerated; and the tension of the cord is always the same, through the whole descent.*

Scholium.—(*Fig.* 10. *Pl.* VIII.) Let W = weight of a body, S = space described by a body falling freely. Then the spaces described by rotation or whirling, in the following bodies, as SBF, in the same time, are,

1. In the circumference of a circle, SBF, or surface of a cylinder, space = $\frac{1}{2}$ S; tension of the string = $\frac{1}{2}$ W.
2. In the circumference of a circle, SBF, without weight, and the weight be in the centre C; space = S, tension of the string = 0.
3. In the plane of circle SBF, or a cylinder, space = $\frac{2}{3}$ S, and the tension of the string AS = $\frac{1}{3}$ W.
4. In the surface of a sphere SBF, space = $\frac{3}{5}$ S, and the tension of the string AS = $\frac{2}{5}$ W.
2. In a sphere SBF, space = $\frac{5}{7}$ S, and tension of the thread = $\frac{2}{7}$ W.



SECTION SEVENTH.

THE QUANTITY AND DIRECTION OF THE PRESSURE OF BEAMS OF TIMBER, BY ANY WEIGHTS, AND THE FORCES NECESSARY TO SUSTAIN THEM.

PROP. LXII. (*Fig.* 1. *Pl.* IX.)

IF A BEAM OF TIMBER BE SUPPORTED AT C AND B, LYING UPON THE WALL ACE, WITH ONE END. AND IF G BE THE CENTRE OF GRAVITY OF THE WHOLE WEIGHT SUSTAINED; AND THE LINE FGH BE DRAWN PERPENDICULAR TO THE HORIZON, AND CF AND BH TO CB, AND BF DRAWN; I SAY,

THE WEIGHT OF THE WHOLE BODY	FH
PRESSURE AT THE TOP C	BH
THRUST OR PRESSURE AT THE BASE B, ARE, RESPECTIVELY, AS	FB, AND IN THESE SEVERAL DIRECTIONS.

If the beam support any weight, the beam and weight must be considered as one body, whose centre of gravity is G. Then

the end C is supported by the plane BCE; and (by Cor. 3. Prop. LIII.) the other end B may be supposed to be sustained by a plane perpendicular to BF; therefore (by Cor. 1. Prop. LIII.) the weight and forces at C and B, are, respectively, as FH, BH, and BF.

Cor. 1. *Produce* FB *towards* Q, *then* BQ *is the direction of the pressure at* B; *and the pressures at* B *in directions* BQ, FD, DB, *are as* FB, FD, DB.

Cor. 2. *Draw* D*r perpendicular to* BC, *and draw* CD. *Then the weight, pressure at the top, direct pressure at bottom, and horizontal pressure at bottom, are, respectively, as* CB, BD, DC, *and* D*r*.

For since the angles BCF, BDF are right; a circle described upon the diameter BF, will pass through C, D. Therefore ∠ BCD = BFD standing on the same arch BD. And because the ∠GBH and the angles at D are right, BHF = CBD; therefore the triangles FHB and CBD are similar, and the figure BHDF similar to the figure, DB*r*C, whence FH : BH : BF : BD :: are as CD : BD : DC : and D*r*.

Cor. 3. *All this holds true for any force instead of gravity, acting in direction* GD.

PROP. LXIII. (*Fig.* 2. *Pl.* IX.)

If BC be any beam, bearing any weight, G the centre of gravity of the whole; and if it lean against the perpendicular wall CA, and be supported in that position, draw BA, CF parallel, and FGD perpendicular to the horizon, and draw FB; then

the whole weight	FD
pressure at the top C	BD
thrust or pressure at the bottom B are, respectively, as	FB, and in the same directions.

For the end C is sustained by the plane AC; and if the end B be supposed to be sustained by a plane perpendicular to FB; then (by Cor. 1. Prop. LIII.) the weight, and pressure at top and bottom, are as DF, DB, FB. If you suppose the end B is not sustained by a plane perpendicular to FB, the body will not be supported at all, by Cor. 2. Prop. LIII.

Cor. 1. *If* FB *be produced to* Q, *then* BQ *is the direction of the pressure at* B; *and the perpendicular pressure at* B (FD) *is equal to the weight; and the horizontal pressure at* B (BD), *is equal to the pressure against* C.

PROP. LXIV. (*Fig.* 3. *Pl.* IX.)

IF A HEAVY BEAM, OR ONE BEARING A WEIGHT, BE SUSTAINED A C, AND MOVEABLE ABOUT A POINT C; WHILST THE OTHER E B LIES UPON THE WALL BE; AND IF HGF BE DRAW THROUGH THE CENTRE OF GRAVITY G, PERPENDICULAR THE HORIZON, AND BF, CH PERPENDICULAR TO BC, AND C BE DRAWN; THEN

THE WHOLE WEIGHT ⎫ HF
PRESSURE AT B ⎬ HC
FORCE ACTING AT C, ⎪ CF,.
ARE, RESPECTIVELY, AS ⎭ AND IN THESE DIRECTIONS.

For the end B is sustained by the plane CB, and (by Cor. Prop. LIII.) the end C may be supposed to be sustained by plane perpendicular to FC, or by a cord in direction CF. Th since HC is parallel to BF, the weight, force at C, pressure at are, respectively, as HF, CF, HC; by Cor. 1. Prop. LII., Cor. 1. Prop. LIII.

Cor. But if, instead of lying upon the inclined plane at B, *the e* B *laid upon the horizontal plane* AB, *then the weight, the press at* B *and* C, *are, respectively, as* BC, GC, *and* BG; *and in this ca there is no lateral pressure.*

For BF will be perpendicular to BA, and parallel to HF, an consequently, CF is, also, parallel to HF; therefore (by Cor. Prop XIX.) the forces at C, G, B, are as BG, BC, and CG.

PROP. LXV. (*Fig.* 4. *Pl.* IX.)

IF A HEAVY BEAM BC, WHOSE CENTRE OF GRAVITY IS G, BE SU

Cor. Hence, whether a body be sustained by two ropes BH, CH, *or by two posts* AB, CD, *or by two planes perpendicular to* BA, CD; *the body then can only be at rest, when the plumb line* HGF *passes through* G, *the centre of gravity of the whole weight sustained. Or, which is the same thing, when* AB, DC *intersect in the plumb line* HGF *passing through the centre of gravity.*

SCHOLIUM.—By the construction of these four last propositions, there is formed the *triangle of pressure*, representing the several forces. In which, *the line of gravity* (or plumb line passing through the centre of gravity) always represents the absolute weight, and the other sides the corresponding pressures.

PROP. LXVI. (*Fig.* 5. *Pl.* IX.)

IF SEVERAL BEAMS AB, BC, CD, &c. BE JOINED TOGETHER AT B, C, D, &c., AND MOVEABLE ABOUT THE POINTS A, B, C, &c. BE PLACED IN A VERTICAL PLANE, THE POINTS A, F, BEING FIXED, AND THROUGH B, C, D, DRAWING *ri*, *sm*, *tp*, PERPENDICULAR TO THE HORIZON; AND, IF SEVERAL WEIGHTS BE LAID ON THE ANGLES B, C, D, &c., SO THAT THE WEIGHT ON ANY ANGLE C MAY BE AS $\frac{S.BCD}{S.mCB \times S.mCD}$. THEN ALL THE BEAMS WILL BE KEPT IN EQUILIBRIO BY THESE WEIGHTS.

Produce DC to r. Then (by Cor. 2. Prop. VIII.) S. $\angle$ABC : S.$\angle$ABr :: weight B : force in direction BC $= \frac{B \times S.ABr}{S.ABC}$; and S.BCD : S.DC$s$ :: weight : force in direction CB $= \frac{C \times S.DCs}{S.BCD}$; which, to preserve the equilibrium, must be equal to the force in direction BC, that is, $\frac{B \times S.ABr}{S.ABC} = \frac{C \times S.DCs}{S.BCD}$; whence B : C :: $\frac{S.ABC}{S.ABr} : \frac{S.BCD}{S.DCs}$. And, by the same way of reasoning, C : D :: $\frac{S.BCD}{S.BCs} : \frac{S.CDE}{S.EDt}$. Therefore, ex equo, weight B : weight D :: $\frac{S.ABC}{S.ABr \times S.BCs} : \frac{S.CDE}{S.DCs \times S.EDt} :: \frac{S.ABC}{S.ABi \times S.CBi} : \frac{S.CDE}{S.CDp \times S.EDp}$

Cor. 1. *Produce* CD, *so that* Dw *may be equal to* Cr, *and draw*

wx parallel to D*p*, *cutting* DE *in x*. *Then the weight* C, *the for in directions* CB *and* CD, *are as r*B, CB *and* C*r respectively*. *A weight* C *is to the weight* D, *as* B*r to wx*.

Cor. 2. *The force or thrust at* C, *in direction* CB, *or at* B, *in rection* BC, *is as the secant of the elevation of the line* BC *above t horizon*.

For, force in direction CB : force in direction CD :: CB Cr :: S.CrB or rCm or sCD : S. rBC :: cosine elevation of CD cosine elevation of CB :: secant elevation of CB : sec elevation CD; because the secants are, reciprocally, as t cosines.

Cor. 3. *Draw* C*p*, D*m parallel to* DE, CB; *then the weights* C *and* D *to preserve the equilibrium, will be as* C*m to* D*p*. *A therefore, if all the weights are given, and the position of two li* CD, DE; *then the positions of all the rest* CB, BA, &c. *will successively found*.

For, let the force in direction CD or DC be CD, then C*p* the force in direction DE, and D*m*, in Direction CB. And D or the weight D, is the force compounded of DC, C*p*; and C*m* the weight C is the force compounded of CD, D*m*, by Cor. Prop. VII.

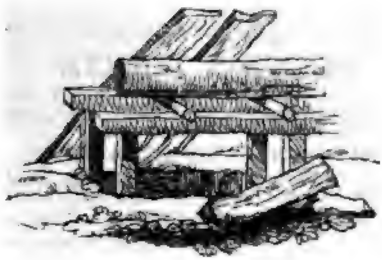
Cor. 4. *If the weights lie not on the angles* B, C, D, &c. *the places of their centres of gravity be at g, h, k, l*. *And let h, k, l, also express their weights*. *And take the weight* B $\frac{Ag}{AB}g + \frac{hC}{BC}h$, C $= \frac{Bh}{BC}h + \frac{kD}{CD}k$, D $= \frac{Ck}{CD}k + \frac{lE}{DE}l$, &c.; *then* C, D, &c. *will be the weights lying upon the respective angles*.

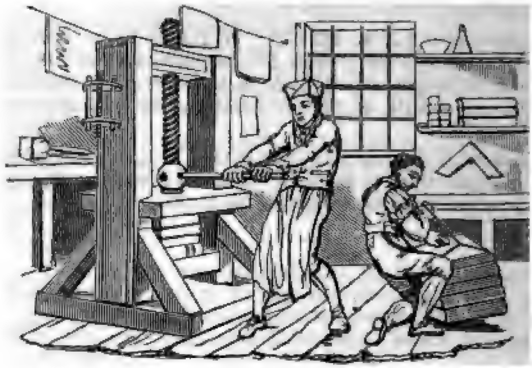
This is evident by Cor. 5. Prop. XIX.

supposed before to thrust against C, the same forces now do pull from it.

Scholium.—(*Fig*. 6. *Pl*. IX.) If DABF be a semi-circle, whose diameter is DF; draw AG perpendicular to DF; then the force or weight at any place A, to preserve the equilibrium, will be, reciprocally, as AG^3, or directly as the cube of the secant of the arch BA.

Likewise it follows from Cor. 5. that if any cords of equal lengths be stretched to the same degree of curvature, the stretching forces will be as the weights of the cords.





SECTION EIGHTH.

THE STRENGTH OF BEAMS OF TIMBER IN AI POSITIONS, AND THEIR STRESS BY AN WEIGHTS ACTING UPON THEM, OR BY AN FORCES APPLIED TO THEM.

PROP. LXVII. (*Fig.* 7. *Pl.* IX.)

[redacted]

Aa, ab, bc, &c. and imagine QAa, QAb, QAc, &c. so many bended levers whose fulcrum is at A; and let us see what will be the sum of all the forces applied at Q to break the timber at A. Now, (by Cor. 1. Prop. XIX) the power applied at Q to equal or overcome the resistances at A, a, b, c, &c. will be $\frac{o}{AQ}$, $\frac{Aa}{AQ}$, $\frac{Ab}{AQ}$, $\frac{Ac}{AQ}$, $\frac{Ad}{AQ}$, &c. to $\frac{AF}{AQ}$; that is as $\frac{o}{AQ}$, $\frac{1}{AQ}$, $\frac{2}{AQ}$, $\frac{3}{AQ}$, . . . $\frac{n}{AQ}$. Therefore, the effect of all the forces applied to Q, or the whole strength of the beam at A, will be $\frac{o+1+2+3 \quad . \quad . \quad . +n}{AQ}$ or $\frac{nn}{2AQ}$; that is, because AQ is given, as nn or AF^2. Now, if the breadth FG be increased in any proportion, it is evident the strength of every part Aa, ab, &c., will be increased in the same proportion, and, therefore, the absolute lateral strength will be as $AF^2 \times FG$.

Cor. 1. *In square timber, the lateral strength is as the cube of the breadth or depth.*

Cor. 2. *And, in general, the lateral strength of any pieces of timber, whose sections are similar figures, are as the cubes of the similar sides of the sections.*

Cor. 3. *And in any pieces of timber, whose sections are such figures that the correspondent ordinates, parallel to the horizon, are proportional, the strengths are as the breadths and squares of the depths, or as the sections multiplied by the depths.*

Cor. 4. (*Fig.* 1. *Pl.* XI.) *The strength of cylindrical pieces, or of any similar pieces of timber, being forced or twisted round the axis, will also be as the cubes of the diameters.*

For, let $AD = r$, circumference of the section DEFG $= c$, $Ap = x$, then the circumference $pqr = \frac{cx}{r}$, and if the cohesion of a particle at p be $= 1$; then the force applied at Q to overcome it, will be $\frac{x}{AQ}$; and the force applied at Q to overcome the cohesion of all the parts, in the circumference pqr, will be $\frac{x}{AQ} \times \frac{cx}{r}$, or $\frac{c}{r \times AQ} \times xx$; and the total force at Q to overcome the cohesion of all the particles in the whole section DEFG is $= \frac{c}{r \times AQ} \times$ sum of all the $xx = \frac{c}{r \times AQ} \times 1^2 + 2^2 \times 3^2 + 4^2 \ldots$

to $r^2 = *\frac{c}{r \times AQ} \times \frac{r^3}{3}$. Therefore, because AQ is given, and the ratio $\frac{c}{r}$, and this force is the strength of the beam; therefore the strength is as r^3 or AD^3.

SCHOLIUM. (*Fig.* 7. *Pl.* IX.)—What is here said of timber is true of any homogeneous bodies, whatever sort of matter they are of. But the absolute strength of any beam, lever, rope, &c. when drawn in direction of its length, will be as the section of it. For every part does in this case bear an equal stretch, and the sum of all the parts is equal to the whole; and that is as the section.

PROP. LXVIII. (*Fig.* 8. *Pl.* IX.)

THE LATERAL STRENGTH OF A TUBE OR HOLLOW CANE AB, TO THAT OF A SOLID ONE CD, IS AS THE SECTION OF THE TUBE (EXCLUDING THE HOLLOW,) TO THE SECTION OF THE SOLID CANE, AND THE WHOLE DIAMETER OF THE TUBE TO THE DIAMETER OF THE SOLID CANE, NEARLY.

For, (by Cor. 2. of the last Prop.) the strength of the solid cylinder BF is AF^3, and the strength of the inner solid cylinder, whose fulcrum is at G, is EG^3, and whose fulcrum is at F, is greater than EG^3, and less than EF^3, and is nearly $\overline{EG + \frac{1}{2}GF}|^3 = \overline{AF - \frac{3}{2}AE}|^3$, that is $AF^3 - \frac{9}{2}AF^2 \times AE$ nearly. Therefore, the strength of the tube AFIGE, is the difference of the strength of these cylinders, that is, $AF^3 - AF^3 + \frac{9}{2}AF^2 \times AE$ or $\frac{9}{2}AF^2 \times AE$. Likewise the strength of the solid cylinder DCH, is CH^3. Therefore the strength of the tube FB : strength of the cylinder HD :: $\frac{9}{2}AF^2 \times AE : CH^3$. But the section of the tube is as $AF^2 - EG^2$ or $AF^2 - \overline{AF - 2AE}|^2 = 4AF \times AE$ nearly. Whence,

[redacted]

from the proportion of their diameters. Let the diameter of that circle be R: then, strength of the ring or tube : strength of an equal circle :: AF : R. And the strength of R : to that of CH :: R^2 : GH^2. Therefore, ex equo, strength of the tube BF : strength of the cylinder HD :: $AF \times R^2$: $R \times CH^2$ :: $AF \times R^2$: $CH \times CH^2$ :: AF × area ring : CH × area of the circle CH.

Cor. Hence, the strength of different tubes are as their sections, and diameters, nearly.

PROP. LXIX. (*Fig.* 1. *Pl.* X.)

If any force be applied laterally to a lever or beam, the stress upon any place is directly as the force and its distance from that place.

For, suppose PAF to be a bended lever; it is evident the greater the power at P, the greater force is applied at F to separate the parts of the wood. Also, the greater the distance AP, the greater power has any given force applied at P, to overcome the cohesion of the wood at F. And, therefore, the whole stress depends on both.

Cor. 1. (Fig. 2. Pl. 10.) *If two equal weights lie upon the middle of two beams, or upon any other similar places, the stress in these planes will be as the lengths of the beams.*

For, if C be the middle point, then A bears half the weight; therefore, the stress at C is as AC × ½ weight. And, because half the weight, or the force acting at A is given; therefore, the stress is as AC or half AB, and, therefore, as AB. And if C be in any other similar situation in both beams, the same thing will follow.

Cor. 2. *If two beams bear two weights proportional to their lengths, and in a like situation, the stress upon each will be as the square of its length.*

Cor. 3. *And if two beams bear two weights reciprocally as their lengths, in a similar situation, the stress where the weights lie, is equal in both.*

PROP. LXX. (*Fig.* 3. *Pl.* X.)

Let AB be any beam of a given length, supported at A and B, and any given weight either suspended at any point C, or equally diffused through the whole length of the beam AB; I say, in either case, the stress of the beam in C, is as the rectangle AC × CB.

Case I.

Let the given weight be represented by the given length of the lever AB. Then (by Cor. 5. Prop. XIX.) the weight at A, and the re-action equal thereto, will be CB. And, by the last Prop.,

the stress at C will be as the force acting at A × distance A(that is, AC × CB.

CASE II.

Let AB be divided into an infinite number n of equal par each $= 1$. Then, as AB represents the whole weight, 1 will the weight supported upon 1 part of the beam, let it rest at then $\frac{Ap}{AB} =$ its pressure on B. Therefore, (by the last Prop.) t stress at p is $\frac{Ap \times pB}{AB}$; and the stress at C is $\frac{Ap \times BC}{AB}$, arisi from the weight at p. Consequently, the stress at C arisi from the sum of all the weights between A and C, will

$$\frac{0+1+2+3\ldots AC}{AB} \times BC;$$

that is, (because AC is the numl of them) $\frac{AC^2 \times BC}{2AB}$. And, by a like reasoning, the stress at arising from the whole weight between B and C, will $\frac{CB^2 \times AC}{2AB}$. Consequently, the whole stress at C

$$\frac{AC^2 \times BC + CB^2 \times AC}{2AB} = \frac{AC+CB}{2AB} \times AC \times CB = \frac{AC \times CB}{2}.$$

Cor. 1. *The greatest stress of a beam is in the middle; the wei being either suspended there, or equally disposed over the wh length of the beam.*

Cor. 2. *The stress of the beam at any point p, by a weight plied to any other point* C, *is as* $Ap \times CB$.

For, $AC \times CB$ is the stress at C, and (by Prop. last) $Ap \times$(will be the stress at p.

$Ap \times CB$, and at C, is $AC \times CB$. Therefore, the whole stress at C, by the whole weight on all the points of pC, is the sum of all the $Ap \times CB = Ap + \overline{Ap+1} + \overline{Ap+2} \ldots AC : \times CB = \frac{Ap+AC}{2} \times pC \times CB$. But the stress of the whole weight at C, is $AC \times CB \times pC$, and the former is to the latter as $\frac{Ap+AC}{2}$ to AC.

Cor. 6. *If a weight press equally on all the parts of* Ap, *the stress at any point* C *by that weight, is to the stress at* C *if suspended there* :: *as* Ap *to* 2AC.

For the stress at C by all the weight on Ap, is $0 + 1 + 2 \ldots Ap \times CB = \frac{Ap^2}{2} \times CB$. And the stress by the weight Ap at C is $AC \times CB \times Ap$.

Cor. 7. *The stress at p, by a weight at* C, *is equal to the stress at* C, *by the same weight at p.*

PROP. LXXI. (*Fig.* 4. *Pl.* X.)

IF CD BE A PROMINENT BEAM, FIXED HORIZONTALLY AT THE END C, AS IN A WALL; AND IF A WEIGHT PROPORTIONAL TO THE LENGTH OF THE BEAM, BE DISPERSED UNIFORMLY ON ALL THE PARTS OF THE BEAM; THE STRESS AT ANY POINT F, WILL BE AS DF^2, THE SQUARE OF THE DISTANCE FROM THE EXTREMITY.

For, let FD be divided into an infinite number of equal parts at p, q, r, s, &c., and let each be $= 1$, and sustain the weight 1.; then (by Prop. LXIX.) the stress at F, by the weights at F, p, q, r, &c., will be 1×0, $1 \times Fp$, $1 \times Fq$, &c. or as 0, 1, 2, 3, &c., respectively: therefore, the whole stress at C will be $0+1+2+3 \ldots FD = \frac{FD^2}{2}$.

Cor. 1. *Hence, the stress at* F, *by any weight suspended at* D, *will be double the stress at the same point* F, *when the same weight presses uniformly on all the parts between* F *and* D.

For, (by Prop. LXIX.) the stress at F by the weight DF, is $DF \times DF$, or FD^2.

Cor. 2. *The stress at the end* BC, *by the weight* P, *is the same as the stress upon the middle of a beam of twice the length* DC, *with twice the weight* P *laid on its middle, this beam being supported at both ends.*

For the stress now at C, is the same as if DC was continued to the same length beyond C, and a weight equal to P sus-

pended at the end; and then the fulcrum C will be acted on wi twice the weight P. And this is the same as if the beam w turned upside down, and twice the weight P laid on the mi dle C.

PROP. LXXII. (*Figures* 5 *and* 6. *Pl.* X.)

IF THERE BE TWO BEAMS STANDING A SLOPE, AND BEARING T WEIGHTS UPON THEM, EITHER IN THE MIDDLE, OR IN A GIVEN SITUATION, OR EQUALLY DIFFUSED OVER THE WHO LENGTH OF THE BEAMS; THE STRESS UPON THEM WILL BE I RECTLY AS THE WEIGHTS, AND THE LENGTHS, AND THE COSIN OF ELEVATION.

For (by Cor. 1. Prop. XXXI.) the weight is to the pressu upon the plane, as radius to the cosine of elevation. Therefo the pressure is as cosine elevation × weight; and this is the for acting against the beam. Therefore, (by Prop. LXVIII.) t stress will be as its length and this force; that is, as the leng the weight, and cosine elevation.

Cor. 1. *If the weights and length of the beams be the same,* t *stress will be as the cosine of elevation, and, therefore, greatest wh it lies horizontal.*

Cor. 2. *If the beams lie horizontal, or at any equal inclinatio and the weight be as the length, then the stress is as the square of length.*

Cor. 3. (Fig. 5. Pl. X.) *If the weights are equal, on the ho zontal beam* AB, *and the inclined one* AC, *and* BC *be perpendicu to* AB; *then the stress will be equal upon both.*

For the length × cosine elevation is the same in both, or AC cosine A = B × radius.

PROP. LXXIII.

IF ANY BEAM OF TIMBER BE TO SUPPORT ANY WEIGHT, OR PRESSURE, OR FORCE, ACTING LATERALLY UPON IT, THE BREADTH MULTIPLIED BY THE SQUARE OF THE DEPTH, OR IN SIMILAR SECTIONS, THE CUBE OF THE DIAMETER, IN EVERY PLACE, OUGHT TO BE PROPORTIONAL TO THE LENGTH MULTIPLIED BY THE WEIGHT OR FORCE ACTING ON IT, OR AS THE STRESS IN THAT PLACE. AND THE SAME IS TRUE OF SEVERAL DIFFERENT PIECES OF TIMBER COMPARED TOGETHER.

For every several piece of timber, as well as every part of the same timber or beam, ought to have its strength proportioned to the weight, force, or pressure it is to sustain. And, therefore, the *strength* ought to be universally as the *stress* upon it. But (by Prop. LXVII.) the strength is as the breadth $\times$ square of the depth. And (by Prop. LXIX.) the stress is as the weight or force $\times$ by the distance it acts at. And, therefore, these must be in an invariable ratio.

Cor. 1. (Fig. 8. Pl. X.) *If* AEB *be a prominent beam fixed at the end* AE, *and sustaining a weight at the other end* B. *And if the sections in all places be similar figures, and* CD *be the diameter in any place* C, *then* CB *will be every where as* CD^3. *And if* ACB *be a right line,* EDB *will be a cubic parabola. Therefore* $\frac{2}{5}$ *of such a beam may be cut away without any diminution of the strength.*

But if the beam be bounded by two parallel planes, perpendicular to the horizon, then CB *will be as* CD^2, *aud then* EDB *will be the common parabola. Whence, a third part of a beam may be thus cut away.*

Cor. 2. (Fig. 8. Pl. X.) *But if a weight press uniformly on every part of* AB, *and the sections in all points at* C, *be similar; then* BC^2 *will be every where as* CD^3, *and* EDB *a semi-cubical parabola.*

But if the beam (Fig. 7. Pl. X.) *be bounded by parallel planes, perpendicular to the horizon, then* BC *will be as* CD, *and* EDB *a right line. Here half a beam may be cut away without losing any strength.*

Cor. 3. (Fig. 10. Pl. X.) *If* AB *be a beam supported at both ends, and if it bear a weight in any variable point* C, *or uniformly on all the parts of it; and if all the sections be similar figures, and* CD *be the diameter in that place* C; *then will* CD^3 *be every where as* AC $\times$ CB.

But if it be bounded by two parallel planes, perpendicular to the horizon, then will CD^2 *be every where as* AC $\times$ CB, *and, therefore, the curve* ADB *is an ellipsis, supposing* AB *a right line.*

Cor. 4. (Fig. 11. Pl. X.) *But if a weight be placed at any given point* P, *and all sections are similar figures, and if* CD *be any diameter, then will* BC *be as* CD^3, *and* AQ *and* BQ *are two cubic parabolas.*

But if the beam be bounded by two parallel planes, perpendicul to the horizon, then BC *is as* CD^2, *and* AQ *and* BQ *are two cor mon parabolas.*

Cor. 5. *All circular plates, whether great or little, being of t same matter and thickness, and supported all round on the edges, w bear equal weights. The same is true of square plates, or any sir lar ones.*

For let AD, *ad*, (*Fig.* 2. *Pl.* XI.) be two squares, and : them first be only supported at the ends AB, CD, and *ab*, Then, by this Prop., (since the thickness is given) AB : AC weight on AD :: *ab* : *ac* × weight on *ad*, and weight on AD weight on *ad* :: $\frac{AB}{AC} : \frac{ab}{ac}$, but $\frac{AB}{AC} = \frac{ab}{ac}$, therefore, weig on AD = weight on *ad*, when the plates are only supported AB, CD, and *ab*, *cd*. And, for the same reason, the weights w be equal, when only supported by the sides AC, BD, and *ac*, And, consequently, the weights will still be equal, when the pla are supported by all four sides, in which case, twice the weig will be supported; and the same holds equally true of all simi figures. For,

Let AD, *ad*, (*Fig.* 3. *Pl.* XI.) be two hollow circles; draw t inscribed squares ABCD, *abcd*; these squares are supported up the four sides AB, BD, DC, CA, and *ab*, *bd*, *dc*, *ca*, by the con nuity of the plates; therefore, the weights will be equal, as w proved before. And that the continuity of the plates will equa support the weights in both circles is plain, because the streng of both segments AB, *ab*, are equal, the length being as t breadth.

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PROP. LXXIV. (*Fig* 9. *Pl.* X.)

IF A WEIGHT A BE SUPPORTED UPON THE END OF A CROOKED PIECE OF TIMBER ABD, AND, FROM THE ENDS, A LINE AB BE DRAWN PERPENDICULAR TO THE HORIZON, AND FROM THE ANGLE B, THE LINE BC PERPENDICULAR TO AD; THE STRESS AT B WILL BE AS THE PERPENDICULAR BC.

For, as the weight A acts not in direction AB, but in direction AD, therefore, it is the same as if it were applied at the point C; but a force applied at C, has a greater power to break the timber at B, in proportion as the lever BC is longer. This force therefore, or the stress at B, is as BC.

Cor. 1. (Fig. 4. Pl. XI.) *Hence, if any two forces acting from or against one another, at the ends* A, F, *of any crooked beam* ABDEF, *and keep one another in equilibrio; and the line* AF, *or the direction of the forces being drawn, the stress at any point is as the perpendicular upon* AF. *So the stress at b is bc; at* B, BC, *at* D, DI; *at* E, EK; *and at* G *and* H, *nothing.*

Cor. 2. *Hence, also, that the strength in any part b, may be proportional to the stress there, the breadth multiplied by the square of the depth, must be as the perpendicular bc, reckoning that the depth, which is in the plane passing through* AF.

PROP. LXXV. (*Figures* 5 *and* 6. *Pl.* XI.)

HAVING THE LENGTH AB AND WEIGHT W, OF A CYLINDER OR PRISM, THAT CAN JUST SUPPORT THE WEIGHT P AT THE END, TO FIND THE LENGTH OF ANOTHER BEAM FG, SIMILAR TO THE FORMER, AND OF THE SAME MATTER, THAT WILL JUST BREAK WITH ITS OWN WEIGHT, OR ONLY SUPPORT ITSELF.

Since the weights of similar solids of the same matter are as the cubes of the lengths, it will be, $AB^3 : W :: FG^3 : \frac{FG^3}{AB^3}W =$ the weight of the beam GH. Then, by Cor. 2. Prop. LXVII. the strength of the beam AB is AC^3; and of FG, is FH^3. And by Prop. LXIX. the stress at A is $\overline{\frac{1}{2}W+P} \times AB$. And the stress at F $\frac{FG^3}{2AB^3}W \times FG$. And since the beams are both supposed to break with these weights, therefore the strength must be as the stress; that is $\overline{\frac{1}{2}W+P} \times AB : \frac{FG^4}{2AB^3}W\ (:: AC^3 : FH^3) :: AB^3 : FG^3$. Whence $\frac{FG^4 \times W}{2} = FG^3 \times AB \times \overline{\frac{1}{2}W+P}$. Or $FG \times W = AB \times \overline{W+2P}$. Whence $W : W+2P :: AB : FG$, the length required.

Cor. 1. *If* eW $=$ P. *Then* FG $=$ AB $\times \overline{1+2e}$.

Cor. 2. *Hence, there is one and only one beam, that will just bre by its own weight, or just sustain itself.*

Cor. 3. *The same Prop. will likewise hold good, in regard to t beams supported at both ends and breaking in the middle, by Cor. Prop.* LXX.

Cor. 4. *If the beam* FG *break by its own weight, a beam of tw the length of* FG, *and supporting at both ends, will also break by own weight; or if one sustain itself, the other will.*

For the stress is the same in both of them, by Cor. 3. Pr LXX. and Cor. 1. Prop. LXXI. each of them being equal to stress of a beam, twice the length of FG, and suspended in middle.

PROP. LXXVI. (*Figures* 7 *and* 8. *Pl.* XI.)

If any weight be laid on the beam AB, as at C, or a force applied to it at C; the beam will be bent throu a space CD proportional to the weight or force appl at C. And the resistance of the beam will be as space it is bent through nearly.

In order to find the law of resistance of beams of timber, such like bodies, against any weights laid upon them, or str ing them, I took a piece of wood planed square, and supp ing it at both ends A, B, I laid, successively, on the middle c at C, 1, 2, 3, 4, 5, 6, 7, and 8 pounds; and I found middle point C to descend through the spaces 1, 2, 3, 4, 5 7, and 8, respectively. And, repeating the same experin with the weights 3, 6, 9 pounds, they all descended thrc

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that they have the least resistance when least bent, and in all cases are bent through spaces nearly proportional to the weights or forces applied. And, therefore, I think this law is sufficiently established, that the resistance, any of these bodies makes, is proportional to the space through which it is bent, or that it exerts a force proportional to the distance it is stretched to.

The knowledge of this property of springy bodies is of great use in mechanics, for, by this means, a spring may be contrived to pull at all times with equal strength, as in the fusee of a watch; or it may be made to draw in any proportion of strength required.

The action of a spring may be compared to the lifting up a chain of weights, lying upon a plane, or to the lifting a cylinder of timber out of the water endways.

PROP. LXXVII. (*Fig.* 9. *Pl.* XI.)

TO FIND THE LATERAL STRENGTH OF ANY BEAM OF TIMBER, WHOSE TRANSVERSE SECTION IS ANY FIGURE WHATSOEVER.

Let ERG be the section of the beam in the place where it breaks. Draw the ordinates IN, *in*, infinitely near each other, and parallel to the base RG.

Put ER $= d$ EI $= x$
RG $= b$ IR $= v$
IN $= y$

The absolute strength of one fibre of the wood $= 1$.

When the beam breaks, it is done by the separation of the parts of the wood at E. Therefore, QRE must be esteemed a bended lever, whose fulcrum is at R. When the beam breaks, the fibres at E are stretched to their full strength, but those nearer R are less stretched, and exert less force or resistance in proportion to their distance from R (by the last Prop.); and, therefore, the resistance of the fibre at I $= \frac{v}{d}$; and the resistance of all the fibres in the parallelogram I*n*, $= \frac{v}{d} \times \text{I}n$; and the power of all the fibres in the parallelogram, in regard to the brachium IR, is $= \frac{vv}{d} \times \text{I}n$. And the sum of all the powers in the whole section $=$ sum of all the $\frac{vv}{d} \times \text{I}n$.

Let *g*, *p*, be the distance of the centre of gravity and percussion from RG, as the axis of motion.

Then (by Cor. 1. Prop. LVII.) the sum of all the $vv \times$ In $=$ $\times$ sum of all the I$n = gp \times$ section ERG. And the sum all the $\frac{vv}{d} \times$ I$n = \frac{gp}{d} \times$ section ERG. Therefore, the stren of the beam at E, is $= \frac{gp}{d} \times$ section ERG.

Cor. 1. (Fig. 10. Pl. XI.) *If there be taken* RO $= \frac{gp}{d}$; *t all the fibres of the wood being supposed to be collected in* O, *acting there with their full strength; their total strength at* O, *be equal to the strength of the beam, at the section* ERG.

For, suppose O to be such a point; then the strength of beam, or $\frac{gp}{d} \times$ section ERG $=$ RO $\times$ section ERG; and $= \frac{gp}{d}$.

Cor. 2. *If the section be a parallelogram,* $g = \frac{1}{2}d$, *and* $p =$ *therefore* RO $= \frac{1}{3}d$.

In a circle whose diameter is ER, $g = \frac{1}{2}d$, $p = \frac{5}{8}d$, *and* RO $\frac{5}{16}d$, $= \frac{1}{3}d$ *nearly, as in the parallelogram.*

In the periphery of a circle (the beam being a hollow cane) $g =$ *and* $p = \frac{3}{4}d$, *whence* RO $= \frac{3}{8}d = \frac{1}{3}d$ *nearly; as in the paralle gram.*

In a triangle whose base is at E *parallel to the horizon, and ve tex at* R; $g = \frac{2}{3}d$, *and* $p = \frac{3}{4}d$, *and* RO $= \frac{1}{2}d$. *And its streng* $= \frac{1}{2}d \times$ *area of the triangle.*

Cor. 2. (Fig. 1. Pl. XI.) *The strength of a cylinder when twist wrested, or wrung round its axis, is equal to the lateral strength o triangular beam whose height = radius, base = circumference of*

And in the triangle, the area being $\frac{cr}{2}$, and $d = r$, the strength $= \frac{1}{2}r \times \frac{cr}{2} = \frac{crr}{4}$, the same as for the twisted cylinder.

Scholium.—This Cor. 2. does not agree so well with timber as metal, for the texture of wood is not the same in length as breadth. For all wood is composed of long slender tubular fibres, joined together by a glutinous matter, which is easily separated; and, therefore, wood is much more easily split than broken.

PROP. LXXVIII. (*Fig.* 11. *Pl.* XI.)

Given the weight that will break a beam laterally, to find how much will break it when drawn in direction of its length.

Let DR *be the beam.*
Put l = *its length* DE,
W = *weight applied at* D, *that can break it at* E,
d = *depth* ER,
g = *distance of the centre of gravity of the section* ERG *from* R,
p = *distance of the centre of percussion of* ERG *from* RG,

Then $\frac{dl}{gp}$ W = *weight that will break it when drawn in direction of its length.*

But if the beam be supported at both ends, and the weight breaks it in the middle, g and p must be measured from the upper side, and take l for half its length, W *for half the weight that breaks it.*

For, by Cor. 1. of the last Prop. if RO $= \frac{gp}{d}$, then all the fibres of the beam acting at O, will be equal to the strength of the beam; and since W, applied at D, can break it in either case, therefore, by the nature of the lever, it will be, $l \times$ W = RO $\times$ absolute strength $= \frac{gp}{d} \times$ absolute strength; therefore, the absolute strength $= \frac{ld\mathrm{W}}{gp}$, or the weight that can break it, when drawn in length.

Cor. Hence, if there be taken RL $= \frac{gp}{d}$, *then the weight which being applied at* L, *will just break the beam horizontally, the same will just pull it asunder, when applied lengthwise.*

For, then $l = \frac{gp}{d}$, and weight W $=$ resistance at O $=$ streng of the whole beam.

Therefore, if a piece of oak, an inch square and a foot lo supported at both ends, bears 315 lb. before it breaks, it w bear, when drawn in length, 2835 lb. or 1 ton, 5 hundreds stones, 7 pounds; that is, above a ton and a quarter.

Scholium.—Here we all along suppose, that the fulcrum at remains fixed; but if it should vary by the denting in of t parts at R, it will cause a little variation in the strength, a make the beam something weaker, laterally. And that it w yield a little this way, is evident from experiments; for t hardest bodies, such as glass in small threads, may be extend in length, and, consequently, may be contracted by a contra force; and balls of glass or wood, let fall upon a hard bo will rebound; which they cannot do without the denting in of t parts.

PROP. LXXIX. (*Fig.* 12. *Pl.* XI.)

If a weight be laid upon the straight beam AB, su ported at both ends, its bending or curvature wi be nearly as the weight and length directly; a as the breadth and cube of the depth reciprocally.

It is found (by Prop. LXXVI.) that if several weights be la successively upon a horizontal beam AB, the space CDB, throu which the point D descends, will be as the weight it bears. No the parts at e, f, which are contiguous at the beginning of the

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and C the correspondent curvature, and it will be as $\frac{b \times Df}{L} : \frac{1}{Df} :: W : C = \frac{LW}{b \times Df^3}$.

Cor. 1. *The quantity of deflexion* CD *of any beam is as the weight and cube of the length directly, and the breadth and cube of the depth reciprocally.*

For when CD is very small, ADB is very near a circle, or nearer a parabola; suppose it a circle, and let its radius be R, then $2R \times CD = AD^2$ or $\frac{1}{4}$ LL; therefore $\frac{CD}{LL} = \frac{1}{8R}$, whence $\frac{CD}{LL}$ is as the curvature C, that is, as $\frac{LW}{b \times Df^3}$, or CD is as $\frac{WL^3}{b \times Df^3}$. And if ACB, the original position of the beam, is not a right line, yet CD will still be of the same quantity.

Cor. 2. *In similar homogenous straight bodies the curvature is as the weight directly, and cube of the depth reciprocally; but the deflexion* CD *is as the weight directly, and depth reciprocally.*

Cor. 3. *In similar bodies bending from a straight line by their own weight, the curvature is given, and the deflexion is as the square of the depth.*

Cor. 4. *In the utmost strength of beams, or their breaking position, the curvature is reciprocally as the depth, and the deflexion as the square of the length directly, and depth reciprocally.*

For then $b \times Df^2$ is as LW.

Cor. 5. *What is said of straight beams is equally true of any beams, in regard to the increase or variation of curvature, and to the deflection from their original position.*

Scholium.—What is said of beams of timber in this section, is equally applicable to any solid bodies, acted on in a like manner as by weights. There are some bodies in which a very little bending may have a great effect, as in the glasses of large telescopes. For (by Cor. 3. of the last Prop.) the deflexion from their true figure, arising from their own weight, is as the square of the diameter, when the glasses are similar. And though this be insensible in small glasses, it may produce some sensible error in large ones; and the same may happen to them in grinding, by too much pressure.

From the foregoing propositions it follows, that if a certain

beam of timber be able to support a given weight, another be
of the same timber, similar to the former, may be taken so gr
as to be able but just to bear its own weight; and any big
beam cannot support itself, but must break by its own weig
and any less beam will bear something more. For the stren
being as the cube of the depth, and the stress being as the mat
and length, is as the fourth power of the depth; it is plain
stress increases in a greater ratio than the strength. Whenc
follows, that a beam may be taken so large, that the stress n
far exceed the strength. And that of all similar beams ther
but one that will support itself, and nothing more. Likewise
any beam can bear ten times its own weight, no other simi
beam will do the same. And the like holds in all machines, a
in all animal bodies. And hence, there is a certain limit, in
gard to magnitude, not only in all machines and artificial str
tures, but also in natural ones, which neither art nor nature
go beyond, supposing them made of the same matter, and in
same proportion of parts.

Hence, it is impossible that mechanic engines can be increa
to any bigness. For when they arrive at a particular size, th
several parts will break and fall asunder by their weight; neit
can any buildings of vast bigness be made to stand, but must f
to pieces by their great weight, and go to ruin. Vast colum
and pyramids will break by their weight and tumble down.

It is, likewise, impossible for nature to produce animals of a
vast size at pleasure, or any such thing as giants, or men of pr
digious stature; except some sort of matter can be found
make the bones of, which is so much harder and stronger th
any hitherto known; or else that the proportion of the parts
so much altered, and the bones and muscles made thicker in pr
portion, which will make the animal distorted and of a monstr

And, therefore, a small animal will carry far more than its own weight, whilst a great one cannot carry so much as its weight. And hence it is, that small animals are more active, will run faster, jump farther, or perform any motion quicker, for their weight, than large animals; for the less the animal the greater the proportion of the strength to the stress. And nature seems to know no bounds as to the smallness of animals, at least in regard to their weight.

Neither can any two unequal and similar machines resist any violence alike, or in the same proportion; but the greater will be more hurt than the less. And the same is true of animals, for large animals, by falling, break their bones, whilst lesser ones falling higher receive no damage. Thus, a cat may fall two or three yards high and be no worse, and an ant from the top of a tower.

It is likewise impossible, in the nature of things, that there can be any trees of immense bigness; if there were any such, their limbs, boughs, and branches, must break and fall down by their great weight. Thus, it is impossible there can be an oak a quarter of a mile high; such a tree cannot grow or stand, but its limbs will drop off by their weight. And hence, likewise, lesser plants can better sustain themselves than large ones can do.

Neither could a tree of an ordinary size be able to stand if it was composed of the same tender matter that some plants consist of; nor such a plant if it were much bigger than common. And that plants made of such tender matter may better support themselves, nature has made the trunks and branches of many of them hollow, by which means they are both lighter and stronger.

The propositions before laid down concerning the strength and stress of timber, &c., are also of excellent use in several concerns of life, and particularly in architecture; and upon these principles a great many problems may be resolved relating to the due proportion of strength in several bodies, according to their particular positions and weights they are to bear, some of which I shall briefly enumerate.

If a piece of timber is to be holed with a mortise hole, the beam will be stronger when it is taken out of the middle, than if it be taken out of either side. And in a beam supported at both ends, it is stronger when the hole is taken out of the upper side than the under one, provided a piece of wood is driven hard in to fill up the hole.

If a piece is to be spliced upon the end of a beam to be supported at both ends, it will be stronger when spliced on the under side of a beam, than on the upper side. But if the beam is sup-

ported only at one end, to bear a weight on the other, it stronger when spliced on the upper side.

When a small lever, &c., is nailed to a body to remove it suspend it by, the strain is greater upon the nail nearest the ha or point where the power is applied.

If a beam is supported at both ends, and the two ends res over the props, and be fixed down immoveable, it will bear tw as much weight as when the ends only lie loose or free upon t supporters.

If a slender cylinder is to be supported by two pieces, the d tance of the pins ought to be $\frac{5858}{10000}$ parts of the length of the c linder, that is $\frac{3}{5}$ its length, the pins equi-distant from its en and then the cylinder will endure the least bending or strain its weights.

By the foregoing principles it also follows, that a beam fix at one end, (*Fig.* 13. *Pl.* XI.) and bearing a weight at the oth if it be cut in the form of a wedge, and placed, with its paral sides, parallel to the horizon, it will be equally strong every whe and no sooner break in one place than another.

If a beam has all its sides cut into the form of a concave par bola, (*Fig.* 14. *Pl.* XI.) whose vertex is at the end, and base square, a circle, or any regular polygon, such a beam fixed ho zontal at one end, is equally strong throughout for supporting own weight.

By the same principles, if a wall faces the wind, and if t section of it be a right angled triangle, or the foreside be perpe dicular to the horizon, and the backside terminated by a slopi plane, intersecting the other plane in the top of the wall, such wall will be equally strong in all its parts to resist the wind, the parts of the wall cohere strongly together; but if it be bu of loose materials, it is better to be convex on the backside

And if such a pillar be turned upside down, and suspended at the thick end in the air, it will be no sooner pulled asunder in one part than another by its own weight. And the case is the same if the small end be cut off, and, instead of it, a cylinder be added, whose height is half the subtangent.

Lastly, let AE (*Fig.* 15. *Pl.* XI.) be a beam in form of a triangular prism, and if AD $= \frac{1}{3}$ AB, and AI $= \frac{1}{3}$ AC, and the part ADIF be cut away parallel to the base, the remaining beam DICEF will bear a greater weight P, than the whole ABCEG, or the part will be stronger than the whole, which is a paradox in mechanics.

And upon the same principles, an infinite number of questions of like kind may be resolved, which are curious enough, and of great use in the common affairs of life.

All I shall here add, is the strength of several sorts of timber, and other bodies, as I have collected from experiments.

In the first edition of this book I had inserted the strengths of some sorts of wood, such as I had made experiments upon; in all which, I gave the least weight which the worst of them was just able to bear; lest any body, computing the strength of a beam, should overcharge it with too much weight. And, since that time, I have made a great many more experiments, not only upon many different sorts of wood, but several other bodies, the result of which I shall here set down. A piece of good oak, an inch square, and a yard long, supported at both ends, will bear in the middle, for a very little time, about 330 pounds *averd.* but will break with more than that weight. This is at a medium; for there are some pieces that will carry something more, and others not so much: but such a piece of wood should not in practice be trusted, for any length of time, with above a third or a fourth part of that weight. For, since this is the extreme weight which the best wood will bear, that of a worse sort must break with it. For, I have found by experience, that there is a great deal of difference in strength, in different pieces of the very same tree; some pieces I have found would not bear half the weight that others would do. The wood of the boughs and branches is far weaker than that of the body; the wood of the great limbs is stronger than that of the small ones, and the wood in the heart of a sound tree is strongest of all. I have also found by experience, that a piece of timber, which has borne a great weight for a small time, has broke with a far less weight, when left upon it for a far longer time. Wood is likewise weaker when it is green, and strongest when thoroughly dried, and should be two or three years old at least. If wood happens to be sappy it will be weaker upon that account, and will likewise decay sooner. Knots in wood weaken it very much, and this often causes it to break where a knot is. Also, when

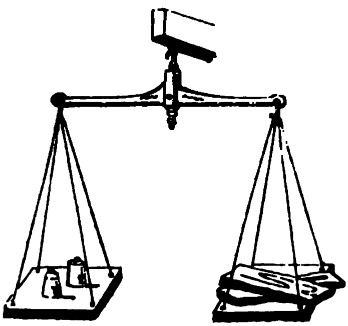
All this supposes these bodies to be sound and good throughout; but none of these should be put to bear more than a third or a fourth part of the weight, especially for any length of time.

From what has been said, if a spear of fir or a rope, or a spear of iron of d inches diameter, was to lift $\frac{1}{4}$ the extreme weight, then,

The fir would bear $8\frac{3}{4}$ *dd* hundred weight.

The rope 22 *dd* hundred weight.

The iron $6\frac{3}{4}$ *dd* ton weight.



SECTION NINTH.

THE PROPERTIES OF FLUIDS, THE PRINCIPLES OF HYDROSTATICS, HYDRAULICS, AND PNEUMATICS.

PROP. LXXX. (*Fig.* 1. *Pl.* XII.)

MOTION OR PRESSURE IN A FLUID IS NOT PROPAGATED IN RIGHT LINES, BUT EQUALLY ALL AROUND IN ALL MANNER OF DIRECTIONS.

If a force act at *a* in direction *ab*, that motion can be directed no further than these particles lie in a right line as to *c*. But the particle *c* will urge the particles *d*, *f* obliquely, by which that motion is conveyed to *e*, *g*. And these particles *e*, *g*, will urge the particles *n*, *p*, and *r*, *s* obliquely, which lie nearest them. Therefore the pressure, as soon as it is propagated to particles that lie out of right lines, begins to deflect towards one side and the other; and that pressure being farther continued, will deflect into other oblique directions, and so on. Therefore, the pressure and motion is propagated obliquely ad infinitum, and will, therefore, be propagated in all directions.

Cor. (Fig. 2. Pl. XII.) *If any part of a pressure propagated through a fluid, be stopped by an obstacle, the remaining part will deflect into the spaces behind the obstacle. Thus, if a wave proceeds from* C, *and a part goes through the hole* A, *it expands itself, and forms a new wave beyond the hole, which moves forward in a semicircle whose centre is the hole.*

For any part of a fluid pressing against the next is equally re-acted on by the next, and that by the next to it, and so on; from whence follows a lateral pressure (equal to the direct pressure) into the places behind the obstacle.

PROP. LXXXI. (*Fig.* 3. *Pl.* XII.)

A FLUID CAN ONLY BE AT REST WHEN ITS SURFACE IS PLACED IN A HORIZONTAL SITUATION.

For, let ABCD be a vessel of water or any fluid; and let AB be parallel to the horizon. Suppose the surface of the liquor to be in the position FE. Then, because the parts of the fluid are easily moveable among themselves; therefore (by Ax. 7.) the higher parts at E will, by their gravity, continually descend to the lower places at F. Also the greater pressure under E and the lesser under F, will cause the parts at E to descend, and those at F to ascend. And thus the higher parts of the fluid at E descending, and spreading themselves over the lower parts at F, which are at the same time ascending; the surface of the fluid will at last be reduced to a horizontal position AB. But being settled in this position, since there is no part higher than another, there is no tendency in any one part to descend, more than in another; and, therefore, the fluid will rest in an horizontal position.

Cor. 1. (Fig. 4. Pl. XII.) *If the fluid does not gravitate in parallel lines, but towards a fixed point or centre* C; *then the fluid can only be at rest when its surface takes the form of a spherical surface* AB, *whose centre is* C.

For if any parts of the surface of the fluid A or B, were further from C than the rest, they would continually flow down to the places nearer C, towards which their weights are directed; till at last they would be all equi-distant from it.

Cor. 2. *Any fluid being disturbed, will of itself return to the same level, or horizontal position.*

Cor. 3. (Fig. 5. Pl. XII.) *Hence, also, if a different fluid* ABEF *rest upon the fluid* ABCD; *both the surface* FE, *and the surface* AB *that divides them, will lie in a level or horizontal situation, when at rest.*

For, if any part of the surface AB be higher than the rest, will descend to the same level; and since FE is also level, ar therefore, the heights AF, BE in every place equal; the press of it on all the parts of the horizontal surface AB, will be equ And, therefore, it cannot descend in one place more than an ther, but will continue level.

Cor. 4. *Hence, water communicating with two places, or any w conveyed from one place to another, will rise to the same level both places; except so far as it is hindered by the friction of t channel it moves through, or, perhaps, some very small degree of te city or cohesion.*

PROP. LXXXII. (*Fig.* 6. *Pl.* XII.)

IN ANY FLUID REMAINING AT REST, EVERY PART OF IT, AT T SAME DEPTH, IS IN AN EQUAL STATE OF COMPRESSION.

For, let the plane EF be parallel to the surface AB. The since the height of the fluid at all the points of EF, is equa therefore the weights standing upon any equal parts of EF a equal; and, therefore, the pressure in all the points of EF equal also.

Cor. 1. *A fluid being at rest, the pressure at any depth is as t depth.*

For this pressure depends on the weight of the superincumbe fluid, and, therefore, is as its height.

Cor. 2. *In any given place, a fluid presses equally in all direc tions.*

For (by Prop. LXXX.) as the pressure in any place acts in directions, it must be the same in all directions. For if it we

Case I.

Let ABCD be a cylinder or prism, then (by Cor. 1. Prop. LXXXII.) the pressure upon a given part of the base (as a square inch) is as the depth. And the pressure upon the whole base is as the number of parts, or inches, contained in it; and, therefore, is as the base and altitude of the fluid.

Case II. (*Fig.* 7. *Pl.* XII.)

Let the heights and bases of the vessels ABC, DEF be equal to those of the cylinder ABCD (*Fig.* 6. *Pl.* XII.); then, since any part of the bases AB or DE is equally pressed, as an equal part of the base CD; (*Fig.* 6. *Pl.* XII.) therefore the whole pressure upon the bases AB or DE is equal to the whole pressure upon the base CD, (*Fig.* 6. *Pl.* XII.); and, therefore, is as the base and perpendicular height.

Cor. 1. *If two vessels* ABC, DEF, *of equal base and height, though never so different in their capacities, be filled with any the same fluid, their bases will sustain an equal quantity of pressure, the same as a cylinder of the same base and height.*

Cor. 2. *The quantity of pressure at any given depth, upon a given surface, is always the same, whether the surface pressed be parallel to the horizon, or perpendicular, or oblique; or whether the fluid continued upwards from the compressed surface, rises perpendicularly into a rectilinear direction, or creeps obliquely through crooked cavities and canals; and whether these passages are regular or irregular, wide or narrow.* And hence,

Cor. 3. (Fig. 8. Pl. XII.) *If* ABDCF *be any vessel containing a fluid; and* BL, ED, HFOK, *and* GC *be perpendicular to the horizon, and* GHAB *the surface of the liquor; and* FL, COD *parallel to* AB; *then the pressure at* L *and* F *is as* BL *or* HF; *at* D, O *and* C, *as* ED; *at* K *as* HK; *and, therefore, the pressure at* L *and* F *is the same; and the pressures at* D, O, C *are equal.*

Cor. 4. (Fig. 8. Pl. XII.) *The pressure is every where directed perpendicularly against the inner surface of the vessel. Therefore at* K *it is directed downward, at* L *sideways, and at* F *upwards.* By Prop. IX.

Cor. 5. (Fig. 9. Pl. XII.) *If two vessels* AB, CD, *communicate with one another by the tube* BC, *and if any liquor be poured into one* AB, *it will rise to the same height in the other* CD, *and will stand at equal heights in both; that is,* AD *will be a horizontal line.*

For if the fluid stand at unequal heights, the pressure in the higher will be greater than in the lower, and cause it to move towards the lower.

Cor. 6. (Figures 10 and 11. Pl. XII.) *If two different flu sustain one another at rest in two vessels* AB, CD, *that commu cate; their height above their place of meeting, will be reciproca as their densities or specific gravities.*

Let the fluids join at C, and take the perpendicular height *e*C, equal to that of AB.; then, if the densities of the flui were equal, they would sustain one another at the equ heights AB, C*e*. Therefore, that the pressure of the other flu may be the same at C, its height must be so much greater as i density is less; that is, CD : C*e* or AB :: density of AB : de sity of CD.

Scholium.—(*Fig*. 12. *Pl*. XII.) The truth of the foregoing pr positions may be easily proved experimentally. Take several tub open at both ends, some straight, some crooked, with their lo ends turned in all directions, and of several sizes, regular and regular. Put these into a vessel of water to any depth, and t water will rise up to the height of the external surface of the w ter in them all. But this is to be understood of such tubes as a sufficiently wide; for in capillary tubes immersed in a vessel water, it rises something above the level, and that to heights rec procally as the diameters. Likewise, if water can rise and suspended at the height B in the capillary tube AB, it will suspended at the same height B, whilst the part of the tube at remains the same, whatever be the figure or wideness of the und part CD. And the ascent and suspension of water is the very sar in vacuo. The same holds for any other fluids, but different flui rise to different heights. But quicksilver, instead of ascending a capillary tube, sinks in it, and has its surface depressed belo the common surface, to depths which are reciprocally as the d ameters of the tubes. But the forces by which fluids are suspen ed in capillary tubes come under no hydrostatic laws.

below as much as a column of the fluid of the same depth. Therefore the pressure of the body at F against the fluid is equal to the pressure of the fluid at F against the body. And, therefore, these two pressures will remain in equilibrio, and the body will be at rest.

CASE II.

If the body is more dense, the pressure against the fluid underneath is greater than that of an equal quantity of the fluid. Therefore the weight of the body will overcome the pressure of the fluid under it, and it will sink. But if the body be lighter, the pressure of the fluid will overcome the weight of the body, and it will rise to the top.

Cor. 1. *If several fluids of different densities be mixed together in the same vessel, the heaviest will get to the lowest place, and the lightest to the top, and those of a mean density to the middle. And, in any bodies whatever the heaviest will be the lowest.*

Cor. 2. *Hence, bodies placed in fluids have a twofold gravity, the one true and absolute, the other apparent or relative. Absolute gravity is the force with which bodies tend downward; by this, all sorts of fluid bodies gravitate in their proper places, and their weight taken together compose the weight of the whole; for the whole is heavy, as may be experienced in vessels full of liquor.*

Relative gravity is the excess of the gravity of the body above that of the fluid. By this kind of gravity fluids do not gravitate in their proper places; that is, they do not preponderate, but hindering one another's descent, remain in their proper places as if they were not heavy.

Cor. 3. *Hence, an irregular body, or one that is heterogeneous, descending in a fluid, or if it move in any direction, and a line be drawn connecting the centre of gravity and centre of magnitude of the body, the body will so dispose itself as to move in that line; and that the centre of gravity will go foremost and the centre of magnitude behind.*

For, there being more matter and less surface near the centre of gravity, that part will be less resisted than near the centre of magnitude; therefore, the centre of magnitude will be more retarded than the centre of gravity, and will be left behind.

Cor. 4. *Hence, no body can be at rest within a fluid, unless it be of the same specific gravity as the fluid.*

SCHOLIUM.—What is here said of bodies of greater density sinking in a fluid, must be understood of such as are solid. For, if a body be hollow, it may swim in a fluid of less density. But, if the hollows or cavities be filled with the fluid, it will then sink. Likewise, bodies of greater specific gravity being reduced to ex-

tremely small particles, may then be suspended in the fl
But the forces by which this is done belong not to any laws
hydrostatics.

PROP. LXXXV. (*Fig.* 13. *Pl.* XII.)

BODIES IMMERSED IN A FLUID, AND SUSPENDED IN IT, LOSE T WEIGHT OF AN EQUAL BULK OF THE FLUID.

For, (by the last Prop.) if the body EF be of the same dens as the fluid, it loses all its weight, and neither endeavours to cend or descend. Therefore, if it be lighter or heavier it o endeavours to ascend or descend with the difference of weights of the body and the fluid, and has, therefore, lost weight of as much of the fluid.

Cor. 1. *The fluid acquires the weight which the body loses.*

For the sum of the weights of the solid and fluid is the san both before and after emersion.

Cor. 2. *All bodies of equal magnitude immersed in a fluid l equal weights, and unequal bodies lose weights proportional to th bulks.*

Cor. 3. *The weights lost by immerging one and the same body different fluids are as the densities of the fluids, or as their spec gravities.*

Cor. 4. *Hence, also, if two bodies of unequal bulks be in equilib in one fluid, they will lose their equilibrium in another fluid of diff ent density.*

SCHOLIUM.—Since a body immersed in a fluid loses so mu weight as that of an equal quantity of the fluid, therefore it ten downwards only with the difference of these weights; and this

fluid underneath, is just the same as the pressure of the fluid in the room of the immersed part; and, therefore, the weight of one is equal to the weight of the other.

Cor. 1. *If the body be homogeneous, the weight or magnitude of the whole floating body is to the weight or magnitude of the part immersed :: as the density or specific gravity of the fluid is to the density or specific gravity of the body.*

For the density of the fluid : density of the body :: weight (of the fluid equal to the immersed part, or the weight) of the whole whole body : weight of the immersed part.

Cor. 2. *If one and the same body float, or swim upon different liquids, the immersed part in each liquid will be reciprocally as their densities; and, therefore, a body will sink deeper in a lighter fluid than in a heavier.*

PROP. LXXXVII. (*Fig.* 14. *Pl.* XII.)

If a floating body AFBE, or system of bodies, be at rest in a fluid, and D be the centre of gravity of the whole body, and C the centre of gravity of the fluid AFB, equal to the immersed part of the body; then, I say, the line CD will be perpendicular to the horizon.

For, as C is the centre of gravity of the fluid AFB, it is the centre of all the forces or weights of the parts of the water in AFB, tending downwards; but because the body is at rest, the same point C is also the centre of all the pressures of the fluid underneath tending upwards, by which the weight of the fluid AFB or of the body AFBE (equal to it by Prop. last) is sustained. Therefore the sum of all the forces tending upwards to C, is equal and contrary to the sum of all the forces tending downwards from D, (by Ax. 11.) because that pressure sustains the body. But the weight of the body tending from D is perpendicular to the horizon; therefore, CD is perpendicular to the horizon.

Cor. If the whole body be as heavy or heavier than water, and be immersed in it, the centre of gravity will be the lowest, and descend the foremost.

PROP. LXXXVIII. (*Fig.* 1. *Pl.* XIII.)

If a fluid, considered without weight, be enclosed in a vessel, and strongly compressed on all sides, every part within it will be in the same compressed state.

For, if any particle was less pressed than another, the greater

pressure would move the fluid towards the less compressed p
till their compression became every where equal; and then
equal pressures would balance one another, and remain at rest.

Cor. 1. *Hence, any soft body as* GHI, *whose parts cannot be c
densed, being immersed in a fluid enclosed in a vessel, and stron
compressed on every side, the body will retain its figure, and suffer
change from the compression of the ambient fluid. And all its pa
will remain at rest among themselves, and in the same compres
state as the fluid.*

Cor. 2. *The motion of any included body as* E, *or of any num
of bodies, will not be at all changed by the compression of the fl
but will remain the same as before.*

For, the compression acting every way alike, can make no al
ration in the motion of bodies.

Cor. 3. *In an inflexible vessel, a fluid will not sustain a stron
pressure on one side than another; but will give way to any excess
pressure in a moment of time, and be reduced to an equality of pr
sure.*

PROP. LXXXIX. (*Fig.* 13. *Pl.* XII.)

If air, or any elastic fluid of small density, be shut
in a close vessel, every part of it will be in the sa
compressed state.

For, let ABCD be a vessel full of enclosed air, then the air,
equal altitudes within the vessel will be in the same state of co
pression; and the compression in the bottom of the vessel c
only exceed that at the top, by the weight of a column of air of t
height of the vessel AC, (by Cor. 1. Prop. LXXXII.) but t

For the spring or elasticity of the air is the force it exert against the force of compression, and, therefore, must be equal to it.

Scholium.—That the air is a heavy, elastic, compressible body, is confirmed by many experiments made for that purpose. Its properties are these.

1. The air has some, though a very small degree of weight, which is so small, that it hardly becomes sensible, but in the weight of the whole atmosphere, or body of air enclosing the earth.

2. The air is an elastic fluid, and capable of being condensed and rarified. And when it is condensed or forced into a less space, its spring, or the force it exerts to unbend itself, is proportional to the force that compresses it. And the space any given quantity takes up, is reciprocally as the compressing force; or its elasticity is as its density.

3. All the air near the earth is in a compressed state, by the weight of the atmosphere or body of the air above, which compresses it. And, from hence, the density of the air grows continually less, the higher it is above the surface of the earth. The weight of the atmosphere at the surface of the earth is, at a medium, about $14\frac{1}{4}$ lb. averd. upon every square inch. But at different times it differs, by reason of winds, hot or cold weather, &c. But the height of the atmosphere is uncertain, by reason it grows continually more rare towards the top till it vanishes. The weight of the atmosphere is equal to the weight of water 11 yards high.

4. The spring or elasticity of the air is increased by heat, and decreased by cold, so that if any quantity of air be enclosed in a vessel, it will have a greater spring or pressure when heated, and will lose part of its spring by cold.

PROP. XC.

To find the specific gravity of bodies.

Case I.

If it be a solid body heavier than water, weigh it exactly, first in air, and then in water, or some fluid whose specific gravity you know; and let

The absolute weight of the body = A,
The weight in water, &c. = B,
The specific gravity of water, &c. = C,
The specific gravity of the body = D,

Then will $D = \frac{A}{A - B}C$, *the specific gravity of the body.*

CASE II.

For a solid body lighter than water. Take any piece of me and tie it to a piece of the light body, so that the compound m sink in water; and putting A, C, D, as in Case I. and

$E =$ *weight of the metal in water.*
$F =$ *weight of the compound in water.*

Then $D = \frac{AC}{A + E - F}$, *the specific gravity of the light body.*

CASE III.

For a fluid. Take a solid body of known specific gravi which will sink in the fluid. And putting the same letters as Case I. *Then will* $C = \frac{A - B}{A}D$, *the specific gravity of the flu*

Or thus:

Take a body that will sink in the fluid, and also in water, an let

$A =$ *absolute weight of the body,*
$B =$ *its weight in water,*
$G =$ *its weight in the fluid,*
$C =$ *specific gravity of water,*
$Z =$ *specific gravity of the fluid required,*

Then $Z = \frac{A - G}{A - B}C.$

But for mercury, or for powders, dust, or small fragments

an equal quantity of water is $A - \overline{F - E}$ or $A + E - F$, therefore, (as in Case I.) it is $A + E - F : A :: C : D$. And the rule is equally true, whether D be lighter or heavier than water.

In Case III., since $D = \frac{A}{A - B} C$, therefore $C = \frac{A - B}{A} D$. Or in the other rule; $A - B =$ weight of as much water, and $A - G =$ weight of as much of the fluid, and the specific gravities being as the weights of equal quantities of the matter; therefore, $A - B : A - G :: C : Z$.

Cor. 1. Hence, if a piece of metal, or any sort of matter is offered to know what sort it is of, find its specific gravity by the rule above, which seek in the following table; and the nearest to it gives the name of the body, or what kind it is of.

Cor. 2. And to find the solid content of a small body heavier than water. Weigh it in air and water, and the difference of the weights reduced to grains, being divided by 256, the quotient is the cubic inches it contains.

For a cubic inch of water weighs 256 grains; or a cubic foot weighs 76 lb. troy, or 62½ lb. averdupoise, which is but 254 grains to an inch.

Cor. 3. Hence, also, the solidity of a body being known, the weight may be found, and the contrary. Thus, put $n = 0.5275$ ounces troy, or 0.5787 ounces averdupoise; and $D =$ specific gravity of the body by the following table; then as $1 : nD ::$ solid content in inches : weight in ounces; and one being given finds the other.

For the weight of a cubic inch of water is .5275 oz. troy, or .5787 oz. averdupoise.

A TABLE OF THE SPECIFIC GRAVITIES OF BODIES.

SOLIDS.

Fine gold	19.640	Cast iron	7.000
Standard gold	18.888	Lead ore	6.200
Lead	11.340	Copper ore	5.167
Fine silver	11.092	Lapis calaminaris	5.000
Standard silver	10.536	Loadstone	4.930
Copper	9.000	Crude antimony	4.000
Copper halfpence	8.915	Diamond	3.517
Fine brass	8.350	White lead	3.160
Cast Brass	8.100	Island crystal	2.720
Steel	7.850	Marble	2.707
Iron	7.644	Pebble stone	2.700
Pewter	7.471	Coral	2.700
Tin	7.320	Jasper	2.666

Rock crystal	2.650	Coal	1.2
Pearl	2.630	Jet	1.2
Glass	2.600	Coral	1.2
Flint	2.570	Ebony	1.1
Onyx stone	2.510	Pitch	1.1
Common stone	2.500	Rosin	1.1
Glauber salt	2.250	Mahogany	1.0
Crystal	2.210	Amber	1.0
Oyster shells	2.092	Brazil wood	1.0
Brick	2.000	Box wood	1.0
Earth	1.984	Common WATER	1.0
Nitre	1.900	Bees' wax	.9
Vitriol	1.880	Butter	.9
Alabaster	1.874	Logwood	.9
Horn	1.840	Ice	.9
Ivory	1.820	Ash (dry)	.8
Brimstone	1.800	Plumtree (dry)	.8
Chalk	1.793	Elm (dry)	.8
Borax	1.717	Oak (dry)	.8
Allum	1.714	Yew	.7
Clay	1.712	Crabtree	.7
Dry bone	1.660	Beech (dry)	.7
Humane calculus	1.542	Walnut-tree (dry)	.6
Sand	1.520	Cedar	.6
Gum arabic	1.400	Fir	.5
Opium	1.350	Cork	.2
Lignum-vitæ	1.327	New-fallen snow	.0

FLUIDS.

Quicksilver	14.000	Serum of human blood	1.0
Oil of vitriol	1.700	Ale	1.0

[redacted]

reason of their different goodness, fineness, compactness, texture, dryness, being more or less free from mixture, &c. And sometimes by a greater degree of heat or cold, which affect all bodies a little, from whence there will arise a sensible difference in different parcels of the same sort of matter, in almost all bodies, whether solid or fluid. Particularly in wood there is great difference, for green wood is far heavier than dry wood, and some green wood will sink in water, as elm.

PROP. XCI. (*Fig.* 2. *Pl.* XIII.)

THE CENTRE OF PRESSURE OF ANY PLANE SUSTAINING A FLUID PRESSING AGAINST IT, IS THE SAME AS THE CENTRE OF PERCUSSION, SUPPOSING THE AXIS OF MOTION TO BE AT THE INTERSECTION OF THIS PLANE WITH THE SURFACE OF THE FLUID.

The centre of pressure is that point against which a force being applied equal and contrary to the whole pressure, it will just sustain it, so as the body pressed on, will incline to neither side.

Let AF be the surface of the water, O the centre of pressure; draw AO, and parallel to AF, draw cbd. Then the pressure against any small part cd, is as cd and the depth of the fluid, that is, as $cd \times Ab$. And the force to turn the plane about O, is $cd \times Ab \times bO$, or $cd \times Ab \times AO - cd \times Ab^2$. And the sum of them all must be equal to 0. Therefore, $AO = \dfrac{\textit{sum of all } cd \times Ab^2}{\textit{sum of all } cd \times Ab}$, and, therefore, (by Prop. LVII.) O is the same as the centre of percussion.

Cor. 1. *The centre of pressure, upon a plane parallel to the horizon, or upon any plane where the pressure is uniform, is the same as the centre of gravity of that plane.*

For the pressure acts upon every part, in the same manner as gravity does.

Cor. 2. *The quantity of pressure upon any plane surface, is equal to that of the same plane, placed parallel to the horizon, at the depth where its centre of gravity is; and the same is true of any number of surfaces taken together.*

For the whole pressure is as the sum of all the $Ab \times cd$; and upon the whole figure placed at the centre of gravity, it is ABC $\times$ distance of the centre of gravity, from A. But, (by Cor. 3. Prop. XLIV.) these products are equal. And the same may be proved for several surfaces, or the surface of any solid, taking the centre of gravity of all these surfaces.

PROP. XCII. (*Fig.* 1. *Pl.* VII.)

To find the centre of equilibrium of a body, or a syst of bodies, immersed in a fluid.

The centre of equilibrium is the same with respect to bod immersed in a fluid, as the centre of gravity is to bodies in fi space; it is a certain point, upon which, if the body or bodies suspended, they will rest in any position.

Let A, B, C be three bodies, or the quantities of matter in the p, q, r their relative gravities in the fluid; $1 =$ absolute gravi Then pA, qB, rC are the weights of A, B, C in the fluid. I G be the centre of equilibrium. Then, by the same reasoning in Prop. XLVII., the sum of the forces of A, B, C is $pA \times$. $+ qB \times Bb + rC \times Cc = Gg \times \overline{pA + qB + rC}$, the sum the forces or weights when situated in G. *Whence* Gg $\frac{Aa \times pA + Bb \times qB + Cc \times rC}{pA + qB + rC}$, *the distance of the centre equilibrium from* ST, *in the fluid*. And if any body, as A, lighter than the fluid, then its relative gravity p will be negati And if any body is situated on the other side of the plane, its d tance from it must be taken negative.

Cor. If the body or bodies be homogeneous, the centre of equ brium is the same as the centre of gravity.

Scholium.—The relative gravity is found thus: Take the sp cific gravity of the fluid from that of the body, and divide the mainder by the specific gravity of the body. And these speci gravities are had by Prop. XC.

[redacted]

Then pA, qB, rC are the weights of the bodies in the fluid. Let G be the centre of equilibrium; and O the centre of oscillation sought. Put $s = A \times SA^2 + B \times SB^2 + C \times SC^2$. Then (by the same reasoning, and construction, as in Prop. LVIII.) the angular velocities which the bodies A, B, C generate in the system are, $\frac{-Se \times pA}{s}$, $\frac{Sn \times qB}{s}$, $\frac{Sd \times rC}{s}$; and the whole angular velocity generated by them all, is $\frac{-Se \times pA + Sn \times qB + Sd \times rC}{s}$.

Likewise the angular velocity which the particle P, situated in O, generates in the system, is $\frac{Sr \times P}{P \times SO^2}$ or $\frac{Sr}{SO^2}$. But their vibrations are performed alike; therefore their angular velocities must be equal. That is $\frac{-Se \times pA + Sn \times qB + Sd \times rC}{s}$ $= \frac{Sr}{SO^2} = \frac{Sg}{SG \times SO}$. Whence $SO = \frac{Sg}{SG} \times \frac{s}{-Se \times pA + Sn \times qB + Sd \times rC}$. But (by Prop. XCII.) $-Se \times pA + Sn \times qB + Sd \times rC = Sg \times \overline{pA + qB + rC}$; therefore $SO = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{SG \times \overline{pA + qB + rC}}$, *the length of an isocronal pendulum, out of the fluid.*

Cor. 1. (Fig. 6. Pl.VIII.) *Hence, if the bodies are homogeneous, then* $p = q = r$; *and* $SO = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{p \times SG \times \overline{A + B + C}}$.

Cor. 2. *The system makes an exceeding small vibration in the fluid, in the same time that a simple pendulum, whose length is* $\frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{SG \times \overline{pA + qB + rC}}$, *makes a vibration in vacuo.*

For the velocity of the system being very small, the resistance is inconsiderable.

Cor. 3. *Hence, if* SA *be the length of a simple pendulum* (A), *vibrating in a fluid; then* $\frac{SA}{p}$ *is the length of an isocronal pendulum in vacuo.*

For in a single body, $SO = \frac{A \times SA^2}{SA \times pA}$ or $\frac{SA}{p}$.

Cor. 4. *And if p be negative, or the pendulum specifically lighter*

than the fluid; the pendulum will turn upside down, and vibrate 1 *wards in the fluid. And the length of an isocronal pendulum out the fluid, will be* $\frac{SA}{p}$, *as before.*

SCHOLIUM.—The centre of percussion is the same in a fluid out of it. For there is nothing concerned in that, but the quan ties of matter and velocities; both of which are the same in 1 fluid, as out of the fluid.

The relative gravities p, q, r are found by the Scholium of 1 last Prop.

PROP. XCIV. (*Fig.* 3. *Pl.* XIII.)

IF A FLUID RUNS THROUGH ANY TUBE, PIPE, OR CANAL, A ALWAYS FILLS IT, ITS VELOCITY IN ANY PLACE WILL RECIPROCALLY AS THE AREA OF THE SECTION IN THAT PLACI

Let AB, CD be two sections at A and C; and let the qu tity of the fluid ABDC, in a very small time, be translated i the part *abdc* of the pipe. Draw Pp, Qq perpendicular to A CD, or parallel to the motion of the fluid; then Pp, Qq bei indefinitely small, will be the velocities of the fluid at P and or the spaces described in that small part of time. Then, l cause the pipe is always full, the quantity ABDC = *abdc*. T from both, the part *ab*DC, which is common; and there mains AB*ba* = CD*dc*, that is the section APB × Pp = secti CQD × Qq.

Cor. 1. *The quantity of motion of the fluid in the pipe* AD *any section* CD, *is equal to the motion of a cylinder of that fl whose base is* CD, *and length the same with the pipe from the ginning to that section, and velocity that of the fluid at* CD.

quantity of water discharged at R, *in a second, or some given time, will be as the section at* R.

For if the force is given, the motion generated in a given time will be given; and this motion, being as the quantity of water × velocity at R; therefore the quantity forced through R, will be reciprocally as the velocity, or directly as the section at R, by this Prop.

Cor. 3. *The velocity and quantity of motion, is the same very near in crooked tubes as in straight ones; and in pipes divided into several branches, taking the section of all the branches as the section of one tube.*

PROP. XCV. (*Fig.* 4. *Pl.* XIII.)

IN ANY PIPE WHOSE SECTION IS ABCD, THE STRESS OR FORCE TO SPLIT ANY PART OF THE PIPE AT B, IS EQUAL TO HALF THE PRESSURE OF THE FLUID UPON THE PLANE BD, DRAWN PERPENDICULAR TO THE CURVE IN B, AND OF THE SAME LENGTH AS THAT PART OF THE PIPE.

Let E*e* be any small part of the surface; draw EO perpendicular to the curve E*e*; draw E*n*, *en* perpendicular to BO, and *er* perpendicular to EN. And let OE represent the pressure of a particle of the fluid, then EO × E*e* = pressure upon E*e*. The force OE may be divided into the two ON, NE; of which ON tends only to split the tube at A, but that in direction NE is the force to separate the parts at B. Therefore EN × E*e* is the stress at B. But the triangles E*er*, EON are similar, and E*e* × EN = EO × *er*, or EO × N*n*. Therefore the part of the pressure on E*e*, in direction NE, from whence the stress at B arises, is = EO × N*n*, that is = to the pressure upon the plane N*n*. Consequently the stress arising from the pressure on BE is = pressure on BN, and from the pressure on BA is = pressure on BO. And the stress at D by the pressure on AD, is equal to the pressure on OD.

Also, we suppose the same forces acting in the semi-circle BCD: but these serve only to keep the forces acting upon BAD in equilibrio.

Cor. 1. *The stress on any part of a pipe full of water, is as the diameter of the pipe, and the perpendicular height of the water above that place; and, consequently, the thickness of the metal ought to be in that ratio.*

Cor. 2. *In any concave surface, cask, or vessel, generated by revolving round an axis, and filled with a fluid, the stress as to splitting is equal to half the pressure upon the plane passing through its axis. And the stress on both sides at* B *and* D, *equal to the whole pressure on that plane.*

Cor. 3. *Hence, the internal pressure on any length of the pipe to the stress it suffers as to splitting* :: *as* 2 × 3.1416, *to* 1.

Cor. 4. *Hence it follows, that the stress, arising from any pr sure, upon any part, to split it longitudinally, transversely, or in i direction, is equal to the pressure upon a plane, drawn perpendicu to the line of direction. Thus the stress arising from the press on* BE *is* = *pressure on* BN.

Cor. 5. *And if the pipe be flexible, it will, by the pressure, be into a cylindrical form, or such that the section is a circle.*

For, if BD be greater than AC, there will be a greater pr sure in direction OA than in direction OB; and the grea pressure will drive out the sides A and C, till AC become eq to BD, and ABCD be a circle. Besides, a circle is more ca cious than any other figure. And if a pipe be not flexible, the pressure of the fluid will always endeavour to put it int circular figure.

Cor. 6. *And if an elastic compressed fluid be inclosed in a ves flexible, and capable of being distended every way, it will form self into a sphere, for the same reason.*

PROP. XCVI. (*Fig.* 5. *Pl.* XIII.)

If a close flexible tube AB full of air, be immers wholly or in part in the water CDEF, the force split it, in any place O, is proportional to AO, height from A.

For the air compressed at A is in equilibrio with the exter pressure of the water. At B and O the air is in the same co pressed state as in A; but the external pressure at B, is less

PROP. XCVII. (*Fig.* 6. *Pl.* XIII.)

THE QUANTITY OF A FLUID FLOWING IN ANY TIME THROUGH A HOLE IN THE BOTTOM OR SIDE OF A VESSEL, ALWAYS KEPT FULL, IS EQUAL TO A CYLINDER WHOSE BASE IS THE AREA OF THE HOLE, AND ITS LENGTH THE SPACE A BODY WILL DESCRIBE IN THAT TIME, WITH THE VELOCITY ACQUIRED BY FALLING THROUGH HALF THE HEIGHT OF THE LIQUOR ABOVE THE HOLE.

Let ADB be a vessel of water, B the hole, and take BC = BD the height of the water; and let the cylinder of water BC fall by its weight through half DB, and it will by that fall acquire such a motion, as to pass through DB or BC uniformly in the same time, by Cor. 3. Prop. XIV. But (by Prop. LXXXIII. and Cor. 2.) the water in the orifice B is pressed with the weight of a column of water, whose base is B and height BD or BC; therefore this pressure is equal to the cylinder BC. But equal forces generate equal motions; therefore the pressure at B will generate the same motion in the spouting water, as was generated by the weight of the cylinder of water BC. Therefore, in the time of falling through half DB, a cylinder of water will spout out, whose length (or the space passed uniformly over) is BC or BD. And in the same time repeated, another equal cylinder BC will flow out, and in a third part of time, a third, &c. Therefore the length of the whole cylinder run out, will be proportional to the time, and, consequently, the velocity of the water at B is uniform. Therefore, in any time, the length of a cylinder of water spouting out, will be equal to the length described in that time, with the velocity acquired by falling through half DB.

Cor. 1. *Hence, in the time of falling through half* DB, *a quantity of the fluid runs out, equal to a cylinder whose base is the hole; and length, the height of the fluid above the hole.*

Cor. 2. *The velocity in the hole B is uniform, and is equal to that a heavy body acquires by falling through half* DB.

Cor. 3. (Fig. 7. Pl. XIII.) *But at a small distance without the hole, the stream is contracted into a less diameter, and its velocity increased; so that if a fluid spout through a hole made in a thin plate of metal, it acquires a velocity nearly equal to that, which a heavy body acquires by falling the whole height of the stagnant fluid above the hole.*

For since the fluid converges from all sides towards the centre of the hole BF; and all the particles endeavouring to go on in right lines, but meeting one another at the hole, they will compress one another. And this compression being every where directed to the axis of the spouting cylinder, the parts of the

fluid will endeavour to converge to a point, by which means t fluid will form itself into a sort of a conical figure, at some distar from the hole, as BEGF. By this lateral compression, the pa cles near the sides of the hole are made to describe curve lines HE, KG; and, by the direct compression, the fluid from the h is accelerated outwards at EG; and thus the stream will be co tracted at E, in the ratio of about $\sqrt{2}$ to 1, and the velocity creased in the same ratio.

It must be observed, however, that the particles of the flu do not always move right forward; but near the edge of t hole, often in spiral lines. For no body can instantly chan its course in an angle, but must do it gradually, in some cu line.

Cor. 4. *The fluid at the same depth, spouts out nearly with same velocity, upwards, downwards, sideways, or in any directi And if it spout vertically, ascends nearly to the upper surface of fluid.*

Cor. 5. *The velocities of the fluid, spouting out at different dept are as the square roots of the depths.*

For the velocities of falling bodies are as the square roots of t heights.

Cor. 6. *Hence, if* $s = 16\frac{1}{12}$ *feet,* D = *depth of the vessel to centre of the hole,* F = *area of the hole, all in feet,* t = *time seconds; then the quantity of water running out in the time* t, *this Prop. will be* $tF\sqrt{2Ds}$ *feet, or* $6.128tF\sqrt{2Ds}$ *ale gallons.*

Scholium.—There are several irregularities in spouting flui arising from the resistance of the air, the friction of the tubes, t bigness and shape of the vessel, or of the hole, &c. A flu spouts farthest through a thin plate; if it spout through a tube [redacted]

PROP. XCVIII. (*Fig.* 8. *Pl.* XIII.)

IF A NOTCH OR SLIT, *fghi*, IN FORM OF A PARALLELOGRAM, BE CUT OUT OF THE SIDE OF A VESSEL FULL OF WATER, ADE, THE QUANTITY OF WATER FLOWING OUT OF IT, WILL BE TWO-THIRDS THE QUANTITY FLOWING OUT OF AN EQUAL ORIFICE, PLACED AT THE WHOLE DEPTH *gi*, OR AT THE BASE *hi*, IN THE SAME TIME; THE VESSEL BEING SUPPOSED TO BE ALWAYS KEPT FULL.

For, draw the parabola *goh*, whose axis is *gi*, and base *hi*, and ordinate *ro;* then, since the velocity of the fluid at any place *r*, is as $\sqrt{gr}$, (by Cor. 5. of the last Prop.) that is, (by the nature of the parabola) as the ordinate *ro*; therefore *ro* will represent the quantity discharged at the depth or section *rn*. Also *hi* will represent the quantity discharged at the depth or base *hi*. Consequently the sum of all the ordinates *ro*, or the area of the parabola, will represent the quantity discharged at all the places *rn*. And the sum of all the lines *hi* or *rn*, or the area of the parallelogram *fghi*, will represent the quantity discharged by all the sections *rn*, placed as low as the base *hi*. But the parabola is to the parallelogram, as $\frac{2}{3}$ to 1.

Cor. 1. *Let* $s = 16\frac{1}{12}$ *feet.* D $=$ *gi, the depth of the slit.* F $=$ *area of the slit, fhig. Then the quantity flowing out in any time or number of seconds t, is* $= \frac{2}{3}\, t\mathrm{F}\, \sqrt{2\mathrm{D}s}$.

This follows from Cor. 6. of the last Prop.

Cor. 2. *The quantity of fluid discharged through the hole rnhi, is to the quantity which would be discharged through an equal hole placed as low as hi, as the parabolic segment rohi, to the rectangle rnhi.*

This appears from the reasoning in this proposition.

SECTION TENTH.

THE RESISTANCE OF FLUIDS, THEIR FORCES AND ACTIONS UPON BODIES; THE MOTION OF SHIPS, AND POSITION OF THEIR SAILS.

PROP. XCIX.

A BODY DESCENDING IN A FLUID, ADDS A QUANTITY OF WEIGHT TO THE FLUID, EQUAL TO THE RESISTANCE IT MEETS WITH IN FALLING.

For the resistance is equal to the gravity lost by the body. And, because action and re-action are equal and contrary, the gravity lost by the body is equal to that gained by the fluid. Therefore, the resistance is equal to the gravity gained by the fluid.

Cor. 1. *If a body ascends in a fluid, it diminishes the gravity of the fluid, by a quantity equal to the resistance it meets with.*

Cor. 2. *This increase of weight arising from the resistance, is over and above the additional weight mentioned in Cor.* 1. *Prop.* LXXXV.

Cor. 3. *If a heterogeneous body descend in a fluid; it will endeavour to move with its centre of gravity foremost, leaving the centre of gravity of as much of the fluid, behind.*

For the side towards the centre of gravity contains more matter, and will more easily make its way through the fluid, and be less retarded in it.

PROP. C.

IF ANY BODY MOVES THROUGH A FLUID, THE RESISTANCE IT MEETS WITH, IS AS THE SQUARE OF ITS VELOCITY.

For the resistance is as the number of particles struck, and the velocity with which one particle is struck. But the number of particles of the fluid which are struck in any time, is as the velocity of the body. Therefore the whole resistance is as the square of the velocity.

Cor. 1. *The resistances of similar bodies moving in any fluids, are as the squares of their diameters, the squares of their velocities, and the densities of the fluids.*

For the number of particles struck with the same velocity, are as the squares of the diameters, and the densities of the fluids.

Cor. 2. *If two bodies* A, B, *with the same velocity, meet with the resistances* p *and* q, *their velocities will be* $\frac{1}{\sqrt{p}}$ *and* $\frac{1}{\sqrt{q}}$, *when they meet with equal resistances.*

For let b be the common velocity, then $p : bb :: q : \frac{bbq}{p}$, and $b\sqrt{\frac{q}{p}}$ = velocity of A to have the resistance q; and since b = velocity of B to have the same resistance q; therefore velocity A : velocity B $:: b\sqrt{\frac{q}{p}}\ b :: \frac{1}{\sqrt{p}} : \frac{1}{\sqrt{q}}$.

PROP. CI.

THE CENTRE OF RESISTANCE OF ANY PLANE MOVING DIRECTLY FORWARD IN A FLUID, IS THE SAME AS THE CENTRE OF GRAVITY.

The centre of resistance is that point, to which, if a contrary force be applied, it shall just sustain the resistance.

Now, the resistance is equal upon all equal parts of the plane, and, therefore, the resistance acts upon the plane after the same manner, and with the same force, as gravity does; therefore the centre of both the resistance and gravity must be the same.

Cor. 1. *In any body moving through a fluid, the line of direction of its motion will pass through the centre of resistance, and centre of gravity of the body.*

For, if it do not, the forces arising from the weight and resist-

ance, will not balance one another, which will cause the body librate or oscillate in the fluid; till, by degrees, the situation these two centres will fall into the line of their motion.

Cor. 2. *And, for the same reason, if a globe, moving in a fl oscillates or turns round its axis; that side, which in oscillat moves against the fluid, suffers a greater force or resistance; therefore the body is driven from that part, and made to recede fr that side, and deflect to the other side; and, perhaps, describe a cu line in the fluid.*

PROP. CII. (*Fig.* 9. *Pl.* XIII.)

If a non-tenacious fluid, such as the wind, &c., mo against the sail SA, or any plane surface, in directi WS, it shall urge it in a direction WA perpendicul to that surface, with a force, which is as the square the velocity, the square of the sine of the angle of cidence, the magnitude of the sail, and the density the fluid.

Draw WA, and AC perpendicular to SA, and SW; and force of the fluid upon SA, is as the force of one particle, and number of them falling on SA.

But (by Cor. 1. Prop. IX.) the force of one particle is as velocity × S. incidence WSA.

And the number of them (supposing the density to be given) as their velocity × CA, or (supposing the sail SA given) as velocity × S.WSA.

Therefore, the force of the fluid upon the sail SA, is as square of the velocity, and the square of the sine of WSA.

Increase the density of the fluid, and the magnitude of the sa in any ratio, and it is evident the force of the fluid against

Cor. 3. *The force of a fluid in direction* WS, *to move the sail or body* SA *in the same direction* WS; *is* (*ceteris paribus*) *as the cube of the sine of incidence* WSA.

For then WSB will be one continued straight line.

Cor. 4. *But the force of a given stream of a fluid, against any sail* SA, *to move it perpendicular to its surface, is simply as the* S. *angle of incidence; but to move it in the same direction with itself, as the square of the* S. *incidence, all things else remaining the same.*

This follows from Cor. 1. Prop. IX. and Cor. 2. of this.

Scholium. I.—If the angle WSB be given, the fluid will have the greatest force possible against the sail, to move it in direction SB; when its position is such, that the sine of the difference of the angles $\overline{\text{WSA} - \text{ASB}}$, may be $\frac{1}{3}$ the sine of the sum WSB.

Scholium II.—(*Fig.* 9. *Pl.* XIII.) If the fluid be tenacious it will urge the body in the same direction with itself, and with a force which is as the sine of incidence; or universally, as the sine of incidence, the square of the velocity, the magnitude of the sail, and density of the fluid.

For, by reason of the tenacity of the fluid, the sail is acted on by both the forces WA, AS, which are equivalent to WS.

PROP. CIII. (*Fig.* 11. *Pl.* XIII.)

If a very thin and light body SA, plain on both sides, be placed in a very dense fluid, which moves in direction WS, and the body can make little or no way through the fluid, but only in the direction of its length SA. And if the body be obliged to move parallel to itself in a given direction SD; I say, the body will be so moved in the fluid, that its absolute velocity will be $= \frac{\text{S.WSA}}{\text{S.ASD}} \times$ velocity of the fluid.

Draw DT parallel to AS, and produce WS to T. Then, whilst a part of the fluid moves from S to T, the body will be moved into the line TD; and since SD is the direction of its motion, the point S will be found in D. And, therefore, the velocities of the fluid, and of the body, will be as ST to SD; that is, as S.TDS or DSA, to S.STD or WSA.

Cor. 1. *If* WS *the direction of the fluid, is perpendicular to* SD *the direction of the body, then the velocity of the body* SA *will be* = *tangent,* WSA × *velocity of the fluid.*

For $\frac{\text{S.WSA}}{\text{S.ASD}} = \frac{\text{S.WSA}}{\textit{cos}.\text{WSA}}$ = tan. WSA, radius being =

Cor. 2. *And hence, if the body* SA *continually turn round an* ɩ *parallel to* WS, *then the velocity of* SA, *in direction perpendicula*ı WS, *will be as the tangent* WSA × *velocity of the fluid.*

For SA, in this case, will always have the same position to direction of the fluid, as before.

Cor. 3. (Fig. 12. Pl. XIII.) *If a very thin body* SA *be oblig to move parallel to itself, through a very dense fluid at rest; and it be drawn with a given velocity in direction always parallel to* S' *its absolute velocity in the fluid, will be reciprocally as the cos* WSA, *and in direction* SA.

Draw AC perpendicular to SW. Then, by reason of the de sity and resistance of the fluid, the body will not be able to mc laterally, but only in direction SA. But the velocities of 1 point S in directions SW, SA are as SC to SA, or as cosine CS to radius. Therefore velocity in direction SA = $\frac{\textit{rad.}}{\textit{cos.}\ \text{CSA}}$ × v locity in direction SW.

PROP. CIV. (*Fig.* 13. *Pl.* XIII.)

If a plane surface SA, moving parallel to itself, wi' velocity and direction SD, be acted upon by a flu moving with velocity and direction WS; and if W be drawn parallel and equal to SD, and FS drawn; say, the fluid acts upon the plane in the angle FS. with the relative velocity FS.

SD, with the greatest force, when it has such a position, that the sine of the difference of the angles, $\overline{FSA - ASD}$, may be ½ the sine of the angle FSD.

And when the angle WSA is given, the fluid will have the greatest force upon the sail SA, to move it in direction SD, when the S. angle ASD is equal to $\frac{WS}{WF} \times$ ½ the S. of WSA.

PROP. CV. (*Fig.* 14. *Pl.* XIII.)

Let SA be the sail of a ship, SD the position of her keel; SK, DK perpendicular to SA, SD; and if DE, DS be as the resistances the ship has ahead and aside, with equal velocities; and if DC is a mean proportional between DE and DK; then, SC will be the way of the ship nearly.

For, let SK perpendicular to SA represent the force of the wind upon the sail; the force SK is resolved into the forces SD, DK; SD is the direct force, and DK the force producing her lee-way. By Prop. C. her resistance ahead with velocity SD : resistance ahead with velocity DE :: $SD^2 : DE^2$, and resistance ahead with velocity DE : resistance aside with velocity DE :: DE : SD, and resistance aside with velocity DE : resistance aside with velocity DC :: $DE^2 : DC^2$.

Therefore, ex æquo, resistance ahead with velocity SD : resistance aside with velocity DC :: $SD^2 \times DE^3 : DE^2 \times SD \times DC^2$ :: $SD \times DE : DC^2$.

But the resistances are as the forces producing them, therefore SD : DK :: $SD \times DE : DC^2 = DE \times DK$.

Cor. 1. *Let r = ship's resistance ahead,* R = *ship's resistance aside, with the same velocity. Then* R : *r* :: *radius* × *cotangent* ASD : *tangent square of* DSC, *the leeway.*

For, let radius = 1. tangent DSK = *t*. Then 1 : *t* :: SD : DK = $t \times$ SD; and SD : DC or $\sqrt{DE \times DK}$ or $\sqrt{t \times SD \times DE}$ :: 1 : tangent DSC = $\sqrt{\frac{t \times DE}{SD}}$

Cor. 2. *Hence, the tangent of the lee-way, in the same ship, is as the square root of the cotangent of the angle* ASD, *which the sail makes with the keel. Therefore, if the lee-way be known for any position of the sail, it will be known for all.*

Scholium.—The lee-way of a ship is generally something more than is here assigned; because her hull and rigging will make her drive a little to the leeward, directly from the wind.

PROP. CVI. (*Fig.* 1. *Pl.* XIV.)

IF THE WIND WITH A GIVEN VELOCITY, IN DIRECTION WS, FA ON THE SAIL SA OF A SHIP, MAKING LITTLE OR NO LEE-WAY; WILL URGE THE SHIP IN DIRECTION OF THE KEEL SD, WITH FORCE, WHICH IS AS $S.\overline{WSA}^2 \times$ S.ASD.

Draw SC perpendicular to SA, and CD to SD. And (by Pr CII.) the force acting upon the sail in direction SC, is as t square of the sine of WSA. But the forces in directions SC a SD are as SC to SD, or as radius 1 to the sine of SCD or AS Therefore the force in direction SD $=$ S.ASD $\times$ force in directi SC $=$ S.ASD $\times$ $S.^2$WSA.

Cor. 1. *The force acting in direction* DC *perpendicular to keel, is as* $\overline{S.WSA^2}$ $\times$ *cosine* ASD.

Cor. 2. *The force in direction* SD *will be universally as* S.WS $\times$ S.ASD, *and the square of the velocity of the wind, and mag tude of the sail.*

Cor. 3. *The velocity of the ship in direction* SD, *is as* S.WSA $\sqrt{S.ASD}$ $\times$ *velocity of the wind.*

For the square of the velocity of the ship in any direction, is the resistance in the water, or (its equal) the force of the wi upon the sail in that direction; that is (by Cor. 2.) as $\overline{S.WS}$ $\times$ S. ASD, and the square of the velocity of the wind. T density and sail being given.

Cor. 4. *Let the angle* WSA *be given; and if* SDC *be a se circle described on any given line* SC; *then the force in any di*

PROP. CVII.

IF A STREAM OF ANY FLUID, AS WATER, FLOWS DIRECTLY AGAINST ANY PLANE SURFACE, ITS FORCE AGAINST THAT PLANE IS EQUAL TO THE WEIGHT OF A COLUMN OF THE FLUID, WHOSE BASE IS THE SECTION OF THE STREAM; AND ITS LENGTH TWICE THE HEIGHT DESCENDED BY A FALLING BODY, TO ACQUIRE THE VELOCITY OF THE FLUID.

Let $s = 16\frac{1}{12}$ feet, the height descended by a falling body in one second.

$v =$ velocity of the fluid, or the space it describes in one second.

$B =$ base of the cylinder or column of water.

Then $2s =$ velocity generated by gravity in falling through s.

Therefore, (by Cor. 1. Prop. XIV.)

$4ss : s :: vv : \frac{vv}{4s} =$ height fallen to gain the velocity v. And $\frac{vv}{2s} =$ twice that height. Also $\frac{vv}{2s}B =$ a cylinder of twice that height.

Now, the motion which the cylinder's weight will generate in 1 second, is $2s \times \frac{vv}{2s}B$, or vvB; the motion being as the body $\times$ by the velocity. And the force of the fluid against the plane, is equal to the resistance of the plane. And the motion destroyed in 1 second by the resistance of the plane, is $v \times Bv$ or vvB; which was also the motion generated by the weight of the cylinder $\frac{vv}{2s}B$, in the same time. But equal forces in the same time generate or destroy equal motions. Therefore, the weight of the cylinder $\frac{vv}{2s}B =$ force of the fluid against the plane.

Cor. 1. *The force of a stream of water against any plane, is equal to the weight of a column of water, whose base is the section of the stream, and height* $\frac{vv}{2s}$; *or the height of the water, if it flow through a hole at the bottom of a reservoir.*

It follows from this Prop. and Prop. XCVII. Cor. 2.

Cor. 2. *Moreover, if any part of the water lie upon the plane, the force will be augmented by the weight of so much water.*

Cor. 3. *The forces of different streams of water against a plane, are as their sections and the squares of the velocities.*

Cor. 4. *If the plane be also in motion, the relative velocity the water against the plane, must be taken instead of the absol velocity.*

Scholium.—A cubic foot of water contains 6.128 ale gallo and weighs $62\frac{1}{2}$ lb. avoirdupois.

The density of water to that of air at a mean is as 850 to Sir Isaac Newton found the resistance of water to that of air (the oscillations of a pendulum) to be as 446 to 1, or between 4 and 478 to 1, or at a mean as 460 to 1. See Scholium, Pr XXXI. Book II. Principia.

LEMMA. (*Fig.* 2. *Pl.* XIV.)

If the quadrant EDA revolve about the radius CA, a describe an hemisphere; and from all the points of surface, as D, d, perpendiculars DB, db, be let fall up the base EC; I say, the sum of all the perpendicul BD, in the surface EDA, is to the sum of as many ra CD, as 1 to 2.

For take Dd infinitely small, and complete the square CAH and draw CH, also draw DFGR, $dfgr$ parallel to EC. By similar triangles CDF, Dnd; DF $\times$ Dd $=$ CD $\times$ nd; also I $=$ CF $=$ FG.

The surface of the spherical annulus DdfF is 3.1416 $\times$ 2I $\times$ Dd or 3.1416 $\times$ 2CD $\times$ nd, that is (because 3.1416 $\times$ 2CD given) as nd or Ff. And the sum of all the BD's in the annu is as BD $\times$ by its surface, that is as BD $\times$ Ff, or FC $\times$

[redacted]

der AB, whilst it moves forward, pushes against the several parts of the fluid, and drives them successively before it, in direction of its axis, from the several places through which it passes; so that, in equal times, it moves equal quantities of the fluid, and communicates to them the same velocity that it moves with; it is evident that the cylinder, after it has moved uniformly forward, the length of its axis has removed the cylinder of the fluid FBCG equal to itself ASBF, and has communicated a motion to it equal to its own. And since action and re-action are equal, the force that uniformly generated this motion, is equal to the uniform resistance the cylinder suffered in the mean time. And, therefore, the resistance is equal to the force by which its own motion can be generated, in the time it describes its length.

All this is true, upon supposition that every particle of the fluid is driven directly forward, with the same velocity the cylinder has. But since, in reality, the motion generated in the fluid is not directly forward, but (by Prop. LXXX.) diverges on all sides, and in all manner of directions CD, C*d*, &c. Therefore, if the quadrant AE (*Fig.* 2. *Pl.* XIV.) be divided into an infinite number of equal parts, D*d*, and to all the points D, *d*, the radii CD, C*d*, &c. be drawn, representing the motions of the particles in all directions; and from any one D, the perpendicular DB be drawn on EC; then the motion CD (=CA) is resolved into the two motions CB, BD; of which CB does not affect the cylinder; and the direct motion of the particle D is only BD, which is less than CD. Therefore the force to generate this motion, and, consequently, the resistance of a particle at D (equal to this force) must be less than before in proportion of CD to BD. Therefore the former resistance, when all the particles are driven directly forward, to the resistance when they diverge on all sides, is as the sum of all the radii CD, drawn to every point of the surface of a sphere, to the sum of all the corresponding sines BD; that is (by the Lemma) as 2 to 1. Therefore the resistance the cylinder meets with now, is but half the former resistance. Consequently, since the force to generate any motion is reciprocally as the time, the resistance will be equal to the force that can generate its motion, in the time that it describes twice its length.

Cor. 1. (Fig. 3. Pl. XIV.) *If a cylinder moves in direction of its axis, in a fluid of the same density, and with the velocity acquired by falling in vacuo, from a height equal to its length, it meets with a resistance equal to its weight.*

For the force that generates its motion, in the time of its moving twice its length (or of falling through once its length,) is its gravity.

Cor. 2. *If a cylinder moves uniformly forward in any fluid, resistance is to the force by which its whole motion may be generat in the time of moving twice its length, as the density of the fluid the density of the cylinder.*

For, if the density of the fluid be increased in any ratio, t resistance will be increased in the same ratio.

Cor. 3. *The resistance of a cylinder moving in any fluid, is eq to the weight of a cylinder of that fluid, of the same base, and length equal to the height a body falls in vacuo, to acquire its ve city.* By Cor. 1.

Cor. 4. *Let* $s = 16\frac{1}{12}$ *feet,* B = *base of the cylinder,* v = *velocity, or the space described in one second. Then its resistanc* = *weight of the cylinder* $\frac{vv}{4s}$ *B, of the fluid.*

Scholium.—If the cylinder move in a fluid inclosed in a v sel, instead of the absolute velocity, the relative velocity in t fluid must be taken, in order to find the resistance. And l sides, if the vessel be narrow, the resistance will be increased m or less, because the fluid, being confined by the vessel, cann then diverge in all directions. And if it be so confined, that cannot diverge at all, but is obliged to move directly forwa the resistance then will be double, which is the greatest it c possibly have, or the utmost limit of its resistance. Also, comparing the last Cor. with Cor. 1. Prop. CVII. it appe that the force of a cylinder of water against a plane, is doul the resistance an equal cylinder would meet with, moving water with the same velocity. And this will not appear stran when we consider, that, in the first case, the whole motion of t water is destroyed by the resistance of the plane; but, in t latter case, the water diverges every way from the moving cyl

The sum of all the rr : to the sum of all the yy, in the annulus Bb, is as $2cx \times rr$: to $2cx \times yy$ :: $rrx : yyx$.

And sum of all the rr : sum of all the yy, in the hemisphere, is as sum $rrx \times Bb$: sum $yyx \times Bb$, on the base.

Or as sum of $rrx \times Bb$: sum of $\overline{rr - xx} \times x \times Bb$.

Or as the sum of $rrx \times Bb$: sum $rrx \times Bb$ — sum $x \times Bb$, in the base.

* But the sum of all the $x \times Bb = \overline{1 + 2+3+4, \&c. \text{ to } r} \times 1 = \frac{1}{2} rr$.

* And sum of all the $rrx \times Bb = \frac{1}{2} r^4$. putting $Bb = 1$.

* Also the sum of all the $x^3 \times Bb = \overline{1^3+2^3+3^3+4^3, \&c. \text{ to } r^3} \times 1 = \frac{1}{4} r^4$.

Therefore the sum of all the rr : sum of all the yy, in the hemisphere, is as $\frac{1}{2} r^4 : \frac{1}{2} r^4 - \frac{1}{4} r^4$, or as $\frac{1}{2}$ to $\frac{1}{4}$, that is as 2 to 1.

PROP. CIX. (*Fig* 4. *Pl.* XIV.)

IF A GLOBE MOVE UNIFORMLY FORWARD IN A COMPRESSED INFINITE FLUID, ITS RESISTANCE IS TO THE FORCE BY WHICH ITS WHOLE MOTION MAY BE DESTROYED OR GENERATED, IN THE TIME OF DESCRIBING $\frac{8}{3}$ PARTS OF ITS DIAMETER, AS THE DENSITY OF THE FLUID, TO THE DENSITY OF THE GLOBE, VERY NEARLY.

Let the globe move in the direction CA. Draw the tangent DH, and BDG parallel to CA, and GH perpendicular to DH; and let GD be the force of a particle of the fluid against the base B, in direction GD: then GH will be the force acting against D, in direction DC. And this force is to the force in direction GD as DC to DB. Whence, the force against B, is to the force against D, in direction GD, in a ratio compounded of GD to GH, and DC to DB; that is, as DC^2 to DB^2. Therefore, the force of all the particles of the fluid against the base, is to their force against the convex surface, as the sum of all the DC^2, to the sum of all the DB^2 on the base; that is, (by the Lemma) as 2 to 1. Therefore, the resistance of the surface of the sphere, is but half the resistance of the base, or of a cylinder of the same diameter.

Now, the globe is to the circumscribing cylinder as 2 to 3; and half of that force (which can destroy all the motion of this cylinder, whilst it describes 2 diameters) will destroy all its motion, whilst it describes 4 diameters. And, therefore, the same force that destroys the cylinder's motion, in the time of moving 4 diameters, will destroy the globe's motion whilst it moves $\frac{2}{3}$ of this

* See *Ward's* Math. Guide, Part V.

length, or ⅔ of its own diameter. But (by Cor. 2. Prop. CVII half the resistance of the cylinder, that is, the resistance of t globe, is to this force, as the density of the fluid to the dens of the cylinder or globe.

Cor. 1. *The resistance of a sphere is but half the resistance o cylinder, of the same diameter.*

Cor. 2. *The resistance of a globe moving in any fluid, is equal the weight of a cylinder of that fluid, of the same diameter; and length equal to half the height, through which a body falls in vac to acquire the velocity of the globe.* By Cor. 3. Prop. CVIII.

Therefore, if $s = 16\frac{1}{12}$ *feet,* $v =$ *velocity of a globe, or the sp it moves in* 1 *second,* $D =$ *its diameter; then its resistance is eq to the weight of a cylinder of the fluid, of the same diameter* D, *a its length* $\frac{vv}{8s}$. *And if* $v = 4\sqrt{\frac{Ds}{3}}$, *its resistance is equal to t weight of an equal globe of the fluid.*

Cor. 3. *The greatest velocity a globe can obtain, by descending a fluid, is that which it would acquire by falling in vacuo, throu a space that is to* $\frac{4}{3}$ *the diameter; as the difference between t density of the globe and the density of the fluid, is to the density the fluid.*

For let G, F be the densities of the globe and the fluid, D t diameter of the globe; then, since a globe is equal to a cyli der whose height is ⅔ D.; therefore the weight of the glo $=$ weight of a cylinder of the fluid, whose length is ⅔ D $\times \frac{G}{F}$

And (by Prop. LXXXV.) the weight of the globe in the fluid is weight of a cylinder of the fluid, whose length is ⅔ D $\times$ G—F

medium, will, in times that are reciprocally as the first velocities, describe equal spaces, and lose a given part of their motions.

For the motion lost, in describing two very small equal spaces, is as the resistance and time; that is, (because the space is given,) as the square of the velocity directly and the velocity inversely; that is, directly as the velocity. And so, in describing any space, the motion lost will always be as the first motion; and the time reciprocally as the first velocity.

Cor. 5. *Two homogeneous globes, moving with equal velocities in a fluid, lose equal velocities in describing spaces proportional to their diameters.*

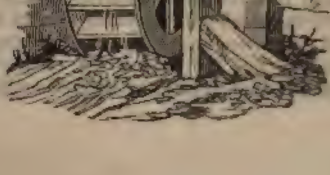
For the velocity lost in each, by describing two small spaces proportional to the diameters, will be as the resistance and time directly, and the body inversely; that is, (because the resistance is as the square of the diameter, and the time as the diameter,) as the cube of the diameter directly, and the cube of the diameter inversely: therefore the velocity lost is equal in both. And the like for any succeeding correspondent parts.

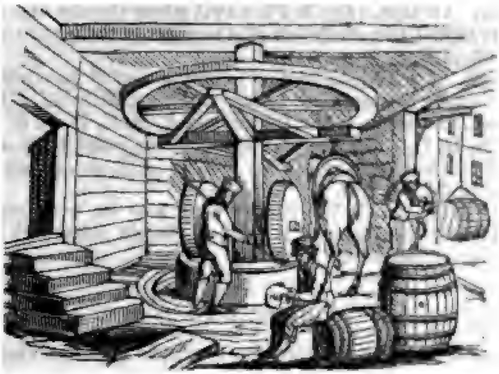
Scholium.—The resistance of fluids is of three kinds. 1. *Tenacity* or *cohesion* of the parts of the liquor, which is the force by which the particles of the fluid stick together, and causes them not to separate easily; and this is the same for all velocities. 2. *Friction* or *attrition*, where the parts of the fluid do not slide freely by one another; and this is as the velocity. 3. The *density* or *quantity of matter* to be removed; and this is as the square of the velocity. The two former kinds are very small in all fluids, except viscid and glutinous ones; and upon this account the foregoing theory regards only the last kind. And therefore the resistance there described is the very least the body can possibly meet with. But since all fluids have some small degree of friction and tenacity, they will increase the resistance a little. Also, when the velocity is very great, the compression of the fluid ought to be so too, to cause the fluid to return with equal ease behind the moving body; and when this does not happen, the resistance is increased upon that account. For a fluid yielding to a projectile, does not recede ad infinitum; but, with a circular motion, comes round to the places which the body leaves. Likewise, when bodies move in a stagnant fluid near the surface, the fluid cannot dilate itself upwards, to give way to the moving body; and this will considerably increase the resistance. Also, if a body moves in a fluid inclosed in a vessel, the relative velocity of the body in the fluid must be esteemed its true velocity. But the resistance it meets with will be increased, because the fluid has not liberty to diverge every way. And the straiter the vessel, the more is the resistance increased; and it may, by this means, be increased till

it be near double; beyond which it cannot go. For all that a body can do is to drive the fluid wholly before it, without any diverging. So that the least resistance a globe can have is the same as is laid down in Cor. 2. of the last Prop. and the greatest can never exceed the double of it; so that it will always be between these limits. If the fluid in which the body moves be elastic and spring from the body, the resistance will be greater than if it was non-elastic. But these irregularities are not considered in the foregoing theory.

There are some bodies that may be reckoned in a middle state between solids and fluids. And in some of these the tenacity and friction is so great, as in many cases far to exceed the resistance arising from their density only. For example, it appears by experiments, that if a hard body be suspended at several heights, and be let fall upon any soft substance, such as tallow, soft clay, wax, snow, &c. it will make pits or impressions, which are as the heights fallen, that is as the squares of the velocities. Likewise, nails give way to a hammer in a ratio which is as the square of the velocity. Comparing this with Schol. Prop. XIV. it appears that, in these cases, the resistance is the same for all velocities: which argues a very great degree of tenacity. Again, bodies projected into earth mixed with stones; the impressions are found to be between the simple and duplicate ratio of the velocities. Therefore, in this case, the resistance is in a less ratio than the simple ratio of the velocity; and, therefore, these sort of bodies have both friction and tenacity. And, in different sorts of bodies, there is great difference and variety in their nature and constitution.

Tenacity may be compared to the force of gravity, which is always the same; with this difference, that tenacity acts always contrary to the motion of the body, and when the body is at rest, it is nothing. *Attrition* may be compared to the motion of a body striking always a given number of particles of matter in a given time, with any velocity; and, therefore, the resistance of such a body will be as the velocity.





SECTION ELEVENTH.

METHODS OF COMMUNICATING, DIRECTING, AND REGULATING ANY MOTION IN THE PRACTICE OF MECHANICS.

PROP. CX.

To communicate motion from one body to another, or from one place to another.

1. (*Fig*. 5. *Pl*. XIV.) The easiest and simplest method of communicating motion from one thing A to another B, is by a rope or a lever AB, reaching between the two places, or things.

2. (*Fig*. 6. *Pl*. XIV.) Motion is communicated from one wheel or roller DC to another AB, by a perpetual or endless rope ABCD, going once, or oftener, about them; or, if you will, by a chain. That the rope slip not, make knots on it, and channels in the wheels, if necessary.

3. (*Fig*. 7. *Pl*. XIV.) Motion is communicated from one wheel

ABC, to another DEF; by the teeth in the two wheels working together. Or thus, (*Fig.* 8. *Pl.* XIV.) where the axis of A having but one tooth; one revolution of it answers to the motion of only one tooth in B.

4. (*Fig.* 9. *Pl.* XIV.) Motion is communicated from one place to another, by one or more beams or levers, MB, BC, CE, EF, FH, &c., moveable about the centres A, B, C, D, E, F, G; of which A, D, G, &c. are fixed. Here, if the point M be moved, the point H will be moved; for MB, BC, CE, &c. all move one another to the last, FH.

5. (*Fig.* 10. *Pl.* XIV.) Motion may also be communicated from A to B, by a pinion at A, and a straight ruler with teeth, which bite one another.

PROP. CXI.

By help of one uniform motion given, to produce another, either uniform or accelerated.

1. A uniform motion is produced in the wheel DEF, (*Fig.* 7. *Pl.* XIV.) by moving the wheel ABC uniformly, which carries it. Also a uniform motion is produced in wheels moving by cords, as AB, CD; (*Fig.* 6. *Pl.* XIV.) for one being moved uniformly, moves the other also uniformly.

2. The wheel BF (*Fig.* 11. *Pl.* XIV.) may be made to move uniformly about the centre C, by the motion of the wheel BD. On the base BF with the generating circle BD, describe the epicycloidal tooth BE. Then the point B of the wheel AB, moving uniformly about the centre A, and passing over the tooth BE, will move the wheel BF uniformly about C. Here the acting tooth AB ought to be made crooked as A*b*, that it touch not the end E, of the tooth BE, if it act on the concave side. Or else the plane of the wheel BD must be raised above the plane of BF, and a tooth made at B to bend down perpendicular to the plane of the wheel, as AG, to catch the tooth BE.

3. The lever AB (*Fig.* 1. *Pl.* XV.) may be made to move up and down with either a uniform or accelerated motion, after this manner. Let AE be a wheel whose axis is parallel to the lever, and directly above it. Take any arch N4, and divide it into any number of equal parts at 1, 2, 3, &c. through which from the centre O, draw O*a*, O*b*, O*c*, O*d*; and make 1*a*, 2*b*, 3*c*, 4*d*, &c. respectively equal to 1, 2, 3, 4 equal parts. And through the points N, *a*, *b*, &c. draw the curve N*abcd*. Then the part N*d*F being made of solid wood, and fixed to the wheel; and the wheel being turned uniformly about, in the order ENA; the part NF will give a uniform motion to the lever AB, about the centre of

motion C. And you may fix as many of these teeth to the wheel as you will.

Again, in the tooth AD, if A1, 12, 23, &c. be taken equal, and 1*a*, 2*b*, 3*c*, 4*d*, &c. be taken equal to 1, 4, 9, 16, &c. equal parts; and the curve *Aabcd* be drawn, and the tooth formed; then the lever will be moved with a uniformly accelerated motion.

The accelerated motion is proper for lifting a given weight at the end B, as a hammer; or for working a pump, by a chain going over the end B.

4. The lever AB (*Fig.* 2. *Pl.* XV.) may also be moved thus, by help of a machine GFD, moving uniformly along GD. Make HI, IF right lines; and make as many such teeth as you will; and these will give a uniform motion to the lever.

Make the curves EFE all parabolas, equal and equi-distant, whose vertices are at F, and their bases meet at E, and these will make the lever rise and fall with an accelerated motion. Such parabolic teeth as these may be placed on a wheel, whose axis is perpendicular to the horizon.

5. One wheel may move another with an accelerative motion thus. On the circle or wheel EF, (*Fig.* 3. *Pl.* XV.) take E*a*, *ab*, *bc*, &c. equal to each other. And on the edge of the wheel BD take B1, a very small part, and 13, 35, 57, &c. 3, 5, 7, &c. times B1, suppose the plane of the wheel EF to be extended as far as the marks 1, 3, 5, 7, &c.; then turn the wheel EF, till E fall on *a*; then mark the point 1 on the plane of the wheel EF; then turn EF till E comes to *b*, and mark the point 3 on the plane of the wheel EF. Likewise, let E come to *c*, *d*, &c., and mark the points 5, 7, &c., on the plane of the wheel EF; then E 1 3 5 7 *r* is the figure of the tooth of the wheel EF, which being uniformly moved, will move DB with an accelerative motion.

PROP. CXII.

To change the direction of any motion.

1. The direction of any motion may be changed by the lever of the first kind, for the two ends have opposite motions. Likewise, a bended lever will change the direction to any other direction, (as *Fig.* 4. *Pl.* XV.)

2. The direction of motion may be changed by the help of pulleys, with a rope going over them. Thus, the direction AB (*Fig.* 5. *Pl.* XV.) is changed successively into the directions BC, CD, DE, EF, FG.

3. The direction may be changed by wheels, whose axles are perpendicular to one another. Thus, the direction AB, (*Fig.* 6.

Pl. XV.) is changed into the direction EF, by the wheel C, working in the crown wheel D.

4. The direction may be changed, by making the lanthorn B, (*Fig.* 7. *Pl.* XV.) inclined in any given angle, to be moved by the cogs of the wheel A. Here, the rungs at F, where they work, must be parallel to the plane of the wheel A, or perpendicular to the cogs. The same thing may be done by wheels with teeth, as C, D. (*Fig.* 1. *Pl.* XVI.)

In both cases, the axles of the two wheels must be in one plane.

PROP. CXIII.

To regulate any motion, or to make it uniform.

1. Any motion is made uniform, by the help of a pendulum AB, (*Fig.* 2. *Pl.* XVI.) suspended at A and vibrating. As the pendulum vibrates, it causes CDE to vibrate also, about the axis DE. The weight I carries the wheel R, and R moves LF. Now, whilst the pendulum vibrates towards M, a tooth of the wheel GF goes off the pallat I, and another catches the pallat H; and when the pendulum returns towards N, it draws the pallat H off the tooth, and another catches the pallat I; and so on, alternately. So that, at every vibration of the pendulum, a tooth goes off one or other of the pallats.

2. A uniform motion is effected by the pendulum CP, (*Fig.* 3. *Pl.* XVI.) vibrating in the arch NM about the centre of motion C. As the pendulum vibrates, it causes the piece ADE to vibrate along with it about the axis of motion DE. By this motion, the leaf *a* catches hold of a tooth of the horizontal wheel GF, in its going, and the leaf *b* of another tooth, in returning. A wheel with a weight is applied to the pinion L, to keep the pendulum going.

3. A pendulum may also be applied thus, for the same purpose. FG (*Fig.* 4. *Pl.* XVI.) is a thick wheel, or rather a double wheel, whose axis is parallel to the horizon. *n*P a pendulum vibrating upon the axis DE, which is parallel to the planes of the wheel FG; *anb* two wings perpendicular to DE, and to *n*P; 1, 1, 1, pins in the rim G; and 2, 2, 2, pins in the rim F. These pins are in the planes of the wheel, but not perpendicular to the circumference, but inclined in an angle of about 45 degrees, and the pins in one end are against the spaces in the other; *ab* is parallel to the axis of the wheel FG, but neither in the same horizontal or perpendicular plane, but almost the radius of the wheel below, and something more forward. Whilst the pendulum P vibrates in the arch MN, about the axis DE, the wing *a* catches

hold of a tooth in the end F, and, when it returns, the wing *b* catches hold of a tooth in the end G. Thus, the pins acting alternately against the wings, *a*, *b*, keep the pendulum going, by help of the weight W.

4. A steady motion is continued by applying the heavy wheel ABC, (*Fig.* 5. *Pl.* XVI.) to the machine; or the cross bar DE (*Fig.* 7. *Pl.* XVI.) loaded with two equal weights at D and E; or a cylinder of some heavy matter may be applied, being made to revolve about its axis. By these the force of the power, which would be lost, is kept in the wheel, and is equally distributed in all parts of the revolution. Such a wheel is of great use in such machines as act with unequal force at different times, or in different parts of a revolution. For, by its weight, it constantly goes on at the same rate, and makes the motion uniform, and every where equal. By reason of its weight, a little variation of force will not sensibly alter its motion; and its friction, and the resistance of the air will hinder it from accelerating. If the machine slackens its motion, it will help it forward; if it tends to move too fast, it will keep it back.

Every such regulating wheel ought to be fixed upon that axis, where the motion is swiftest; and ought to be the heavier, the slower it is designed to move; and the lighter, the swifter the motion is. And, in all cases, the centre of motion must be in the centre of gravity of the wheel. And the axis may be placed parallel to the horizon, as well as perpendicular to it.

If the machine be large, and the axis of the heavy wheel be perpendicular to the horizon, the heavy wheel may be made to roll on the ground, round that axis, by putting the wheel upon another axis fixed in the former at right angles to it, and thus the weight is taken off the first axis. And two such wheels may be applied on opposite sides.

5. Any swift motion may be moderated by a fly AB, (*Fig.* 6. *Pl.* XVI.) moveable about the axis CD. This is made of thin metal: at *s* is as a spring to keep the axis and fly pretty stiff together. This bridles the rapidity of the motion of the machine, to which it is applied, by reason of its great resistance in the air; and, therefore, it hinders the motion from accelerating beyond a certain degree. This sort of fly is used in clocks, and is also useful in any motion that requires to stop, or move a contrary way.

None of these regulating wheels or flies add any new power to the machine; but rather retard the motion by their friction and resistance.

PROP. CXIV.

To describe several sorts of knots.

As ropes are made use of in several sorts of machines, and especially aboard of ships, it is proper for a mechanic to know how to tie them together; therefore, I shall here describe several sorts of knots, not so much to teach how to tie them, as to shew the form they appear in when they are tied. For the method of tying them is best learned from those that can tie them already.

1. *A thumb knot.* (*Fig.* 1. *Pl.* XVII.) This is the simplest of all, and is used to tie at the end of a rope, to hinder its opening out. Also it is used by tailors at the end of their thread.

2. *A loop knot.* (*Fig.* 2. *Pl.* XVII.) This is used to join pieces of rope together.

3. *A draw knot* (*Fig.* 3. *Pl.* XVII.) is the same as the last, only one (or both) of the ends returns the same way back, as *a b c d e*. By pulling at *a* the part *bcd* comes through, and the knot is loosed.

4. *A ring knot.* (*Fig.* 5. *Pl.* XVII.) This serves also to join pieces of rope together.

5. *Another knot* (*Fig.* 4. *Pl.* XVII.) for tying ropes together. This is made use of when any rope is often to be loosed.

6. *A running knot*, (*Fig.* 6. *Pl.* XVII.) to draw any thing close. By pulling at the end *a*, the rope is drawn through the loop *b*, and the part *cd* is drawn close about a beam, &c.

7. *Another knot*, (*Fig.* 7. *Pl.* XVII.) to tie any thing to a post; here the end may be put through as often as you will.

8. *A very small knot.* (*Fig.* 8. *Pl.* XVII.) There is a thumb knot made at the end of each piece, and the end of the other is to go through it. Thus the rope *ac* runs through the loop *d*, and *bd* through *c*; and then drawn close by pulling at *a* and *b*: if the ends *e*, *f* be drawn, the knot will be loosed again.

9. *A fisher's knot*, or *water knot.* (*Fig.* 9. *Pl.* XVII.) This is the same as the 4th, only the ends are to be put twice through the ring, which, in that, was but once, and then drawn close.

10. *A mashing knot* (*Fig.* 10. *Pl.* XVII.) for nets; and is to be drawn close.

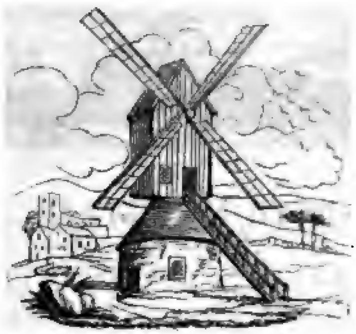
11. *A barber's knot*, (*Fig.* 11. *Pl.* XVII.) or a knot for cawls of wigs. This must be drawn close.

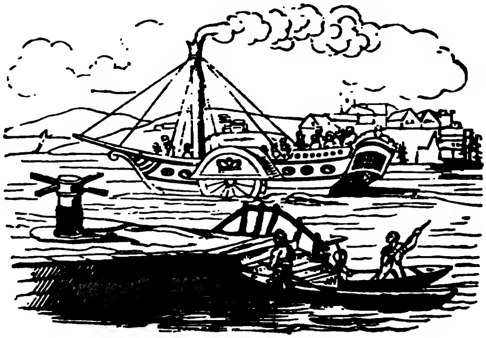
12. *A bowline knot.* (*Fig.* 12. *Pl.* XVII.) When this is drawn close, it makes a loop that will not slip. This serves to hitch over any thing.

13. *A wale knot* (*Fig.* 13. *Pl.* XVII.) is made with the three strands of a rope, so that it cannot slip. When the rope is put

through a hole, this knot keeps it from slipping through. If the three strands are wrought round once or twice more, after the same manner, it is called *crowning*. By this means, the knot is made bigger and stronger. A thumb knot, art. 1, may be applied to the same use as this.

Concerning the strength of ropes, see the latter end of Section VIII.





SECTION TWELFTH.

THE POWERS AND PROPERTIES OF COMPOUN ENGINES; OF FORCES ACTING WITHIN TH MACHINE; OF FRICTION.

PROP. CXV.

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weight in the second be taken for the power in the third, and this power be to the weight as C to D; then the first power A is to the last weight D, in the compound ratio of A to B, B to C, and C to D, and so on through the whole.

Cor. In any machine, composed of wheels, the power is to the weight, in the compound ratio of the diameter of the axle where the weight is applied, to the diameter of that were power is applied; and the number of teeth in the pinion of each axis, beginning at the power, to the number of teeth in each wheel they work in, till you come at the weight.

Or, instead of the teeth, you may take their diameters.

PROP. CXVI.

IF THE POWER AND WEIGHT BE IN EQUILIBRIO ON ANY MACHINE, IF THEY BE PUT IN MOTION, THE VELOCITY OF THE WEIGHT WILL BE TO THAT OF THE POWER, AS THE POWER IS TO THE WEIGHT.

For, since they are in equilibrio, one of them cannot of itself move the other; therefore, if they be put into motion, the momentum, or quantity of motion of the weight, will be equal to that of the power; and, therefore, their velocities will be reciprocally as their quantities.

Cor. 1. *Hence, it follows, that if any weight is moved by help of a machine, what is gained in power is lost in time.*

For in whatever proportion the power is less than the weight, in the same proportion will the weight be slower than the power.

Cor. 2. *Hence, the motion of the weight is not at all increased by any engine, or mechanical instrument; only the velocity of the weight is so much diminished thereby, that the quantity of motion of the weight may not exceed the quantity of motion of the power. And, therefore, it is a vain fancy for any one to think that he can move a great weight with a little power, and with the same velocity as with a greater power.*

For the advantage gained by the power is lost by the velocity. If any power is able to raise a pound with a given velocity, it is impossible, by the help of any machine whatever, that the same power can raise two pounds with the same velocity. Yet, it may, by help of a machine, be made to raise 2 pounds with half that velocity, or even 1000 pounds with the thousandth part of the velocity. But still there is no greater quantity of motion produced, when 1000 pounds weight is moved, than when 1 pound: the 1000 pounds being proportionally slower. The power and

use of machines consists only in this, that by their means the velocity of the weight may be diminished at pleasure, so that a given weight may be moved with a given power, or that with a given force any given resistance may be overcome. Mechanic instruments being only the means whereby one body communicates motion to another, and not designed to produce a motion that had no existence before.

Cor. 3. *Hence, also, it is plain, that a given power or quantity of force, applied to move a heavy body by help of a machine, can produce no greater quantity of motion in that body, than if that force was immediately applied to the body itself; nor not quite so much, by reason of the friction and resistance of the engine. And, if the power be given, you may chuse whether you will move a greater weight with a less velocity, or a less weight with a greater velocity. But to do both, is utterly repugnant to the eternal laws of nature.*

PROP. CXVII. (*Fig.* 1. *Pl.* XVIII.)

If any machine CD, is to be moved by the help of levers, wheels, &c; and if the power that moves it acts intirely within the machine, and exerts its force against some external object B; then the force applied within to move the machine, will be just the same as if the machine was at rest, and the object B was to be moved; supposing B to be as easily moved as the machine.

For, suppose first, the lever AFB to be fixed, and to make a part of the machine; and let the external force acting at B, which is capable to move the machine, be 1. Now, suppose the lever AFB, moveable about F, and a force applied at A, so great as to act at B, with the force 1; then the action and re-action at B being the same as before, it is plain the machine will be moved as before. But the force now acting at A, is $\frac{BF}{AF} \times 1$; just the same as if the point F were fixed, and B was to be moved. And if more levers, or any number of wheels be added, the thing will still be the same.

Otherwise.

Let the absolute force to move the machine be 1, and the force acting at A be f; and let us first consider it as acting out of the machine; then B being fixed, is the fulcrum; therefore the force acting at F, is $\frac{AB}{FB} \times f$. Now, if the acting force be considered in the machine, it will not be urged forward with all this force,

for the re-action will be equal to f, the power at A. Therefore the absolute force the machine is moved with, is $\frac{AB}{FB} \times f - f$ or $\frac{AB - FB}{FB} \times f$, that is $\frac{AF}{FB} \times f$, but this is $= 1$, therefore $f = \frac{FB}{AF} \times 1$.

Cor. 1. *Hence, if the absolute external force, to move any body or machine, be given, and the machine is to be moved by an internal power; that power may be found, by supposing the machine at rest, and the external object B was to be removed, and to require the same absolute force to move it.*

For it is the same thing, as to the power, whether the machine, or the external object, be moved, whilst the other is at rest.

Cor. 2. *If the power acting within the machine be not communicated to some external object, it will have no force at all to move the machine. And any force that both begins and ends within it, does nothing at all to move it.*

For the power acting only against some part of the machine, will be destroyed by the contrary and equal re-action. And the body being acted on by these equal and contrary forces, will not be moved at all. Thus, if a man, sitting in the head of a boat, pull the stern towards him by a rope, the boat will not be moved at all out of its place by that force.

PROP. CXVIII.

To determine the friction, and other irregularities in mechanical engines.

The propositions, hitherto laid down suppose all bodies perfectly smooth, that they slide over one another without any friction, and move freely without any resistance. But, since there is no such thing as perfect smoothness in bodies, therefore, in rubbing against one another, they meet with more or less friction, according to their roughness; and, in moving in any medium, will be resisted according to the density of the medium. Even ropes going over pullies cannot be bended without some force.

Among machines, some have a great deal more friction than others, and some very little. Thus, a pendulum has little or no friction, but what arises from the resistance of the air. But a carriage has a great deal of friction. For, upon plain ground, a loaded cart requires the strength of several horses to draw it along, and all, or most, of this force is owing to its friction.

All compounded machines have a great deal of friction, and so much the more, as they consist of more parts that rub against one another; and there is great variety in several sorts of bodies, as to the quantity of friction they have, and even in the same bodies under different circumstances; upon which account, it will be impossible to give any standing rules by which its quantity can be exactly determined. All we can do is to lay down such particular rules, as have been deduced from experiments made upon particular bodies, which rules will require some variation under different circumstances, according to the judgment and experience of the artist.

1. Wood and all metals, when oiled or greased, have nearly the same friction; and the smoother they are the less friction they have. Yet metals may be so far polished as to increase friction, by the cohesion of their parts.

Wood slides easier upon the ground in wet weather than in dry, and easier than iron in dry weather: but iron slides easier than wood in wet weather. Lead makes a great deal of resistance. Iron, or steel, running in brass, makes the least friction of any. In wood acting against wood, grease makes the motion twice as easy, or rather two-thirds easier. Wheel naves greased or tarred, go four times easier than when wet.

Metals oiled make the friction less than when polished, and twice as little as when unpolished.

In general, the softer or rougher the bodies, the greater is their friction.

2. As to particular cases, a cubic piece of soft wood of eight pounds weight, moving upon a smooth plane of soft wood, at the rate of three feet per second, its friction is about one-third the weight of it. But, if it be rough, the friction is little less than half the weight.

Upon the same supposition, other soft wood upon soft wood, very smooth, the friction is about a quarter the weight.

Soft wood upon hard, or hard upon soft, one-fifth or one-sixth the weight.

Hard wood upon hard wood, one-seventh or one-eighth the weight.

Polished steel moving on steel or pewter, a quarter the weight: moving on copper or lead, one-fifth the weight; on brass, one-sixth the weight. Metals of the same sort have more friction than different sorts.

The friction, *cæteris paribus*, increases with the weight, almost in the same proportion. The friction is also greater with a greater velocity, but not in proportion to it, except in very few cases. A greater surface also causes something more friction, with the same weight and velocity. Yet friction may sometimes be increased

by having too little surface to move on, as upon clay, &c. where the body sinks.

3. The friction arising from the bending of ropes about machines, differs according to their stiffness, the temper of the weather, degree of flexibility, &c. but, *ceteris paribus*, the force or difficulty of bending a rope is as the square of the diameter of the rope, and its tension, directly; and the diameter of the cylinder, or pulley, it goes about, reciprocally.

A rope of one inch diameter, whose tension, or weight drawing it, is 5lb. going over a pulley three inches diameter, requires a force of 1lb. to bend it.

4. The resistance of a plane moving through a fluid, is as the square of the velocity; and (putting $v =$ velocity in feet, in a second) it is equal to the weight of a column of the fluid, whose base is the plane, and height $\frac{vv}{64}$. And, in a globe, it is but half so much.

5. The friction of a fluid running through a tube is as the velocity and diameter of the tube.

But the friction is greater in respect to the quantity of the fluid, in small tubes, than in large ones; and that, reciprocally, as their diameters. But the absolute quantity of the friction in tubes, is but very small, except the velocity be very great, and the tube very long.

But, if a pipe be divided into several lesser ones, whose number is n, the resistance arising from the friction will be increased as $\sqrt{n}$; for the area of the section of any one pipe, will be $\frac{1}{n}$; and the friction, being as the circumference, will be as $\frac{1}{\sqrt{n}}$; and, therefore, the friction in all of them will be $\frac{n}{\sqrt{n}}$, or as $\sqrt{n}$.

6. As to the mechanic powers: the single lever makes no resistance by friction; but if, by the motion of the lever in lifting, the fulcrum or place of support be changed further from the weight, the power will be decreased thereby.

7. In any wheel of a machine, running upon an axis, the friction on the axis is as the weight upon it, the diameter of the axis, and the angular velocity. This sort of friction is but small.

8. In the pulley, if p, q be two weights, and q the greater; and if $W = \frac{4pq}{p+q}$, then W is the weight upon the axis of the single pulley, and it is not increased by the acceleration of the weight q, but remains always the same.

The friction of the pulley is very considerable, when the sheaves rub against the blocks, and by the wearing of the holes and axles.

The friction on the axis of the pulley is as the weight W, its angular velocity, the diameter of the axis directly, and the diameter of the pulley inversely. A power of 100lb. with the addition of 50lb. will but draw up 500lb. with a tackle of five.

And 15 lb. over a singe pulley will draw up only 14 lb.

9. In the screw there is a great deal of friction. Those with sharp threads have more friction than those with square threads. And endless screws have more than either. Screws with a square thread raise a weight with more ease than those with a sharp thread.

In the common screw the friction is so great, that it will sustain the weight in any position given, when the power is taken off. And, therefore, the friction is at least equal to the power. From whence it will follow, that, in the screw,

The power must be to the weight or resistance, at least as twice the perpendicular height of a thread to the circumference described by one revolution of the power, if it be able to raise the weight, or only sustain it. This friction of the screw is of great use, as it serves to keep the weight in any given position.

10. In the wedge, the friction is at least equal to the power, as it retains any position it is driven into; therefore, in the wedge,

The power must be to the weight, at least, as the base to the height, to overcome any resistance.

11. To find the friction of any engine, begin at the power, and consider the velocity and the weight at the first rubbing part; and estimate its quantity of friction by some of the foregoing articles. Then proceed to the next rubbing part, and do the same for it. And so on through the whole.

And note, something more is to be allowed for increase of friction, by every new addition to the power.

Cor. Hence will appear the difficulty, or rather impossibility, of a perpetual motion, or such a motion as is to continue the same for ever, or, at least as long as the materials will last that compose the moving machine.

For such a motion as this ought continually to return undiminished, notwithstanding any resistance it meets with, which is impossible; for, although any body once put into motion, and moving freely without any resistance, or any external retarding force acting upon it, would for ever retain that motion. Yet, in fact, we are certain, that no body or machine can move at all without some degree of friction and resistance. And, therefore, it must

follow, that from the resistance of the medium, and the friction of the parts of the machine upon one another, its motion will gradually decay, till, at last, all the motion is destroyed, and the machine is at rest. Nor can this be otherwise, except some new active force, equal to all its resistance, adds a new motion to it. But that cannot be from the body or machine itself; for then the body would move itself, or be the cause of its own motion, which is absurd.

PROP. CXIX.

TO CONTRIVE A PROPER MACHINE THAT SHALL MOVE A GIVEN WEIGHT WITH A GIVEN POWER, OR, WITH A GIVEN QUANTITY OF FORCE, SHALL OVERCOME ANY OTHER GIVEN RESISTANCE.

If the given power is not able to overcome the given resistance, when directly applied, that is, when the power applied is less than the weight or resistance given, then the thing is to be performed by the help of a machine made with *levers*, *wheels*, *pullies*, *screws*, &c. so adjusted, that when the weight and power are put in motion on the machine, the velocity of the power may be at least so much greater than that of the weight, as the weight and friction of the machine, taken together, is greater than the power. For on this principle depends the mechanism or contrivance of mechanical engines, used to draw or raise heavy bodies, or overcome any other force. The whole design of these being to give such a velocity to the power in respect of the weight, as that the momentum of the power may exceed the momentum of the weight. For, if machines are so contrived that the velocities of the agent and resistant are reciprocally as their forces, the agent will just sustain the resistant; but, with a greater degree of velocity, will overcome it. So that, if the excess of velocity in the power is so great as to overcome all that resistance which commonly arises from the friction or attrition of contiguous bodies, as they slide by one another, or from the cohesion of bodies that are to be separated, or from the weights of bodies to be raised, the excess of the force remaining, after all these resistances are overcome, will produce an acceleration of motion proportional thereto, as well in the parts of the machine, as in the resisting body. Now, how a machine may be contrived to perform this to the best advantage, will appear from the following rules.

1. Having assigned the proportion of your power and the weight to be raised, the next thing is to consider how to combine levers, wheels, pullies, &c. so that, working together, they may be able to give a velocity to the power, which shall be to that of

the weight, something greater than in the proportion of the weight to the power. This done, you must estimate your quantity of friction, by the last Prop.; and if the velocity of the power be to that of the weight, still in a greater proportion, than the weight and friction taken together is to the power, then your machine will be able to raise the weight. And note, this proportion must be so much greater, as you would have your engine work faster.

2. But the proportion of the velocity of the power and weight, must not be made too great neither. For it is a fault to give a machine too much power, as well as too little; for if the power can raise the weight, and overcome the resistance, and the engine perform its proper effect in a convenient time, and works well, it is sufficient for the end proposed. And it is in vain to make more additions to the engine, to increase the power any further; for that would not only be a needless expense, but the engine would lose time in working.

3. As to the power applied to work the engine, it may be either a living power, as men, horses, &c., or an artificial power, as a spring, &c., or a natural power, as wind, water, fire, weights, &c.

When the quantity of the power is known, it matters not, as to the effect, what kind of power it is. For the same quantity of any sort will produce the same effect; and different sorts of powers may be applied, in an equal quantity, a great variety of ways.

The most easy power applied to a machine is weight, if it be capable of effecting the thing designed. If not, then wind, water, &c., if that can conveniently be had, and without much expense.

A spring is also a convenient moving power for several machines; but it never acts equally as a weight does; but is stronger, when much bent, than when but a little bent, and that in proportion to the degree of bending, or the distance it is forced to. But springs grow weaker by often bending, or remaining long bent; yet they recover part of their strength by lying unbent.

The natural powers, wind and water, may be applied with vast advantage to the working of great engines, when managed with skill and judgment. The due application of these has much abridged the labours of men; for there is scarce any labour to be performed, but an ingenious artificer can tell how to apply these powers to execute his design, and answer his purpose. For any constant motion being given, it may, by a due application, be made to produce any other motions we desire. Therefore, these powers are the most easy and useful, and of the greatest benefit

to mankind. Besides, they cost nothing, nor require any repetition or renewing, like a weight or a spring, which require to be wound up. When these cannot be had, or cannot serve our end, we have recourse to some living power, as men, horses, &c.

4. Men may apply their strength several ways, in working a machine. A man of ordinary strength, turning a roller by the handle, can act for a whole day against a resistance equal to 30 lb. weight; and, if he works ten hours in a day, he will raise a weight of 30 lb. 3½ feet in a second; or, if the weight be greater, he will raise it so much less in proportion. But a man may act, for a small time, against a resistance of 50 lb., or more.

If two men work at a windlass, or roller, they can more easily draw up 70 lb. than one man can 30 lb. provided the elbow of one of the handles be at right angles to that of the other. And, with a fly or heavy wheel applied to it, a man may do one-third part more work, and, for a little while, act with a force, or overcome a continual resistance of 80 lb. and work a whole day when the resistance is but 40 lb.

Men used to carrying, such as porters, will carry, some 150 lb., others 200 or 250 lb. according to their strength.

A man can draw about 70 or 80 lb. horizontally; for he can but apply about half his weight.

If the weight of a man be 140 lb. he can act with no greater a force in thrusting horizontally, at the height of his shoulders, than 27 lb.

As to horses. A horse is, generally speaking, as strong as five men. A horse will carry 240 or 270 lb.

A horse draws to greatest advantage when the line of direction is a little elevated above the horizon, and the power acts against his breast; and can draw 200 lb. for eight hours in a day, at two miles and a half in an hour. If he draw 240 lb. he can work but six hours, and not go quite so fast. And, in both cases, if he carries some weight, he will draw better than if he carried none. And this is the weight a horse is supposed to be able to draw over a pulley, out of a well. In a cart, a horse may draw 1000 lb.

The most force a horse can exert is, when he draws something above a horizontal position.

The worst way of applying the strength of a horse, is to make him carry or draw up hill. And three men, in a steep hill, carrying each 100 lb. will climb up faster than a horse with 300 lb.

Though a horse may draw in a round walk of 18 feet diameter, yet such a walk should not be less than 25 or 30 feet diameter.

5. Every machine ought to be made of as few parts, and those as simple as possible, to answer its purpose; not only because the expense of making and repairing will be less, but it will also be less liable to any disorder. And it is needless to do a thing with many, which may be done with fewer parts.

6. If a weight, is to be raised but a very little way, the lever is the most simple, easy, and ready machine. Or if the weight be very great, the common screw is most proper. But if the weight is to be raised a great way, the wheel and axle is a proper power, and blocks and pullies are easier still; and the same may be done by help of the perpetual screw.

Great wheels, to be wrought by men or cattle, are of most use and convenience when their axles are perpendicular to the horizon; but, if by water, &c., then it is best to have their axles horizontal.

7. As to the combination of simple machines together; to make a compound one: though the lever, when simple, cannot raise a weight to any great height, and, in this case, is of little service, yet it is of great use when compounded with others. Thus, the spokes of a great wheel are all levers perpetually acting; and a beam fixed to the axis to draw the wheel about by men or horses, is a lever. The lever, also, may be combined with the screw, but not conveniently with pullies, or with the wedge. The wheel and axle is combined with great advantage with pullies. The screw is not well combined with pullies; but the perpetual screw, combined with the wheel, is very serviceable. The wedge cannot be combined with any other mechanical power; and it only performs its effect by percussion; but this force of percussion may be increased by engines.

Pullies may be combined with pullies, and wheels with wheels; therefore, if any single wheel would be too large, and take up too much room, it may be divided into two or three more wheels and trundles, or wheels and pinions, as in clock-work, so as to have the same power, and perform the same effect.

In wheels with teeth, the number of teeth that play together in two wheels, ought to be prime to each other, that the same teeth may not meet at every revolution. For, when different teeth meet, they, by degrees, wear themselves into a proper figure; therefore they should be contrived, that the same teeth meet as seldom as possible.

8. The strength of every part of the machine ought to be made proportional to the stress it is to bear; and, therefore, let every lever be made so much stronger, as its length and the weight it is to support is greater. And let its strength diminish proportionally from the fulcrum, or point, where the greatest stress is, to each

end. The axles of wheels and pullies must be so much stronger, as they are to bear greater weight. The teeth of wheels, and the wheels themselves, which act with greater force, must be proporally stronger. And in any combination of wheels and axles, make their strength diminish gradually from the weight to the power, so that the strength of every part be reciprocally as the velocity it has. The strength of ropes must be according to their tension, and that is as the squares of their diameters, (see the end of Sect. VIII.) And, in general, whatever parts a machine is composed of, the strength of every particular part of it must be adjusted to the stress upon it, according to Sect. VIII. Therefore, in square beams, the cubes of the diameters must be made proportional to the stress they bear. And let no part be stronger or bigger than is necessary for the stress upon it; not only for the ease and well-going of the machine, but for the diminishing the friction. For all superfluous matter in any part of it, is nothing but a dead weight upon the machine, and serves for nothing but to clog its motion. And he is by no means a perfect mechanic, that does not only adjust the strength to the stress, but also contrive all the parts to last equally well, that the whole machine may fail together.

9. To avoid friction as much as possible, the machine ought not to have any unnecessary motions, or useless parts; for a multiplicity of parts, by their weight and motion, increase the friction. The diameters of the wheels and pullies ought to be large, and the diameters of the arbors or spindles they run on, as small as can be consistent with their strength. All ropes and cords must be as pliable as possible, and for that end are rubbed with tar or grease; the teeth of wheels must be made to fit and fill up the openings, and cut in the form of epicycloids. All the axles, where the motion is, and all teeth where they work, and all parts that, in working, rub upon one another, must be made smooth; and, when the machine goes, must be oiled or greased. If a joint is to go pretty stiff and steady, rub a little grease upon it.

The axis *a* (*Fig.* 2. *Pl.* XVIII.) of a wheel may have its friction diminished, by causing it to run on two rollers, B, C, turning round with it, upon two centres.

Likewise, instead of the teeth of wheels, one may place little wheels, as A, B, (*Fig.* 3. *Pl.* XVIII.) running upon an axis in its centre. And this will take away almost all the friction of the teeth. And, in lanterns or trundles, the rounds may be made to turn about, instead of being fixed.

In all machines with wheels, the axles or spindles ought not to shake, which they will do, if they be too short. And their ends ought just to fill their holes.

When the teeth of a wheel are much worn away, it makes that wheel move irregularly about, increases the friction, and requires more force, and may cause the teeth of two wheels to run foul upon one another, and to stop their motion, and endanger breaking the teeth. To prevent this, proper care should be taken to dress the teeth, and keep them to their proper figure.

10. When any motion is to be long continued, contrive the power to move or act always one way, if it can be done. For this is better and easier performed than when the motion is interrupted, and the power is forced to move, first, one way and then another, because every new change of motion requires a new additional force to effect it. Besides, a body in motion cannot suddenly receive a contrary motion, without great violence. And the moving any part of the machine contrary ways by turns, with sudden jerks, tends only to shake the machine to pieces.

11. In a machine that moves always one way, endeavour to have the motion uniform. Some methods of doing this may be seen in Prop. CXIII. and if one uniform motion be required to produce a motion either uniform or accelerated, some light may be had from Prop. CXI. Likewise how to communicate motion, consult Prop. CX. And to change the direction, see Prop. CXII.

12. But when the nature of the thing requires that a motion is to be suddenly communicated to a body, or suddenly stopped, to prevent any damage or violence to the engine, by a sudden jolt, let the force act against some spring, or beam of wood, which may supply the place of a spring.

13. In regard to the size of the machine, let it be made as large as it can conveniently. The greater the machine, the exacter it will work, and perform all it motions the better. For there will always be some errors in the making, as well as in the materials, and, consequently, in the working of the machine. The resistance of the medium in some machines has a sensible effect. But all these mechanical errors bear a less proportion to the motion of the machine, in great machines than in little ones, being nearly reciprocally as their diameters, supposing they are made of the same matter, and with the same accuracy, and are equally well finished. Therefore, in a small machine, they are more sensible, but in a great one almost vanish. Therefore, great machines will answer better than smaller, in all respects, except in strength, for the greater the machine the weaker it is, and less able to resist any violence.

14. For engines that go by water, it is necessary to measure the velocity and force of the water. To get the velocity, drop in pieces of sticks, &c., and observe how far they are carried in a second, on any given time.

But if it flow through a hole in a reservoir or standing receptacle of water, the velocity will be found from the depth of the hole below the surface, by Cor. 2. Prop. XCVII.; and its force by Cor. 1. Prop. CVII.

Thus, let $s = 16\frac{1}{12}$ *feet*, $v =$ velocity of the fluid per second. B = the area of the hole. H = height of the water; all in feet. Then the velocity $v = \sqrt{2sH}$; and its force = the weight of the quantity $\frac{vv}{2s}$ B or HB of water, or $= \frac{62\frac{1}{2}}{112}$ HB hundred weight; because a cubic foot is $= 62\frac{1}{2}$ lb. avoirdupois. Also, a hogshead is about $8\frac{1}{2}$ feet, or 531 lb. and a tun is four hogsheads.

When you have but a small quantity of water, you must contrive it to fall as high as you can, to have the greater velocity, and, consequently, more force upon the engine.

15. If water is to be conveyed through pipes to a great distance, and the descent be but small, so much larger pipes must be used, because the water will come slow. And these pipes ought not to be made straighter in some places than others; for the quantity of water conveyed through them depends upon the bigness of the bore at the straightest place.

Pipes of conduct coming directly from an engine, should be made of iron, with flanches at the ends to screw them together, with lead between, or else of wood; for lead pipes will bulge out at every stroke of the engine and burst; but pipes next a jet must be lead. Pipes should not turn off at an angle, but gradually in a curve; pipes of elm will last twenty or thirty years in the ground; but they must be laid so deep that the frost may not reach them, or else the water must be let out, otherwise the frost will split them.

The thickness of any pipe must be as the diameter of the bore, and also as the depth from the spring. For a lead pipe of 6 inches bore, and 60 or 70 feet high, the thickness must be half an inch; and in wooden pipes 2 inches.

Water should not be driven through pipes faster than four feet per second, by reason of the friction of the tubes. Nor should it be much wire-drawn, that is, squeezed through smaller pipes; for that creates a resistance, as the water-way is less in narrow pipes.

And in pump work, where water is conveyed through pipes to higher places, the bores of the pipes should not be made too straight upwards; for the straighter they are near the top, the less water will be discharged; nor should the pipe that brings the water into the pump be too straight, for the same reason. The wider these are, the easier the pump works.

When pipes are wind bound, that is, when air is lodged in them that the water can hardly pass, it must be discharged thus:

Going from the spring till you come to the first rising of the ground, dig it open till the pipe be laid bare; then, with a nail driven into it at the highest part, or rather a little beyond, make a hole in the top, and all the air will blow out at the hole, and when the water comes, batter up the hole again. Do the same at every eminence, and all the air will be discharged. If the water runs fast through the pipes, the air will be beyond the eminence; but stopping the water, the air will ascend to the highest part. If air be driven in, at first, along with the water, the nail-hole must be left open, or a cock placed there to open occasionally. Sometimes, a small leaden pipe is placed over the other, communicating with it in several places, in which is a cock at top to open upon occasion.

16. When any work is to be performed by a water-wheel moved by the water running under it, and striking the paddles or laddle boards, (*Fig.* 2. *Pl.* XIX.) the channel it moves in ought to be something wider than the hole of the adjutage, and so close to the floats on every side, as to let little or no water pass; and when past the wheel, to open a little that the water may spread. It is of no advantage to have a great number of floats or paddles, for these past the perpendicular are resisted by the back water, and those before it are struck obliquely. The greatest effect that such a wheel can perform, in communicating any motion, is when the paddles of the wheel move with $\frac{1}{3}$ the velocity of the water; in which case, the force upon the paddles is $\frac{4}{9}$ only; supposing the absolute force of the water against the paddles, when the wheel stands still, to be 1. So that the utmost motion which the wheel can generate, is but $\frac{4}{27}$ of that which the force of the water against the paddles at rest would produce. This is when the wheel is at the best; but, oftentimes, far less is done.

Machines to raise water, though well made, seldom lose less than $\frac{1}{3}$ the computed quantity of water to be raised. The best contrived engine is scarce $\frac{1}{5}$ part better than the worst contrived engine, when they are equally well executed.

A man with the best water engine cannot raise above one hogshead of water in a minute, 10 feet high, to work all day.

17. When a weight is to be raised with a given corporeal power, by means of the wheel and axle, so that the weight may receive the greatest motion possible in a given time; the radius of the wheel and axle, and the weight to be raised, ought to be so adjusted, that the radius of the axle (EF) : (*Fig.* 3. *Pl.* III.) may be to the radius of the wheel (AB) :: as $\frac{5}{8}$ the power (P) : to the weight to be raised (W) : or, which comes to the same thing, the velocity gained by the power in descending must be $\frac{1}{2}$ of the velocity which would be gained by gravity in the same time.

This only holds good when the power is a heavy body, as well as the weight; but does not take place, when the power is some immaterial active force, such as that of an elastic medium, the strength of a spring, &c., whose weight is inconsiderable.

18. *These principles, also, are very useful, and necessary to be known, where water-works are concerned.*

The pressure of the atmosphere upon a square inch is 14.7 lb. *avoird.* at a medium.

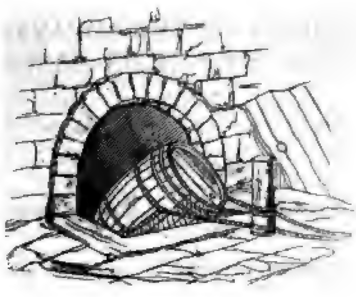
The weight of a column of water, equal to the weight of the atmosphere, is 11 ⅓ *yards.*

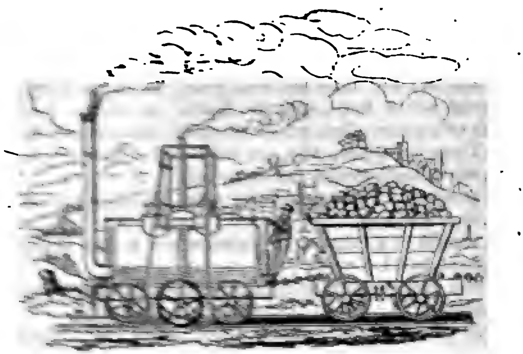
A cubic foot of water weighs 62½ lb. *avoird.* and contains 6.128 *ale gallons.*

An ale gallon of water contains 282 *inches*, and weighs 10.2 lb. *avoird.*

A tun of water, ale measure, weighs 1.1 *tun avoird.* at 63 gallons the hogshead.

A cylinder of water a yard high, and *d* inches in diameter, contains $\frac{1}{10}$ *dd ale gallons*, and weighs $\frac{1}{48}$ *dd pounds avoird.*





SECTION THIRTEENTH.

THE DESCRIPTION OF COMPOUND MACHINE OR ENGINES, AND THE METHOD OF COM PUTING THEIR POWERS OR FORCES; WITI SOME ACCOUNT OF THE ADVANTAGES ANI DISADVANTAGES OF THEIR CONSTRUCTION

PROP. CXX.

[redacted]

with machines of several kinds. I shall, therefore, give the mechanical construction of several sorts of machines, made for several different purposes, which will assist the reader's invention, and give him some idea how he may proceed in contriving a machine for any end proposed. Of which I shall only give a short explanation of the principal parts, not troubling the reader with any description of their minuter ones, nor how they are joined together, or strengthened, &c. It is sufficient here to shew the disposition and nature of the principal members; the rest belongs to carpenters, joiners, smiths, &c. and is easily understood by any one.

To compute their powers.

1. As to simple machines; they are easily accounted for, and their forces computed, by the properties of the mechanic powers.

2. For compound machines; suppose any machine divided into all the simple ones that compose it. Then begin at the power and call it 1; and, by the properties of the mechanic powers, find the force with which the first simple machine acts upon the second, in numbers. Then call this force 1, and find the force it acts upon the third, in numbers. And putting this force 1, find the force acting on the fourth, in numbers; and so on to the last. Then multiply all these numbers together, the product will give the force of the machine, supposing the first power 1.

3. When pullies are concerned in the machine, all the parts of the same running rope, that go and return about several pullies, freely and without interruption, must be all numbered alike for the force. And if any rope act against several others, it must be numbered with the sum of all these it acts against.

4. In a combination of wheels; take the product of the number of teeth in all the wheels that act upon and drive others, for the power; and the product of the teeth in all the wheels moved by them, for the weight. Or, instead of the teeth, take the diameters.

Or thus,

When a machine is in motion, if you measure the velocity of the weight, and that of the power, in numbers; then the first number to the second, gives the proportion of the power to the weight.

Otherwise thus,

In wheel work, there are always two wheels fixed upon one axis, or else one wheel, and a pinion, trundle, or barrel, which supplies the place of a wheel. Of these two, call that wheel the *leader*, which is acted on by the power, or by some other wheel; and the other, on the same axis, called the *follower*, which drives

some other forward. Then, having either the number of teeth, the diameter of each, *take the product of all the leaders, for* *weight; and the product of all the followers, for the power.* H the leader receives the motion, and the follower gives it.

5. And if the velocity of the power or weight be require *Take the product of all the leaders, for the velocity of the pow and the product of all the followers, for the velocity of the weight*

Other things that are more complex and difficult, must be ferred to the general laws of motion.

EXAMPLE I.

Scissars, pinchers, &c. may be referred to the lever of the fi kind. A *handspike* and *crow* are levers of the first kind. *Kni* fixed at one end, to cut wood, bread, &c. are levers of the seco kind. The *bones* in animals, also *tongs,* are levers of the th kind. A *hammer* to draw a nail is a bended lever.

EXAMPLE II.

A *windlass,* and a *capstan* in a ship, and a *crane* to draw goods out of a ship or boat, may be referred to the wheel a axle.

EXAMPLE III.

All *edge tools* and instruments with a *sharp point,* to cut, clea slit, chop, pierce, bore, &c. as *knives, hatchets, scissars, swor bodkins,* &c. may be reduced to the wedge.

EXAMPLE IV.

The *bar* AB (*Fig.* 7. *Pl.* XVIII.) bearing a weight C, may referred to the lever, where the weight upon A : to the wei upon B :: is as BC : to AC.

EXAMPLE V.

Likewise, if two horses draw the weight W. (*Fig.* 5. *Pl.* XVI

If one brachium AC be longer than the other CB, then the weight in the scale E must be greater than that in D, to make an equilibrium. And then you will have a deceitful balance, which being empty, or loaded with unequal weights, shall remain in equilibrio. For AC : CB :: weight in E : weight in D; by the property of the lever. But changing the weights from one scale to the other will discover the deceit; for the balance will be no longer in equilibrio.

Example VII.

The *steelyard* AB (*Fig.* 8. *Pl.* XVIII.) is nothing but a lever, whose fulcrum is C, the centre of motion. If the weight P, placed at D, reduces the beam AB to an equilibrium; and there be taken the equal divisions D 1, 1 2, 2 3, 3 4, &c. then the weight P, placed successively at 1, 2, 3, 4, &c., will equi-ponderate with weights as W, suspended at B, which are, also, as the numbers 1, 2, 3, 4, &c. respectively. Moreover, if the divisions D1, 1 2, 2 3, &c. be each $=$ CB; then, if P be successively placed at 1, 2, 3, &c. the weight W to balance it, will be, respectively, equal to P, 2P, 3P, &c. that is to 1, 2, 3 pounds, &c. if P is a pound.

For, by the property of the lever, $CP \times P + CD \times P = CB \times W$, that is, $PD \times P = CB \times W$. And CB : PD :: P : W, universally. Whence, if DP or D1 $=$ CB, then W $=$ P. If DP or D2 $=$ 2CB, then W $=$ 2P, &c. But if CB be greater than D1, 1 2, &c., then will the constant weight P be greater than W, 2W, &c.

The properties necessary for a steelyard to have, are these:

1. That the fixed weight P being placed at D, where the divisions begin, shall make the beam in equilibrio.

2. That the divisions D 1, 1 2, 2 3, &c. be equal to one another.

3. That CB may be of any length, provided the weight P be rightly adjusted to it, *viz.* so that CB : D1 :: P : 1 pound, if W be pounds. Or CB : D1 :: P : 1 stone, if W be stones.

4. That the beam be straight, and the upper edge in a line with the centres C, B.

5. That it move easily and freely on its centre C.

Many steelyards are likewise graduated on the under side, which may be used by turning them upside down. Generally, one side is for small weights, and the other for great ones. And each side is adjusted by the foregoing rules; and all the crooks hanging at it (except the moveable one for the weight) must go to the weight of the beam.

Example VIII.

Let AB (*Fig.* 9. *Pl.* XVIII.) be a *cheese press;* CE, FG are

levers moveable about the points D, E, F, G, by applying the hand at C. S the stone or weight. H the cheese.

If CD $=$ 5, DE $=$ 2, FG $=$6, GH $=$ 2, FR $=$ 1, FH $=$ 4.; then, in the lever CE, D is the fulcrum. Call the power at C, 1; then the force at E or F is $\frac{5}{2}$. And in the lever FG, whose fulcrum is G, if the power at F be 1, the force at R is $\frac{6}{5}$, therefore the power at C, to the weight S, is as 1 to $\frac{5}{2} \times \frac{6}{5}$ or 3. Also the weight of the stone at R, to the pressure at H, as 2 to 5, or 1 to $\frac{5}{2}$. And the power at C, is to the pressure at H, as 1 to 3 $\times \frac{5}{2}$ or 7½.

Example IX

Let EG (*Fig.* 1. *Pl.* XIX.) be a *spinning wheel.* Diameter of the rim EF $=$ 18. Diameter of the twill *ab* $=$ 2. Diameter of the whorle *cd* $=$ 3 E*ab*F the band going about the twill. E*cd*F the band going about the whorle. Therefore, whilst the rim makes 1 revolution, the twill makes 9, and the whorle and feathers 6. Therefore there are 3 revolutions of the twill, for 2 of the feathers *n*. And, consequently, the difference of the revolutions which is 1, is the quantity taken up by the twill, whilst the thread *trn* is twined by these 2 revolutions of the feathers. The greater the difference of the revolutions of the twill and feathers, the more the wheel takes up. And the nearer an equality, the more she twines If they make equal revolutions in the same time, she will not take up at all. And if the feathers make no revolutions, she will twine none. The greater the proportion of the rim to the whorle and twill, the faster she will do both.

Example X. (*Fig.* 2. *Pl.* XIX.)

A machine to raise a weight by the force of the running water IH, carrying the wheel LK, by means of the floats F, F. Let the diameter of the wheel LK be 10; of GB, 2; of DC, 11; of AE, 3. Let the power of the water against the floats F, be 1. Then the force at B, to move the wheel CD, will be 5; again, if the power at B, be 1, the force at A will be 3 $\frac{2}{3}$. Therefore the force of the water, to the weight W, is as 1 to 5 $\times$ 3 $\frac{2}{3}$, or, as 1 to 18 $\frac{1}{3}$.

When the wheels and axles and weight are so adjusted, that the velocity of the floats at F, is $\frac{1}{3}$ the velocity of the water there, then the weight W will have the greatest motion of ascent possible. For, if any one thing be changed, whether the weight or the diameter of any wheel or axle, whilst the rest remain the same, the motion will be lessened.

Example XI. (*Fig.* 1. *Pl.* XX.)

In the machine FB, which raises the weight W, by means of the wheel EG, and the perpetual screw BE, let the circum-

ference described by the power C be 30 inches, the distance of two threads of the perpetual screw E, be 1 inch, diameter of the wheel EG = 5 feet, of DA = 2. Therefore, if the power at C be 1, the force acting at E to turn the wheel EG will be 30. And if the power at E be 1, the force at D will be $2\frac{1}{2}$. Therefore the power at C, to the weight W, is as 1 to 30 × $2\frac{1}{2}$, or 1 to 75.

Note, it is the same thing, whether CB be straight or crooked, whilst the distance BC, in a straight line, is the same; and, in measuring, you must always take the straight line BC.

Example XII. (*Fig.* 2. *Pl.* XX.)

In a machine compounded of wheels to raise a weight, let AB = 5, diameter of the barrel MN = 2, the number of teeth in the wheels and nuts, as follows; CD = 10, CE = 40, FG = 12, FH = 50, KI = 12, IL = 64. Then the power applied to B, is to the weight W, as 1 × 10 × 12 × 12 to 5 × 40 × 50 × 64; that is, as 1440 to 640,000, or as 1 to 444 $\frac{4}{9}$.

But if the power was at W, to move the weight B, then the ratio will be inverted. For then the power will be to the force at B, as 444 to 1. Or, if the velocity of B was required, you will have the velocity of W to that of B, as 1 to 444.

Example XIII. (*Fig* 1. *Pl.* XXI.)

A machine to raise a weight by help of the *triangle* ABEF, the windlass CC, and two pullies P, Q. Let the diameter HG where the rope goes, be = 2, radius CD = 5. Then, if the power at D be 1, the force at H is 5. And if the force at H, drawn by one rope, be 1, the force at W drawn by two ropes, will be 2. Therefore the power at D, to the weight W, is as 1 to 2 × 5 or 10. If the leg AB be wanting, the other two may be set against a wall, or upheld by ropes, and then it is called a pair of *sheers*.

Example XIV. (*Fig.* 2. *Pl.* XXI.)

If the weight A is to be lifted by the three pullies C, D, E, of which C is fixed; call the power at B, 1. Then the force stretching AE is 1; and both together are equal to the force of DE = 2; and force DA = 2; whence, force DC = 4; likewise, force CA = 4. Therefore the whole force acting at A is 1 + 2 + 4 = 7, and the power at B to the weight A, as 1 to 7.

Example XV. (*Fig.* 4. *Pl.* XXI.)

In this machine, *h*ACD is a running rope fixed at D, B a fixed pulley. Let the power at *h* pulling the rope *h*A be 1. That on AC 1, and CD 1. Then will AB be 2, and BC 2, BE 4. And the weight W opposing AC, BC and DC, will be 1 + 2

$+ 1 = 4$. Whence, the power at h, to the weight W, is a to 4.

Example XVI. (*Fig.* 3. *Pl.* XXI.)

Another machine with pullies. A, a fixed pulley; the ends the several ropes are fixed at B, C, D, E. Suppose the power $M = 1$, then the force on AF, FB is 1; on FG, GC, 2; GH, HD, 4; on HI, IE, 8. But the weight P acts against I IE, and is therefore $= 16$: and the power is to the weight, a to 16.

Scholium.—In a single pulley, (as *Fig.* 4. *Pl.* IV.) if a giv power at P was to be a weight or heavy body, which was to ra some other weight W, there will be the greatest motion generat in W, in any given time, when W = [illegible] P.

And in a combination of pullies, as (*Fig.* 7. *Pl.* IV.) if a weig P was to raise another weight W; and if velocity of W : veloc of P :: [illegible] P : W; then W will be the weight which will acqu the greatest motion in a given time, by that given power P.

Example XVII. (*Fig.* 2. *Pl.* XXII.)

Let DE be a *boat* rowed by oars, and let ABC be one o Here the powers acts at A, and the pin B will be the fulcrur and the force at C, acting against the water, is that which giv her motion. Let the power at A be 1; then the force at by which the boat is moved, is $\frac{AB}{BC}$. Whence, the longer AB, the shorter BC is, so much more power there is at A to move h forward.

Therefore long oars have the disadvantage of losing pow Yet the oars may be too short, as well as too long. For if th be very short, the motion of the boat will allow little time strike, and they will have but small force to act against the wat

C, in direction DB, endeavours to turn the ship round an axis passing through O, with a force which is equal to the absolute force BD × by the distance CB, or CB × BD; and this is the force by which her head is depressed. Likewise, the force BC, in direction BC, endeavours to turn the ship round an axis at O, the contrary way; and that with the force BC × distance BO, or BC × BO; and this is the force that raises her head. Therefore the force to raise her head is to the force to depress it, as CB × BO to CB × BD, or as BO to BD.

Hence, if the point D fall before O, then the sail endeavours to raise the ship's head; if it be behind O, it endeavours to sink it. If it be in O, it will keep her steady. And the height of the sail AS contributes nothing to her progressive motion; the same ratio of the absolute to the progressive force remains still as CD to DB.

EXAMPLE XIX. (*Fig.* 3. *Pl.* XXII.)

EF is a *cart* or *carriage*, BD a rub for the wheel CAD to pass over, AB the horizontal plane; DB, AC perpendicular, and OD parallel to AB. C the centre of the wheel. Then the horizontal force required to pull the wheel over the rub BD, is as $\frac{DO}{CO}$. And the difficulty of going over rubs increases in a greater ratio than that of their heights. Also the higher the wheels, the more easily they pass over them; but then they are more apt to overturn. To draw the cart with the least power over the rub BD, it should not be drawn in the horizontal direction AB or OD, but in the direction AD. The advantage of high wheels is, that they pass the rubs most easily, and they have also less friction, and sink less in the dirt, and more easily press down an obstacle. But their disadvantage is, that they easily overturn; they also make cattle draw too high; for they can apply their strength best when they draw low and upward, as in the direction AD; which is the advantage of low wheels. Yet if the wheels are high, they may be made to draw low, by fixing the limmers or traces as far below the axle as you will, which will then be an equal advantage with low wheels. For the power not pulling at the wheel, but at the carriage, may draw from any part of it. Therere is another advantage in small wheels, that they are better to turn with.

A *waggon* with four wheels is more advantageous than a cart with only two wheels, especially on sand, clay, &c. Narrow wheels and narrow plates are a disadvantage; the broader the wheels, the less they sink, and, therefore, require less draught, and, also, cut the roads less; yet they take up a great deal of dirt, which clogs the carriage. There is a great deal of friction in

all carriages, as is evident by the force required to draw them upon plain ground. And, for that reason, experience can only inform us, how much force is able to draw any carriage. To make the resistance as small as can be, axles of iron, running in brass boxes in the wheel naves, go the easiest.

The spokes in the wheel ought to be a little inclined outwards; that when a wheel sinks into a rut, the spokes (bearing then the greatest weight) may be nearly perpendicular to the horizon.

The underside of the axle-tree, where the wheels run, ought to be nearly in a right line; if they slant much upward towards the ends, the wheel will work against the lin pin. Yet this causes the wheels to be further asunder at top than at bottom in the rut, because the ends of the axle-tree are conical, which is an inconvenience.

Example XX. (*Fig.* 4. *Pl.* XXII.)

Suppose the *waggon* FG is moved forward, by a power acting within it. Which power turns the wheel DE by the spokes AD, AD, &c. and DE turns the wheel IC, which carries the waggon. Let the power at A be 1, then the force acting at E will be $\frac{DA}{DE}$; also, if the power at E be 1, the force at C, by which the waggon is moved, will be $\frac{BE}{BC}$. Therefore, the power at A, to the force by which the waggon can be moved, is as 1 to $\frac{DA \times BE}{DE \times BC}$. Or the power is to that force, as DE $\times$ BC to DA $\times$ EB. It will be the same thing, if, instead of teeth, the wheel DE carries EB by a chain going round them. You must suppose the like wheels on the opposite sides.

Hence, if the absolute force to move the waggon without, be 1, the force within, applied at A, to move it, will be $\frac{DE \times BC}{BE \times DA}$.

Example XXI. (*Fig.* 1. *Pl.* XXIII.)

ABCD are the *sails of a windmill*, all alike inclined to their common axis, and facing the wind, and turning about in the order ABCD. WC the direction of the wind parallel to the axis EH. Since WC is perpendicular to EC, draw CF in the sail perpendicular to EC; then the angle WCF will be the angle of incidence of the wind upon the sail. Therefore the force of the wind to turn the sails about the axis EH, is as the square of the sine of the angle WCF $\times$ by its cosine. And the force act-

ing against the mill, in direction of the axis EH, is as the cube of the sine of WCF. Now, since the force of the wind to turn the sails round, is as $\overline{S.WCF^2} \times$ cosine WCF; therefore, when that force is the greatest, the angle WCF will be 54° : 44'.

And this is the most advantageous position of the sails to move them from rest, and would always be so, if the wind struck them in the same angle when moving as when at rest. But by reason of the swift motion of the sails, especially near the end G, the wind strikes them under a far less angle; and not only so, but as the motion at the end G is so swift, it may strike them on the backside. Therefore, it will be more advantageous to make the angle of incidence WCF greater, and so much more as it is further from E. Therefore, at the places *n*, *o*, G, the tangents of the angles ought to be nearly as the distances, E*n*, E*o*, EG. And, therefore, the sails ought to be twisted, so as at *r* to lie more sharp to the wind, and at G almost to face it. And by that means they will avoid the back wind.

Example XXII. (*Fig* 2. *Pl.* XXIII.)

GB is a common *sucking pump*; GKL the handle; CD the bucket; E, F two clacks opening upwards. When the end L is put down, the end G raises the sucker or bucket CD, and the valve or clack F shuts; and the water above the bucket being raised, the weight of the atmosphere is taken off the water underneath in the pump. Then the pressure of the external air in the pit or well MN, raises the water up the pump, opens the valve E, and ascends through the hole B into the body of the pump DB. Again, when the handle L is raised, the bucket CD descends, the valve F opens, and lets the water ascend through it, and the pressure of the water shuts the valve E, so that the water cannot return through B. Then, whilst the end L is put down again, the sucker CD is raised again, together with the water above it, whilst more ascends through B. So that, at every stroke of the handle, water is raised into the pump, till, at last, it flows through the pipe H.

If the bucket CD be more than thirty or thirty-two foot from the surface of the water MN in the pit, no water will ascend above it; for the pressure of the atmosphere reaches no farther. Therefore it must be always within that distance, or this pump is useless for raising water.

The weight of water which the bucket lifts at each stroke is that of a column of water, whose height is MH, and its diameter that of the bore of the pump at CD, where the bucket goes. Therefore as GK to KL :: so the power applied at L, to that weight. Therefore, it signifies nothing where the bucket is placed, as to the weight of water. If a leak happens in the barrel of the

pump below the bucket CD, the air will get in and hinder working of the pump: If above CD, only some will be lo therefore CD should be placed low; but then it will be bad come at to repair it.

The bucket, sucker, or piston, is to be surrounded with l ther to fit exactly, and must move freely up and down in barrel, and must also exactly fill it. Of valves or clacks, so are flat, made of leather; others are conical: and they must fit very close, and move freely. To balance the weight of wat the handle KL is commonly made heavy, as of iron, with a kn at the end L.

The bore of the pipe at B should not be too strait; the wid it is, the more freely the water ascends, and the easier the pun works. Likewise, the longer stroke the pump makes, the mo water is raised by the same power, there being less water lost, the valves shutting.

Calculation of a common pump.

Suppose LK, 3 feet; KG, 8 inches.

h = HM the height from the water in yards:

Then the diameter of the bore at D will be $= \sqrt{\frac{100}{h}}$ inches.

And a single person will raise $\frac{80}{h}$ hogsheads of water in hour.

In many pumps for common use, it is not necessary to dra a great quantity of water, and then a smaller bore will serv as three or four inches; which will make the pump go so mu the lighter.

Example XXIII. (*Fig.* 4. *Pl.* XXIII.)

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And the same force acting at D, its power to push up the scale is CD × BE. And their difference DB × BE is the absolute force to thrust down the scale. And this force is to the whole thrusting force DE, as DB to DE. And if D were on the other side of C, the force would still be DB × BE, or $\overline{CB + CD} \times$ BE.

But if the scale E was not moveable about B, as if it were tied by the cord DE; then no force acting against any part of the beam FB, could have any effect to destroy the equilibrium.

Example XXIV. (*Fig.* 5. *Pl.* XXIII.)

Suppose a man A standing upon the plank CB, supported only at the end C, and pulling the end B towards him by the rope EB, in order to keep himself and the plank from falling.

Imagine the man and the plank to be one body; then the action and re-action, in direction EB, destroy one another, and his pulling does nothing. It would, therefore, be in vain for him to endeavour to support himself by that force; for both he and the plank must fall down together towards D, by their own weight.

Example XXV. (*Fig.* 1. *Pl.* XXIV.)

CD is a machine with two wheels fixed to an axis DF, round which goes a cord GDFE. There is a power at E endeavouring to draw the machine towards E, in a direction parallel to the horizon HO, by the cord EF going under the axis DF. In the radius AH of the wheel, take AB equal to the radius of the axle DF, towards H, because the string goes below it. Then the force to move the machine, is the same as if the string was fixed at B; where H is the fulcrum, A the weight. Then the force to move the machine towards E, with the given power E, will be as BH. Therefore, it would be in vain, by pulling at the string, to endeavour to make the body roll towards D, the contrary way. But if DF was greater than the diameter of the wheel, that is, if B falls beyond H, then the force drawing towards E, would move the body towards D the contrary way.

If the direction of the power DE be elevated above the horizon, as *fe*, then the machine could approach or recede, till the direction of the string *ef* fell upon the point of contact H, and there it would rest.

Example XXVI. (*Fig.* 2. *Pl.* XXIV.)

AB is an *artificial kite*, kept up by the wind blowing in direction WC. By drawing the string AIBIH, fixed at A and B, the kite will gain such a position, that HI produced will pass through the centre of gravity of its surface at C. Draw CO perpendicular

to BA, and DO perpendicular to the horizon HO. Then OC the direction of the force of the wind acting against the kite; a the force of the wind to keep her up, is as the square of the s of the angle ACW or COD. Now, if DO represent the giv weight of the kite, CO will be the force of the wind acting agai her, and CD the force pulling at the string. The tail EF (wit bullet F at the end) being always blown from the wind, keeps h head always towards the wind.

As the direction of the thread always passes through C, there fore the angle ACH, and, consequently, HCO, will always be t same at all altitudes. And she can never ascend so high, till t angle of altitude CHO be equal to ACH. And hence it follow that the less the angle HCO is made, the higher she will ri And, likewise, the greater the wind is, or the lighter the ki *ceteris paribus*, the higher she will rise.

After a like manner a machine, as *ab* (*Fig.* 3. *Pl.* XXIV.), m be contrived, to keep at the top of a running water, being held the string *de* tied to a stone and sunk to the bottom: *ab* is a th board, *b* a piece of lead to sink the end *b*, but the whole must lighter than water, *cd* an iron pin fixed at C. Or the machi may have a loose tail at *b*, heavier than water, as in the kite.

Example XXVII. (*Fig.* 4. *Pl.* XXIV.)

If AB is a machine to be moved by a power acting at C out the machine, in direction DC. DF, GI two levers within t machine, moveable about the two fixed fulcrums E, H.

Call the power at C, 1; then the force at F to move the lev GI, is $\frac{DE}{EF}$. Then if the force at F be 1, that at the obstacle out of the machine is $\frac{GH}{HI}$. Therefore, if the power at C be

the power 1 must be added to the force at I, and the whole is the force urging forward the machine.

Hence, if the absolute direct force to move the machine be 1, the power applied at D, which is able to move it, will be $\frac{EF \times HI}{DE \times GH - EF \times HI}$. But if the power at D act within the machine, this power could only be $\frac{EF \times HI}{DE \times GH}$; since there is then no force to be deducted for drawing back the machine.

EXAMPLE XXVIII. (*Fig*. 5. *Pl*. XXIV.)

DABH is a *wooden bridge*. AC, AD, AB, BH, BO, beams of timber. DE, EL, SR, RH, braces to strengthen the angles A, B. The stress upon any of the angles, is *ceteris paribus*, so much greater, as the angle is greater. But the strength on any angle A, is as the perpendicular AP.

EXAMPLE XXIX. (*Fig*. 1. *Pl*. XXV.)

AB a *sailing chariot*. CDEF horizontal sails, so contrived that the sails D facing the wind may expand, and those going from the wind may contract. The sails are turned about by the wind coming from any point of the compass. These sails turn the axis and trundle GH. And the trundle turns the wheel IL by the cogs in it; therefore the chariot may move in any direction. R is a rudder to steer with.

Suppose the chariot to go against the wind. Let D be the centre of pressure of the two sails C, D, the wind blows on. And let the power, (that is the force of the wind acting against the sails) be 1, then the force acting against the teeth in IL, is $\frac{GD}{OH}$. And this force being 1, the force at L is also 1. Therefore the power at D to the force at L, is as 1 to $\frac{GD}{OH}$; or as OH to GD. Now, since the mast is strained by the power falling on the sails, therefore, by this power OH, the chariot is urged backward. And by the force at L, which is GD, it is urged forward. Let R be the force of the wind upon the body of the chariot, together with the friction in moving. Therefore, if GD is greater than the radius OH + R, the chariot will move forward against the wind; if less, backward. But if they be equal, it will stand still.

EXAMPLE XXX. (*Fig*. 2. *Pl*. XXV.)

FG a *chariot* or waggon to sail against the wind. S the sails

of a windmill, turning in the order 1, 2, 3. As the sails round, the pinion A moves B, and the trundle C moves D whi has both teeth and cogs. D by its teeth moves E, and the tru dle E fixed to the axle-tree carries round the wheels H, I, whi move the waggon in direction HG.

The sails are set at an angle of 45°, so the force to turn the and the force in direction of the axis, will be equal. This wa gon will always go against the wind, provided you give the sa power enough, by the combination of the wheels. But then h motion will be so much slower.

EXAMPLE XXXI. (*Fig.* 1. *Pl.* XXVI.)

Let AB be part of a *rope*, *cd*, *cd*, *&c.* the particular stran running about in a spiral manner. Let FH be the axis of t rope, the angle GFH or HFK the obliquity of the stranc Draw KH, GH parallel to FG, FK, and draw GEK. Th the tension of the rope in direction HF, is to the stress on the strands in direction FG, as FH to FG + FK or FG + G that is, as EF to FG. Therefore, the absolute force by whi the rope is stretched, is to the strain or stress upon all t strands, or upon the twisted rope, as FE is to FG; and so is t length of any part of the rope, to the correspondent length of strand.

Hence, ropes the least twisted are strongest and bear the mo weight; and the harder they are twisted the sooner they w break. And, for the same reason, if they be double twisted, th will be weaker still. But as it is very difficult to make all t fibres pull equally without twisting, and impossible to make rope hold together without it, therefore it is necessary it have much, as to prevent the fibres from drawing out; and small degree will not much impair its strength. A rope co sisting of several strands, is thicker when twisted, than when u

the column of water BC being greater than that of BD, the pressure at C is greater than at D; and the pressure of the atmosphere being the same at D and C, therefore, the greater weight at C will make it flow out there, whilst the pressure of the atmosphere at D forces more water up the tube DB; and so keeps it continually running as long as there is any water, and the end C continues lower than the surface at D. But if C is higher than D, the water will return back into BD. But if the height DB exceed the pressure of the atmosphere, which is 30 or 32 feet, then it cannot be made to flow out at the end C; or if there be a hole in the syphon higher than the surface at D, the air will get in, and the water will return through BD. Or if the syphon be very wide, the air will insinuate itself into the end C, between the water and the tube, which will hinder it from running. To prevent which, the end C may be immersed into another vessel of water, lower than the surface at D. If the ends of the syphon be turned up, as F, G, then the water will remain in the syphon, after it has done working, which, in the other, will all run out.

Example XXXIII. (*Fig.* 3. *Pl.* XXVI.)

CDLF a vessel of water; AB a tube open at both ends, and about $\frac{1}{8}$ inch diameter. AE a quantity of mercury put into the tube. Then stopping the end B, let the other end A be immersed deep enough in the water. Then opening the end B, the mercury will sink so deep in the tube, till the height of the water AB be 14 times the height of the mercury AE, and then the mercury will be at rest.

For, the specific gravities of water and mercury being 1 and 14, the column of water AB will be equal in weight with the column of mercury AE. Therefore the pressures at A being equal, they will sustain one another.

Example XXXIV. (*Fig.* 4. *Pl.* XXVI.)

A, B, are two *barometers*; *ed* is a tube, its bore $\frac{1}{4}$ or $\frac{1}{5}$ inch diameter, at least, close at top, and communicating with the vessel C, with mercury in it. C is open to the external air. The use of this instrument is to shew the weight of the atmosphere, and its variations. This tube and vessel with mercury, is put into a frame, and hung perpendicular. Near the top of the tube is placed a scale of inches, by which the height of the mercury in the tube is known, and, likewise, a scale for the weather. At the top of the tube, above the mercury, is a vacuum. Now, the atmosphere pressing upon the surface of the mercury at C, keeps the mercury suspended at the height *d* in the tube, which, therefore, will be higher or lower according to the weight of the atmosphere. The height of the mercury in the tube is generally 28, 29, or 30 inches; seldom more. If any air get into the tube, it spoils

the machine. Lest the quicksilver stick to the glass, it is goo drum a little with the fingers upon it, in making any observat

Rules for observation of the weather.

1. The rising of the mercury presages fair weather. It r and stands highest in serene, sunshiny, droughty weather; an calm frosty weather it generally stands high. In thick foggy w ther it often rises.

2. The falling of the mercury denotes foul weather. It ge rally falls or stands low, in rainy, windy, or snowy weather.

3. In windy weather the mercury sinks lowest of all, and ri fast after storms of wind.

4. In very hot weather, the falling of the mercury foresh thunder.

5. In winter, the rising foretells frost; and falling, in fro weather, foretells thaw.

6. In continual frost, the rising presages snow. At ot times, it generaly falls in snowy weather.

7. When the mercury rises after rain, expect settled sereni if it descends after rain, expect broken showery weather.

8. When foul weather happens soon after the falling of mercury, or fair weather after its rising, expect b .t little of it.

9. In foul weather, rising fast and high, and continuing so t or three days before the foul weather be quite over, expect a co tinuance of fair weather to follow.

10. In fair weather, falling fast and low, and continuing two three days before the rain comes, expect a great deal of wet, an probably, high winds.

11. Unsettled motion of the mercury denotes unsettled weath

12. The greatest height of the mercury is upon easterly a north-easterly winds.

13. The alterations are greater in northerly parts, than in t

spout to that height above the water in C, nearly. But the pipe leading to D must be turned curve.

EXAMPLE XXXVI. (*Fig*. 6. *Pl*. XXVI.)

AB is a *dart* or an *arrow;* at A, three or four feathers are placed nearly in planes passing through the arrow. If the feathers were exactly in this plane, the air could not strike against the feathers, when the arrow is in motion. But, since they are not set perfectly straight, but always a little aslant, whilst the arrow moves forward, the air strikes the slant sides of the feathers; by which force the feathers are turned round, and with the feathers the arrow or reed. So there is generated a motion about the axis of the arrow, which motion will be swifter as they stand more aslant. This motion is like the motion of the sails and axle of a windmill, turned round by the wind. The head B is made of lead or iron, and will, therefore, go foremost in the air; and the feathered end A the hindmost, as being lighter. An arrow will fly about sixty yards in a second.

EXAMPLE XXXVII. (*Fig*. 7. *Pl*. XXVI.)

AB is a vessel which keeps its liquor till filled to a certain height; and if filled higher, lets it all run out. EFG is a crooked pipe, or crane, open at both ends. If water be poured into the vessel, it will continue in it till it rises above F, and ascend to the same height in the pipe EF. But rising above F, the pressure at E will make it run out through the pipe EFG, till the surface of the fluid descends as low as E. This is sometimes called *Tantalus's cup*. The funnel EFG may be put in the handle of this cup, which will look neater.

EXAMPLE XXXVIII. (*Fig*. 1. *Pl*. XXVII.)

BC, CG are two *bones* of an animal, moveable about the joint FK, by help of the muscle KD. The joints of animals are either spherical or circular, and the cavity they move in is accordingly either spherical or circular. And the centre of motion is in the centre of the sphere or circle, as at C. Let W be a weight hanging at B, and draw CP, CK perpendicular to BW, KD. Then, if the weight W be suspended by the strength of the muscle KD, it will be as CK : PC :: W : tension of the muscle KD.

The bone BC is moved about the joint FK, by the strength of the muscle KD. For when the muscle is contracted, the point K is moved towards D, and the end B towards E, about the immoveable centre of motion C. The strength of the muscles is surprisingly great.

Borelli, (in his book, *De Motu Animalium*, Part I. Prop. XXII.)

computes the force of the muscles to bend the arm at the elbow, and says, a strong young fellow can sustain at arm's end a weight of 28 lb., taking in the weight of the arm. And he finds the length of CB to CK to be in a greater proportion than that of twenty to one. Whence, he infers the strength of these muscles to be so great, as to bear a stretch at least of 560 lb.

It is evident, that all animal bodies are machines. For what are the bones but levers, moved by a certain power placed in the muscles, which act as so many ropes, pulling at the bones, and moving them about the joints? Every joint representing the fulcrum, or centre of motion. What are all the vessels but tubes, which contain fluids of different sorts, destined for the use or motion of the several parts of the machine? and which, by opening or shutting certain valves, let out or retain their contents, as occasion requires; or convey them to distant places, by other tubes communicating therewith. And, therefore, all these motions of an animal body are subject to the general laws of mechanics.

Example XXXIX. (*Fig.* 2. *Pl.* XXVII.)

The motion of a *man* walking, running, &c. will easily be accounted for. Let us first suppose a man sitting in a chair; he cannot rise from his seat, till, by thrusting his head and body forward, and his feet and legs backward, the line of direction, or the perpendicular from the centre of gravity, pass through his feet, as the base. Likewise, when we stand upon our feet, the line of direction must fall between our feet; otherwise, we cannot stand, but must fall down towards the side the centre of gravity lies on. And when a man stands firm upon his feet, his legs make an isoceles triangle, the centre of gravity lying between them. And then he is not supported by the strength of the muscles, but by the bones of the legs and thighs, which then stand in a right line with one another.

When a man AC endeavours to walk, he first extends his hindmost leg and foot S almost to a right line, and, at the same time, bends a little the knee H of his fore-leg. Thus, his hind-leg is lengthened, and his fore-leg shortened; by this means, his body is moved forward, till the perpendicular from the centre of gravity, as AV falls beyond the fore-foot B; and then, being ready to fall, he presently prevents it, by taking up the hind-foot, and by bending the joints of the hip, knee, and ancle, and suddenly translating it forward to T, beyond the centre of gravity: and thus he gains a new station. After the same manner, by extending the foot and leg HB, and thrusting forward the centre of gravity beyond the foot S, and then translating the foot B forward, he gains a third station. And thus is walking continued at pleasure.

His two feet do not go in one right line, but in two lines paral-

lel to one another. Therefore, a man walking has a libratory motion from one side to the other; and it is not possible to walk in a right line.

Walking on plain ground is easy, pleasant, and performed with little labour. But in going up hill is very laborious, by reason of the great flexure of the joints required to ascend, and their suffering more stress from the weight of the body in that position. Descending down hill is, for the same reason, more laborious than walking on plain ground, but not so bad as ascending.

The walking of birds is not unlike that of men; only their weight is entirely supported by the strength of the muscles, since their joints are always bent. Also, their feet go in two parallel lines.

A man, in walking, always sets down one foot before the other be taken up; and, therefore, at every step he has both feet upon the ground. But, in running, he never sets one down till the other be up. So that at each step he has but one foot upon the ground, and all the intermediate time none. A good footman will run 400 yards in a minute.

Example XL. (*Fig.* 3. *Pl.* XXVII.)

When a *beast* stands, the line of gravity must fall within the quadrilateral made by his four feet. And when he walks, he has always three feet on the ground, and one up. Suppose he first takes up the hind-foot from C. Before he does this, by extending his leg backwards, he thrusts forward his body and the centre of gravity; then, taking up the foot from C, he moves it forward to F. Then he immediately takes up the fore-foot from B on the same side, and carries it to H; then he takes up the hind-foot D, and translates it forward; and then the fore-foot at A; then F again, and so on.

When he trots, he takes up two together, and sets down two together, diagonally opposite.

When he gallops, he takes up his feet one by one, and sets them down one by one; though some animals strike with the two fore-feet nearly at once, and the two hind-feet near at once; and have not above two feet on the ground at once. A good horse will run half a mile in a minute.

Animals with six or more feet, take up the hindmost first; then the next, and then the next in order, to the foremost, all on one side; and after that, all the feet on the other side, in the same order, beginning at the last. If they were to take up the foremost first, the animals would move backward.

Example XLI. (*Fig.* 4. *Pl.* XXVII.)

AD is a *bird* flying in the air, by help of the wings F, T, and

the tail C. The structure of their wings are such, that, in striki downward, they expand to their greatest breadth, and become most two planes, being something hollow on the under side. A these planes are not then horizontal, but inclined, so that the b part K is higher than the fore part DFG. But in moving wings upward, to fetch a new stroke, they go with the edge D foremost, and the wings contract and become hollow. Th bodies are specifically lighter than men or beasts. Their bo and feathers are extremely porous, hollow, and light. The m cles, by which their wings are moved downwards, are exceedi large, being not less than a sixth part of the weight of the wh body. When a bird is upon the ground and intends to fly, takes a large leap; and stretching his wings right from his bo he strikes them downwards with great force, by which they put into an oblique position; and the resistance of the air acti strongly against them by the stroke, impels them, and the bird, a direction perpendicular to their planes; which is in an obliq direction, or partly upwards, and partly horizontally forward: part of the force tending upwards is destroyed by the weight the bird; the horizontal force serves to carry him forward. T stroke being over, he moves his wings upwards, which being co tracted, and turning their edges upward, they cut through the without any resistance, and being sufficiently elevated, he takes second stroke downwards, and the impulse of the air moves h forwards, as before. And so from one stroke to another, whi are only like so many leaps taken in the air. When he has mind to turn to the right or left, he strikes strongly with the opp site wing, which impels him to the contrary side. The tail a like the rudder of a ship, except only that it moves them upwar or downwards, instead of sideways, because its plane is horizont If a bird wants to rise, he puts his tail in the position LH; or

such a weight. For, in a man, they are not the 60th part of the rest of the muscles of the body; but, in a bird, they are more than all the others put together.

Some birds will fly 1000 yards in a minute.

Example XLII. (*Fig.* 5. *Pl.* XXVII.)

AB is a *fish* swimming; which he does by help of his fins and tail. A fish is nearly of the same specific gravity as water; and most fish have a bladder L, which they can expand or contract, and so make themselves lighter or heavier than water, in order to rise or fall in it. The muscular force by which the tail is moved is very great. The direct motion of a fish is by means of his tail BCD moving from one side to the other, with a vibrating motion, which he performs thus. Suppose his tail in the position FG, being about to move it successively to H, I, and K; he turns the end G oblique to the water, which being moved swiftly through it in that position, the resistance of the water acts obliquely against his tail, and moves him partly forward and partly laterally. The lateral motion is corrected, the next stroke, the contrary way; but the progressive motion is continued always forward. When his tail is arrived at K, he turns its obliquity the contrary way, that, in moving back to G, it may strike the water in the same manner as before. And thus he makes one stroke after another, and moves forward thereby as far as he pleases. The oblique position of his tail is mostly owing to the elasticity of his tail, which, by bending, is put into that form by the resistance of the water. They can exert a very great force with their tail, and which is necessary, to overcome the resistance which their bodies meet with in the water. By help of the tail, they also turn to one side, by striking strongly with it on that side, and keeping it bent, which then acts like the rudder of a ship. The fins of a fish serve to keep him upright, especially the belly-fins E, which act like two feet; without them he would swim with his belly up, for his centre of gravity lies near his back. His fins also help him to ascend or descend, by expanding or contracting them, as he can with pleasure, and so putting them in a proper position. His tail will also help him to rise and fall, by inclining it obliquely, and turning it a little from an erect position to one side. Fish can swim but slow, yet some of them will swim seventy or eighty yards in a minute; but they soon tire.

Brutes can swim naturally, for they are specifically lighter than water, and require to have but a small part of their head out for breathing. Also, they naturally use their legs in swimming, after the same manner as they do when walking.

Birds swim very easily, being much lighter than water; an readily move themselves along with their web feet.

Men cannot swim naturally, though they are specifically light than water. For their heads are very large, and require to be a most all out of the water for breathing. And their way of strikin has no relation to that of walking. Men attain the art of swimmin by practice and industry. And this art consists in striking alte nately with the hands and feet in the water, which, like oars, will ro him forward. When he strikes with his hands, he neither keeps th palms parallel nor perpendicular to the horizon, but inclined. An his hands striking the water obliquely, the resistance of the wat moves him partly upward, and partly forward. Whilst his hand are striking, he gradually draws up his feet; and when the stroke his arms is over, he strikes with his feet, by extending his legs, an thrusting the soles of his feet full against the water. And whi he strikes with his legs, he brings about his arms, for a ne stroke, and so on alternately. He must keep his body a litt oblique, that he may more easily erect his head, and keep his mou above water.

After the same manner may the motions, velocities, powers, an properties of any machine be explained and accounted for, by m chanical principles. I shall proceed to lay down a short descriptio of several other machines, without being so particular in the calc lation of their powers and forces. The mechanism of which bein understood, will assist the invention of the practical mechanic, i contriving a machine for any use.

Example XLIII. (*Fig.* 1. *Pl.* XXVIII.)

AB a machine to raise a weight, and stay it in any position. C a roller turned by the handle E. To the roller is fixed th racket wheel F. GH is a catch made of metal, moveable abo

EXAMPLE XLV. (*Fig.* 3. *Pl.* XXVIII.)

CD another machine to stay a weight in any position. This is only a cylinder of wood, upon which is cut a channel for the rope to go in. If the weight B be lifted up, and A pulled down, then B will remain in any given position, by the friction of the cylinder and rope. And there may be taken as many turns of the rope about the cylinder, as there is occasion for.

EXAMPLE XLVI. (*Fig.* 4. *Pl.* XXVIII.)

C is a clock weight carrying the wheels A and B. D the counterpoise. F a pulley. ADBFA an endless cord. When the weight is down, draw the cord G, till the weight C rise to the top; then the catch *e* keeps the wheel A from turning backwards. This may be serviceable for other uses, besides moving a clock.

EXAMPLE XLVII. (*Fig.* 1. *Pl.* XXIX.)

ADB a machine for reckoning the number of strokes or vibrations made. DH is a wheel moving about a fixed axis, upon the neck of which axis goes a brass spring L, to keep the wheel from shaking. AB a piece of wood or metal, cut away between I and K to receive the wheel. The plane of the piece AIKB is perpendicular to the plane of the wheel. FG are two staples, to guide the motion of the piece AB back and forward. When the piece AB is moved from A towards B, the edge at I catches the tooth C, and sliding along the edge, moves the wheel about in direction CD; this brings the tooth E to the edge K. And when the piece AB is moved back from B to A, the edge at K sliding down the tooth E, moves the wheel from E towards H, which brings another tooth before the edge I; so that, at every motion of AB back and forward, the wheel is moved the breadth of one tooth. And if the teeth be numbered, the index M will shew when the wheel has made one revolution.

EXAMPLE XLVIII. (*Fig.* 1. *Pl.* XXX.)

ABED a machine moving one circle within another, concentrical to it. ABC represents a flat ring of brass; and *abc* a smaller concentric ring lodged in a circular groove, turned within the larger, and kept in the groove by three small plates of brass A, B, C, fixed to the outward ring, and reaching over the edge of the inner one. Upon the inner ring is fixed a concentric arch of a wheel *de*, having teeth in it, which are driven round by the threads of an endless screw DF, turning in a collar at E, and upon a point at F, both fixed to the outward ring. By this mechanism, any point of the circle *abc*, may be set to a given point of the circle ABF, by turning the screw DEF.

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part of the chimney, where the motion of the air is swiftest, a

the water, 3 yards. All the machine, except the water wheel, is within the house.

A hammer may also be made to strike thus: A (*Fig.* 1. *Pl.* XXXII.) is the hammer moveable about the point C. G the axle of a water wheel, in which axis are the pins or curved teeth F, E, &c. As the wheel and axle goes about from F towards E, the pins or teeth F, E thrust down the end B, and raise the end A of the hammer. And when the end B goes off the pin, or tooth, the hammer falls upon the anvil D.

Example LIII. (*Fig.* 2. *Pl.* XXXII.)

I, I a crooked axis, elbow, or crank, for the *suckers of pumps*. IK the pestle or chain of the sucker. Upon the axis is the lantern EF, which is turned by a great wheel, carried either by water, or men, or horses. The pestles IK rise and fall alternately, as the lantern EF goes about, and each gives one stroke of the pump for one turn of the lantern. Place pullies or rolls at *a*, *b*, *c*, *d*, for the chain IK to work against, when it goes out of its perpendicular position, by the obliquity of the motion of the cranks I, I.

Example LIV. (*Fig.* 3. *Pl.* XXXII.)

ABCD a particular combination of *pullies*. T, T, T are posts to which the tackles are fixed. S, S, S are stays to keep them erect. If the power at A be 1, that at B is 3; at C, 9; and at D 27, where the weight is placed.

Example LV. (*Fig.* 1. *Pl.* XXXIII.)

A, B are two *bellows* going by water, and blowing alternately, but neither of them with a continual blast; W the water wheel. DE the direction of the water. FG the axis of the wheel; *a*, *a*, &c., 4 cogs of wood in the axis, forcing down the end of the bellows A. *bb*, &c. 4 cogs forcing down the end of the bellows B. LM, NI two rods of iron fastened to the bellows and to the lever MN, and moveable about the pins M, N. SP a piece of timber moveable about S and P. OP a beam serving for a spring, lying over the piece of timber QR. As the wheel and axle turns round, a cog *b* forces down the end of the bellows B, and makes it blow; this pulls down the end N and raises the end M of the lever MN, which raises the bellows A. And when the cog *b* goes off, the bellows B ceases blowing; and a cog *a* forces down the bellows A, and makes it blow; and at the same time raises the bellows B. And thus the cogs *a*, *b* alternately force down the bellows A, B, and make them blow in their turns. H is the hearth or fire.

A pair of bellows may be moved by water thus: A (*Fig.* 1. *Pl.*

XXXIV.) is a water wheel, carried by the water at W. CD a r
of iron going on the crooked axle-tree, or crank, of the wheel; I
a lever moveable about E. FG a chain going to the bellows
I a weight. As the wheel goes about, the ends D and F of t
lever DF, rise and fall: which motion raises the bellows, and t
weight I carries them down again.

Example LVI. (*Fig.* 1. *Pl.* XXXV.)

A and B are wheels with teeth, and a *roller* to draw up a
weight: H, H, H the handles, which may be wrought by two
three men.

But the easiest and simplest rollers for common use are su
as C and D (*Figures* 1 *and* 2. *Pl.* XXXVI.) In these, as 30
is the weight to be raised, so must the radius of the axle be to t
length of the handle, for a man to work it.

If a given weight P raises another weight W, on such a machi
as (*Fig.* 8. *Pl.* III.) it will generate the greatest motion possible
a given time; when the diameters AB, EF, and weight W are
such quantities, that $W = \frac{[illegible]P \times AB}{EF}$ or when $EF = \frac{[illegible] AB \times}{W}$
For then the motion will be greater, than if any one (H, AB,
EF) be altered, the rest remaining the same.

And in such a machine as (*Fig.* 1. *Pl.* IV.) the greatest moti
will be generated in W; if you make, as velocity of W : veloc
of P :: as [illegible]P : as W.

Example LVII. (*Fig.* 2. *Pl.* XXXV.)

An engine to drive piles. A the rammer, drawn up by the ro
BCD going over the pulley C. DN, DN several small ropes f
several men to pull at. M the pile. EF a brace and ladder to
up. The rammer A is bound at bottom with iron, lest it spl

cavity. DE a lever moveable about the axis GH. At I a weight is laid upon the upper board to make it fall. The bellows is fixed in the frame MK, by two iron pins, which are fast in the middle board. And the pipe P lies upon the hearth. When the end E is pulled down by the rope EF, the end D is raised, and the rope or chain DR raises the lower board CL; this shuts the valve T and opens S, and the air is forced into the upper cavity, which raises the upper board, and blows through the pipe P. And when E is raised, the boards A and C descend, and the valve S shuts, and T opens. And the weight I forces the air still out of the pipe, whilst more air enters in at the valve T; which, when C ascends, is forced again through the valve S as before. And thus the bellows have a continual blast.

Example LIX. (*Fig.* 1. *Pl.* XXXVII.)

An engine to raise water. LMOI a great horizontal wheel. ABP the axis, P the pivot or spindle it turns upon. OQI the waves of the great wheel. QR a small wheel perpendicular to the horizon, and placed under the edge of the great wheel: this wheel is moveable about the centre C, in the end of the lever EFC, which is moveable about the centre D; EF the arch of a circle, drawn from D as a centre, whose plane is perpendicular to the horizon, and in the plane of the wheel QR. EG the chain of a pump.

Whilst the great wheel is turned by the lever NA, from O towards I, the wave Q presses down the wheel QR, and raises the end E, which draws up the water in the pump G. But when the deepest part of the wave is past the wheel QR, the wheel then rises up into the hollow S, and then the chain EG descends, till the next wave raises it again. And thus every wave makes a stroke of the pump.

The wheel QR is placed there only to avoid friction, and so that a perpendicular to its plane may pass through AB. If the number of waves be odd, and another pump wheel and lever be placed diametrically opposite, on the other side of the great wheel, then these acting by turns will keep the motion uniform, and the power at N will always act equally.

Example LX. (*Fig.* 2. *Pl.* XXXVII.)

BFG a *capstan*, to draw great weights. BC the axis, which is driven about by men acting at A, A, by help of the levers AB, AB. Here must only be 3 or 4 spires of the rope DCE folded about the axis BC; for the axis could not hold so much rope as there is sometimes occasion for. And to hinder the rope from slipping back, a man constantly pulls at E to keep it light.

And the axis is made conical, or rather angular at the bott
C, to keep the rope from going any lower, whilst the caps
goes about.

Example LXI. (Fig. 1. Pl. XXXVIII.)

AB is a rack to lift great weights. E is a pinion upon axis FG, GC a toothed wheel, and D a pinion upon the sa axis, working in the teeth of the rack AB. The whole is closed in a strong case KL, all of metal. The handle G goes on the axis FG at the backside of the case.

When a weight is to be lifted, the forked end A is put un the weight; then, turning the handle HI, the pinion E moves wheel GC, with the pinion D; and D raises the rack AB, w the weight.

Example LXII. (Fig. 2. Pl. XXXVIII.)

An engine to raise and let fall two weights with contr motions successively, whilst the moving power acts always o way. GH a great horizontal wheel. N, M two lanterns, placed on the axis AB that the great wheel can only work o of them at once. When the cog wheel GH is turned by t lever LL, it turns the lantern M, and raises the bucket whilst F descends. Then E being raised, move forward t axis AB, that the lantern M may leave the wheel, and N co to it. Then the great wheel moving the same way as befo will now work upon N, and turn the axis the contrary way, a raise the bucket F whilst E descends. Which done, move t axis back towards A, and you will again rise the bucket and so on.

This may also be performed by placing the lanterns M, N, that the great wheel may work them both at once; but maki

slitting iron bars. C, D are 8 inches diameter; A and B about 12. And these cylinders may be taken out and others put in, and may be brought nearer to, or farther from, one another, by help of screws, which screw up the sockets where the axles run. The axles of N, I, K lie all in one horizontal plane. And so does M, G, H. But the cylinders A, B, and also C, D, lie one above another.

For making the plates; if a bar of iron be heated and made thin at the end, and that end put in between the cylinders C, D, whilst the mill is going, the motion of the cylinders draws it through, on the other side, into a thin plate. Likewise a bar of iron, being heated and thinned at the end, and put in between the toothed cylinders A, B, it is drawn through on the other side, and slit into several pieces, or strings. And then, if there be occasion, any of these strings may be put through the plate mill with the same heat, and made into plates.

OPQ is the sheers for clipping bars of cold iron into lengths. V a cog in the axis of the water wheel. OP one side of the sheers made of steel, and moveable about P. The plane LPR is perpendicular to the horizon. When the mill goes about, the cog V raises the side OP, which, as it rises, clips the bar TQ into two, by the edges SP, RP. All the engine, except the water wheels E, F, is within the house.

Example LXIV. (*Fig.* 1. *Pl.* XL.)

AFC a *windmill* to frighten birds from corn or fruit. This is made of wood. The sails F, F a foot long, and their planes inclined to the axis BC, 45 or 50 degrees. The piece B goes upon the end of the axis BC, and is pinned fast on, and the sails and axis turn round together; and the axis goes through the board AD, and is kept from flying out of the hole, by the piece B pinned fast. The whole machine is moveable about the perpendicular staff AG, by which means the wind turns the mill about the axis AG, till the plane AD lies directly from the wind; and then the sails face it. At S, is a spring to clack as it goes about; and the like on the other side.

Example LXV. (*Fig.* 2. *Pl.* XL.)

An *anemoscope*, to show the turnings of the wind. CD is a weather-cock of thin metal, fixed fast to the long perpendicular axis DF, which turns with the least wind upon the foot F, and goes through the top of the house RS. To this axis is fixed the pinion A, which works in the crown wheel B, of an equal number of teeth. The crown wheel is fixed on the axis PI, on the end of which the index NS is fixed. The axis PI goes through the wall LM: against the wall is placed the circle NESW, with the

points of the compass round it. Then, if the vane CD be set to the north, and at the same time the index SN fixed on the axis PI, to point at S; then, however the wind varies, it will turn the vane CD, and pinion A; and A turns the wheel B with the index N; so that the index will always be directed to the opposite point of the compass to the vane DC, or to the same as the wind is in.

EXAMPLE LXVI. (*Fig.* 3. *Pl.* XL.)

DEF is a *rag-pump*, or *chain-pump*. EF the barrel, CD the roller. GH an endless chain, to which are fixed several leather buckets I, I, hollow on the upper side that ascends. AB the handle. The use of this is to cleanse foul waters from dirt and rubbish. The roller is ribbed to hinder the chain from slipping, in working. When the roller is turned, it draws up the chain through the pump, with whatever is in the water, and discharges it at the top. Instead of the roller CD, a wheel like a trundle may be used, called the rag wheel.

EXAMPLE LXVII. (*Fig.* 1. *Pl.* XLI.)

A *dyer's and fuller's mill*. A the great wheel carried about by horses. This turns the trundles B, C, D, together with E, F, G. Then E turns the cog-wheel H, with the axis IK, and the cross pieces L, L; 1, 1, &c. are pullies or rollers. MN, MN wooden beaters, turning upon an axis passing through N, N. Whilst the axis IK turns about, the end *b* slides along the pulley 1; and falling off, the part M strikes against the cloth in the trough at O, O. The lantern F carries the cog-wheel P, and the cranks Q, Q, which work the pumps T, T, by help of the levers RS, moveable about *a*. The trundle G carries the cog-wheels V and W, and W carries the trundle X, with the piston Y that grinds the indigo in the vessel *cd*; from whence it flows to the vessel Z. The ends *m*, *m*, &c. of all the axles, run in pieces of timber going cross the mill, and fastened to one another and to the walls of the house.

EXAMPLE LXVIII. (*Fig.* 4. *Pl.* XL.)

A machine to empty standing waters. This is no more than a large pipe or syphon ABC, being extremely close and tight that no air can get in.

If the pool of water DE is to be emptied over the hill DHG, let the pipe be placed with its mouth A within the water DE, and the mouth C within the water FG, if the pipe be very large. Then stop up A and C, and fill the pipe with water by the cock B at the top. Then, stopping the cock B very close, open A and C; and the water will flow through the pipe from DE

into FG, which may run over at F, at a small height above C, and go away.

Note, the end C must always be lower than A, and the height of the top B above DE must not exceed 11 yards; for if it do, the water will not flow. If the pipe be very strait, the end C need not be immersed in the water; but if large it must; or else the air will insinuate itself into the pipe at C, and hinder the flux of the water.

Example LXIX. (*Fig.* 1. *Pl.* XLII.)

EFGH is a *coal gin*. E the cog-wheel, 11 feet diameter, and 72 cogs; this carries the trundle F, near 2 feet diameter, and 12 rounds, together with the roll G, 4 feet diameter. AH is the start, 20 feet long. The axis AB runs upon the kevy-stock C. There are two cross-trees IK, at the top, through which the axis AB goes. These cross-trees are supported by four posts KL at the four corners. When the coals are to be drawn up out of the pit, two horses are yoked at H, and go round in the path OQD, and draw the wheel about. And whilst the loaded corf N, is drawn up to the top of the shaft M, by the rope going round the roll, the empty one, at the other end of the rope, is descending to the bottom. And the loaded corf N being taken off, and an empty one put on, the horses are turned, and made to draw the contrary way about, till the other corf comes to the top loaded: and so as one corf ascends, the other descends alternately. A corf of coals weighs about five hundred weight, and contains about four and a half bushels. A pit is forty or fifty fathom deep. And fifty fathom of the rope weighs about three hundred weight.

Example LXX. (*Fig.* 1. *Pl.* XLIII.)

A *worm jack* for turning a spit. ABC the barrel round which the cord QR is wound. KL the main wheel of sixty teeth. N the worm wheel of about thirty teeth, cut obliquely. LM the pinion of fifteen or sixteen. O the worm or endless screw, on which are two threads or worms going round, and making an angle with the axis of sixty or seventy-five degrees. X the stud; Z the loop of the worm spindle. P a heavy wheel or fly to make the motion uniform. DG the struck wheel fixed to the axis FD. S, S several holes in the frame, to nail it to a board, which is to be nailed against a wall; the end D going through it. HI the handle, going upon the axis ET, to wind up the weight when down. R are fixed pullies; V moveable pullies with the weight. The axis ET is fixed in the barrel AC, and this axis being hollow, both it and the barrel turn round upon the axis FD, which is fixed to the wheel KL, turning in the order BTA, but cannot turn the

contrary way, by reason of a catch nailed to the end AB, wh
lays hold of the cross bars in the wheel LK.

The weight, by means of the cord QR, carries about the ba
AB, which, by means of the catch, carries the wheel KL, wh
carries the nut LM and wheel N, which carries the worm O v
the fly P. Also the wheel LM carries the axis FD with the wh
DG, which carries the cord or chain that goes about the spit he
(a wheel like DG) which turns the spit. The more pullies at
and V, the longer the jack will go; but then the weight must
greater.

The catch lies between the end AB of the barrel and the wh
KL, and is thus described: *ff* the barrel, *n* the main spindle;
a tumbler moving easy on the centre pin *a*, fastened to an i
plate, nailed to the barrel; *b* a collar of iron, turning a little s
on the spindle: from this proceeds the tongue *bc*, passing throu
the hole *c* in the tumbler; *r* the catch of the tumbler. No
whilst the barrel with the catch is turned about, in the order e
upon the axis *n*, the collar is drawn about by the tongue *bc*; wh
tongue acting backwards, turns the tumbler about the centre *a*, a
depresses the catch *r*. But the barrel being turned the contr
way, the tongue then acts towards *d*; this depresses the end *d*, a
raises the catch *r*, which then takes the cross bars of the ma
wheel, and stops the barrel. This catch would also serve for
clock, and is better than a spring catch, because it makes no noi
in winding-up.

Note, the jack need not be placed so that the axis FD be para
lel to the spit; but any way it can conveniently, For it is
matter whether the chain crosses or not.

Example LXXI. (*Fig.* 2. *Pl.* XLIII.)

DAF the *hydrostatical bellows*. AB, EF, two flat boards of oa

stones, six or seven feet diameter. The lower stone N is the *lyer*, being fixed immoveable upon beams of wood: and the upper stone is the *runner*, and is fastened to the *spindle* LI, by a piece of iron, called the *rind*, fixed in the lower side of the stone, to go square upon the spindle; between which and the stone, there is room left for the corn to fall through upon the lower stone. The spindle goes through the lower stone, and is made so tight with a wooden *bush*, as to turn round in it easily. The upper stone, with the spindle LI, is supported on the end I, upon a horizontal beam of wood FE, called the *bridge;* the end F being fixed, and the end E lying upon the beam HG, fixed at G, called the *brayer* or *bearer*. The end H is supported by the *lifting-tree* HK, by help of a wedge at K. By this means, the upper stone may be raised or lowered. For if KH be raised with the lever K*k*, the end E, the axis LI, with the stone M, and the piece GH are all raised, and may be fixed there by the wedge K. Thus, the stones may be set as near or far off as you will. The lower stone is broader than the upper stone, and is feathered, or cut into small channels, to convey the flour out; and is enclosed with boards all around, as *ab*, close to the lower stone, and above the edge of the under one, to keep the meal in. And through one side of the boards is a hole, called the *mill-eye*, through which the meal runs out into a *trough*.

The surfaces of the mill-stones are not flat, but conical; the upper one an inch hollow, the under one swells up $\frac{3}{4}$ of an inch: so the two stones are wider about the middle, and come nearer and nearer towards the outside; which gives room for the corn to go in, as far as $\frac{2}{3}$ of the radius, where it begins to be ground. The upper stone has a dancing motion up and down, by the springing of the bridge, which helps to grind the corn. The flour, as soon as made, is thrown to the outside, by the circulation of the stone and the air, and driven out at the mill-eye. The quantity of flour ground, is nearly as the velocity and weight of the stone. The stone ought not to go round above once in a second, for bread corn.

The corn is put into the *hopper* S, which falling down into the *shoe* TV, runs into the hole at the top of the stone M. The axis LM is made with six or eight angles; which, as it turns about, shakes the end V of the shoe, and keeps the corn always running down. The axis LM may be taken off. PQ is the direction of the water, which, acting against the floats R, carries about the water-wheel A, and cog-wheel D, which cog-wheel carries the lantern or trundle O, and the upper stone M that grinds the corn.

Sometimes one water-wheel, A, carries two pair of stones, and then two cog-wheels, as D, are put into the axis BC, which carry two trundles with the stones. Otherwise, the cog-wheel D carries

a trundle O, and spur-wheel; which spur-wheel carries two lanterns with the stones, one lantern on each side the wheel. Or, sometimes, the same cog-wheel D carries another lantern and cog-wheel, whose axis is parallel to the horizon; and this cog-wheel carries another lantern with the stones. And the trundle is such, as to make the blue stones, or those that grind wheat flour, go near twice as swift as the grey stones do. In these cases, when one pair of stones is to stand still, there is either a loose rung to be taken out of its lantern, or else the bridge EF is shifted towards H, till the lantern O be clear of D.

The diameter of the water-wheel A must not be too large, for then it will move too slow; nor too little, for then it will want power. When a mill is in perfection, the velocity of the floats, wings, or hands R, upon the water-wheel must be ⅓ the velocity of the stream.

The higher fall the water has, the less of it will serve to carry the mill. In an undershot mill, where the water comes underneath the wheel, it is brought by a narrow channel called the *mill race*. The water is kept up in the mill dam, and let out by the *penstock*, when the mill is to go: and the penstock is raised or let down by help of a lever. The penstock being raised, opens a passage to the water, ten or twelve inches wide, through which it flows to the wheel. And when the mill is to stop, the penstock is let down, and the orifice stopped.

When the water comes underneath the wheel, it is called an *undershot mill*. But if it comes over the wheel (as *Fig.* 2. *Pl.* LV.) it is called an *overfall* or *overshot mill*. This requires less water than an undershot mill; but there is not convenience in all places to make them. The water is brought to the wheel of an overfall mill by a trough, which is turned aside to throw the water off the wheel when the mill does not go.

A *breast mill* is that where the water is delivered into boxes, at about the height of the axis of the wheel, and moves the wheel by its weight. This requires more water than either of the other sorts.

A good overfall mill will grind two and a half or three bushels of corn in an hour; and in that time requires 100 hogsheads of water, having ten or twelve feet fall.

EXAMPLE LXXIII. (*Fig.* 1. *Pl.* XLV.)

AB is a trap to catch vermin, made of boards. GH a piece of wood suspended over the bar IL, by the lever DE, moveable about D, and the thread FE tied, to the start CK. *lm*B a piece of flat wood, moveable about *lm*, and lying on the bottom, whose end B comes through a hole in the side, in which is a catch to take hold of the end K of the start, when the trap is set. When the vermin

go into the trap, they tread upon the board *lm*B, on which a bait is laid, which puts down the end B, and the start CK flies up. This gives liberty to the rod DE to rise up; then the piece of wood GH falls down, and knocks them on the head. If two pieces of board were hailed on the ends G, H, to reach below the piece of wood GH, the trap would take the vermin alive.

ABE (*Fig.* 2. *Pl.* XLV.) is another *trap*, the end B is wire, and the end A slides up and down in two grooves in the sides. When the trap is set, the end A is suspended by the thread CD, tied to the rod DI, moveable about O, the end I being held by the crooked end of the wire IS, moveable about R, the end RS going within the trap. A bait is put on the end at S, and the end E of the trap being open, the vermin goes in and pulls at the bait S; this pulls the catch I from off the end of the lever ID, which lets the end A fall down, and the vermin is taken.

Example LXXIV. (*Fig.* 2. *Pl.* XLIV.)

An engine for moving several *saws* for the sawing of stones, &c. ILLI is a square frame, perpendicular to the horizon, moving in direction LL, in grooves made in the fixed beams AM, CB; and running upon little wheels. IL two rods of iron fixed at I and L. *op* two hands of iron running along these rods; to these are fixed the saws S, S. HIK is a triangle fixed to the axis of a great wheel. As the wheel and triangle go about from H towards I, the point I acting against the piece G, moves the frame towards MB, together with the saws S, S. When I is gone off, the angle K acts against the piece F, and moves the frame back again. Then H acting against G, moves it forward; and so the saws are moved back and forward, as long as the wheel turns round. As these saws work by the motion of the engine, the hands *op* descend. The parts F and G ought to be made curve; and little wheels may be applied at the points of the triangle HIK, to take away the friction against F and G. The axle of the wheel may be made to carry more triangles, and work more saws, if the power is strong enough.

Instead of the triangle HIK, the frame may be moved by the two pieces *ab*, *cd*, going through the axis, across to one another. So that *ab* may only act on F, and *cd* on G. F being only in the plane of *ab*'s motion, and G in that of *cd*. So that F never falls in the way of *cd*, nor G in the way of *ab*.

Example LXXV. (*Fig.* 3. *Pl.* XLV.)

A is an *eolipile*. This is a hollow globe of brass, with only a very small hole at the mouth. Take it by the middle B, and set it on a fire till it is heated; then plunge it in cold water, and the air in it, which was rarified, will be condensed; and water will

go into it, till it be about half full. Then, if it be set on the fire, the water will turn into vapour by the heat, and will blow out at the mouth with great violence, and continue so till the water is spent.

Example LXXVI. (*Fig.* 4. *Pl.* LXV.)

ABD is a *hygroscope*. BC is an index hung by the (therm) string AB, the point B hanging over the centre of a circle, which is divided into equal parts. The string AB twists and untwists by the moisture or dryness of the air. By this means, the index BC turns about, and shews the degrees of drought or moisture, on the circumference DC.

Example LXXVII. (*Fig.* 1. *Pl.* XLVI.)

A *windmill*. AHO the upper room, HOZ the under one. AB the *axle-tree*, going quite through the mill. STVW the *sails* covered with canvas, set obliquely to the wind, and going about in the order STVW, their length about six yards, and breadth two. CD the *cog-wheel* of about forty-eight cogs, *a, a, a*; which carries the *lantern* EF, of eight or nine rounds, *c, c*, and its axis GN. IK the *upper stone* or *runner*; LM the *lower stone*. QR the *bridge*, supporting the axis or *spindle* GN The bridge is supported by the beams *cd*, XY, wedged up at *c, d*, and X. ZY the *lifting tree* standing upright: *ab, ef*, levers whose centres of motion are Z, *u*: *fghi* a cord with a stone *i*, for a balance, going about the pins *gh*. The spindle *t*N is fixed to the upper stone IK, by a piece of iron called the *rind*, fixed in the under side of the stone. The upper stone only turns about, and its whole weight rests upon the bridge QR, and turns upon a hard stone fixed at N. The trundle EF and axis G*t* may be taken away; for it fixes on the lower part at *t*, by a square socket, and the top runs in the edge of the beam *w*. Putting down the end *f* of the lever *fe*, raises *b*, which raises ZY, which raises YX, and this raises the bridge QR, with the axis NG, and the upper stone IK; and thus the stones are set at any distance. The lower stone is fixed immoveable upon strong beams, is broader than the upper, upon which boards are placed round the upper at a small distance, to confine the flour from flying away; and the flour is conveyed through the tunnel *no* down into a chest. P is the *hopper* into which the corn is put, which runs along the *shoe* or spout *r* into the hole *t*, and so falls between the stones where it is ground. The axis G*t* is square, which shaking the spout *r* as it goes about, makes the corn run out: *rs* a string going about the pin *s*, which being turned about, moves the spout nearer or further from the axis, and so makes the corn run in faster or slower, according to the wind. And when the wind is great, the sails S, T, V, W are only part of them,

or one side of them, covered; or, perhaps, only a half of two opposite sails T, W are covered. Towards the end B of the axle-tree is placed another cog-wheel, trundle, and stones, with exactly the same apparatus as before. And the axle carries two pair of stones at once. And when only one pair is to grind, the trundle EF and axis G*t* is taken out from the other: *xyl* is a *girth* of pliable wood, fixed at the end *x*; and the other end *l* tied to the lever *km*, moveable about *k*. And the end *m* being put down, draws the girth *xyl* close to the cog-wheel, and by this means the motion of the mill is stopped at pleasure: *pq* is a ladder going into the higher part of the mill. The corn is drawn to the top, by means of a rope going about the axis AB, when the mill is going.

In mills built of wood, the whole body of the mill turns round to the wind, on a tampin or perpendicular post. But in those of stone, only the upper part turns; the roof is the surface of a cone; there is a wall plate of wood upon the top of the wall: in this a channel is cut quite round, in which are several brass rollers. The roof has a wooden ring for its base, which exactly fits into this channel; and the roof is easily moved round upon the rollers, by help of a rope and windlass.

In the wooden mill, 1 is the mill house, which is turned about to the wind by a man, by help of the lever, or beam 2. 3 is a roller to hoist up the steps 4.

Concerning the position and force of the sails, see Ex. 21, before.

Example LXXVIII. (*Fig.* 1. *Pl.* XLVII.)

AB a *force pump*. C the piston fixed to the rod EC, moveable about E. DF the handle moveable about D. *a*, *d* two clacks or valves opening upwards. The piston C must move freely up and down in the barrel, and exactly fill it, that no air get in. 'Tis made close by circular pieces of leather, cut to fit the barrel, and screwed close between pieces of brass. This pump acts by pressing down; for when the handle F is raised, it raises the piston EC, and the water rises from H, opens the valve *d*, and goes into the barrel, at the same time the valve *a* shuts. But when F is put down, the piston C pressing upon the water, shuts the valve *d*, and opens *a*, and forces the water, that has been raised, through the pipe BG. The piston C must not be above thirty feet from the water in the well.

LM (*Fig.* 2. *Pl.* XLVII.) is *another force pump*, or a lifting pump. N the bucket. *a*, *b*, *c* valves opening upward. This pump is close at the top S, and the small rod of iron plays through a hole made tight with leather. This pump acts by forcing upwards; for when the handle P is put down, it lifts up

the bucket N, the pressure shuts the valve *b*, opens *c*, and forces the water in the barrel NS along the pipe QR. At the same time the valve *a* opens, and lets in more water from M into the barrel. And when P is raised, N descends, the valves *a*, *c* shut, and *b* opens, and lets more water pass into the bucket N, through the upper part. And when the bucket N is drawn up again, the water is forced along the pipe QR as before. This pump is the same as a lifting pump, only there is added the valve *c*, which is not absolutely necessary. No hole or leak must be suffered below the piston or bucket; for air will get in, and spoil the working of the pump. And the bucket must always be within thirty feet of the water.

In these pumps, the bore at H or M, through which the water rises, should not be too strait; the wider it is, the faster the water ascends. Nor should the pipe BG or QR, that discharges the water, be too strait; for then the pump will work slower, and discharge less water in a minute, or require more force to work it. For the calculation of a pump, see Ex. 22.

There are several sorts of *valves* used in pump work, as T, V, W; that at T, being made of two pieces of flat leather, is called a *clack*. These at V, W are made conical, or of any indented figure, and fit exactly into a hole of the same shape. At the bottom of the valve is put a pin across it, to hinder its flying quite out of the hole.

Example LXXIX. (*Fig.* 3. *Pl.* XLVII.)

AB a *hydrometer*, to measure the densities of liquors, especially spirituous liquors. This is a hollow ball of glass, B, partly filled with quicksilver or shot; and hermetically sealed at the top A, when made of a due weight, by trials. The small tube AB is divided into equal parts, and graduated at equal distances. And these divisions noted to which it sinks in different fluids of the best sorts; which points must be taken as standards to compare others with. Then, if the hydrometer be immerged in any fluid, and the point to which it sinks in the surface be marked, it shews the density of it, and its goodness. For it sinks deepest in the lightest liquor, and the lightest liquids are the best.

Example LXXX. (*Fig.* 4. *Pl.* XLVII.)

AB is a *thermometer*, to measure the degrees of heat. B is a glass ball with a long neck AB. The ball and part of the neck is filled with spirit of wine, tinged red with cochineal, and the end A is sealed hermetically; in the doing of which, the end of the tube A, the spirit and included air, are heated, which rarifies the air and spirit; so that when the end A is sealed, and the tube cools, the spirit contracts, and there is a vacuum made in the top

of the tube; and, therefore, the spirit expanding and contracting by heat and cold, has liberty to rise and fall in the tube. This ball and tube is enclosed in a frame, which is divided into degrees. Then, as the top of the spirit rises or falls, the divisions will shew the degrees of heat or cold. These divisions are arbitrary, and, therefore, two thermometers will not go together, or shew the same degrees of heat and cold; except they be made to do so by graduating them both alike by observation. This is commonly put in the same case with the barometer (*Fig.* 4, *Pl.* 26).

There are other sorts of thermometers. CD is a ball with a long neck open at the end D, partly filled with tinged spirit of wine, and put with the open end into the vessel D, near the bottom; which vessel is half full, or more, of the same spirit. The top of the tube CE is air. So in warm weather, when the air in C is rarified by heat, it presses the spirit down into the bucket D, and as the point E descends, the divisions being marked, shew the degree of heat; or, when it ascends, the degrees of cold. But this sort is affected with the pressure of the atmosphere, and therefore is not so true.

Example LXXXI. (*Fig.* 1. *Pl.* XLVIII.)

DA is an *artificial fountain*. AE a strong close vessel of metal, AB a pipe reaching near the bottom of the vessel, and soldered close at A. F, A two cocks. If the cocks be opened and water poured in at A, till the vessel be about half full, then stop the cock F; and, with a syringe, inject the air at A, till it be sufficiently condensed within the vessel. Then stop the cock at A, and take away the syringe. Then, as soon as you open the cock at A, the compression of the air at C will force the water up the tube BA, and spout up to the height D; and a little ball of cork may be kept suspended at the top of the stream D.

But an artificial fountain is most easily made thus: take a strong bottle G, and fill it half full of water. Cork it well, and through the cork, put a tube HI very close, to reach near the bottom of the vessel. Then blow strongly in at H, till the air in the bottle be condensed; then the water will spout out at H to a great height.

Any of these fountains placed in the sun-shine, will shew all the colours of the rainbow; a black cloth being placed behind.

Example LXXXII. (*Fig.* 2. *Pl.* XLVIII.)

CpD is *Archimedes's water screw*. This is a cylinder turning upon the axis CD. About this cylinder there is twisted a pipe, or rather several pipes *no*, *pq*, running spiral ways from end to

end. This cylinder is placed higher at one end D, than at the other. And its use is to screw up the water from the lower end to the higher. AB is a river running in direction AB. *a, b, c, d* several floats fixed to the cylinder. EF the surface of the water. Since the cylinder stands in an inclined position, the upper floats *a, b* are set out of the water, and the under ones *c, d* within it. So that the water acts only upon the under ones *c, d;* and turns about the cylinder in the order *a, b, c, d.* By this motion the water taken into the spiral tubes at the low end, is, by the revolution of the cylinder, conveyed through these pipes, and discharged at the top into the vessel G. If AB is a standing water, there is no occasion for the floats *a b c d.* And then the cylinder is to be turned by the handle at D. Instead of the pipe, a spiral channel may be cut round the cylinder, and covered close with plates of lead. The closer these spiral tubes are, the more water is raised, but it requires more force. Also the more the cylinder leans, the more water it carries, but to a less height.

Example LXXXIII. (*Fig.* 3. *Pl.* XLVIII.)

AL is a *rolling press,* for copper-plate printing. DE, FG, two wooden rollers, of about 12 or 16 inches diameter, running upon the ends of two strong iron axles, that go quite through them, and are fixed in them. To the axis of the upper one DE, is fixed the handle BAC. These rollers run in brass sockets, and must run very true upon their axles, and may be brought nearer, or set further from one another, by wedges, in the frame at P, R. HIK is a flat table or plank, going in between the rollers, and sliding freely upon the frame. LM the frame. NO a shelf to lay the paper upon. When the press is used, the upper roller is folded round with flannel, that every part of the print may take an equal impression; and a paper bottom, spread upon the table .HIK, where the plate is to lie, to prevent the indenting of the plank; then, warming the plate well over a charcoal fire, and rubbing it with the sort of ink proper for it, and laying it upon the paper bottom, on the table at H, take the printing paper, and laying it carefully upon the plate, and turning the handle CAB, the motion of the roller DE turns the roller FG, and draws the table through between the rollers, together with the plate and paper; and the paper is printed.

Note, the paper must be thoroughly wetted in a trough; and after it has laid about a day or two, it is then to be passed through a screw press, to squeeze the water out, and then it is fit for printing.

The ink made use of for printing copper-plates, is made of the stones of peaches and apricots, the bones of sheeps' feet

and ivory, all burnt; this is called *Francfort* black. This must be well ground with nut oil, that has been well boiled; and then it is fit for use. But the best ink is said to be brought from *Holland*.

EXAMPLE LXXXIV. (*Fig.* 1. *Pl.* XLIX.)

The *fire engine* to raise water. LL is a *great beam* or lever, about 24 feet long, 2 feet deep at least, and near 2 feet broad. It lies through the end wall W of the engine house, and moves round the centre *a*, upon an iron axis. CC a hollow *cylinder* of iron, 40 inches diameter, or more, and 8 or 9 feet long; P the *piston* sustained by the chain LP. F the *fire-place* under ground; BB the *boiler*, 12 feet diameter, which communicates with the cylinder, by the hole 2, and throat *pipe* E, 6 or 8 inches diameter. The boiler is of iron, and covered over close with lead: in this, the water is boiled to raise a steam. 4, 5 is the *regulator*, being a plate within the boiler, which opens and shuts the hole of communication 2; this is fixed on the axis 3, 4 coming through the boiler, on which axis is fixed the horizontal piece *h*3, called the *spanner;* so that moving *h* back and forward, moves the plate 4, 5 over the hole 2, and back again. *hl* is a horizontal rod of iron, moveable about the joint *h*. *xyedl* a piece of iron, with several claws, called the *wye*, moving about the axis *de*, in a fixed frame. The claw *sl* is cloven at *l;* and between the two parts, passes the end of *hl*, with two knobs to keep it in its place. AA is the *working beam*, in which is a slit, through which the claws *xy* pass, and are kept there by the pin *q* going between them. DDD is a leaden pipe, called the *injection pipe*, carrying cold water from the *cistern* S, into the cylinder CC, and is turned up at the end within the cylinder. *f* the *injection cock*, to which is fixed the iron rod *fg*, lying horizontal. The end *g* goes through a slit, in the end of the piece *rg*, and on the end is a knob screwed on, to keep it in. *pcbrg* a piece of iron with several claws called the *eff*, moveable about the axis *bc*. The claw *rp* goes through the slit in the beam AA, and is kept there by the two pins *o*, *n*: the claw *rg* goes over the piece *gf:* as the piece *gf* is moved back and forward, the injection cock *f* opens and shuts. 1, 1, &c. are several holes in the beam AA, that, by shifting the pins, serve to set the pieces *p*, *x*, *y*, higher or lower, as occasion requires. N is the *sniffing clack*, balanced by a weight, and opening outwards, to let out the air in the cylinder, at the descent of the piston. In some engines a pipe goes from it to convey the steam out of the house. G is a leaden pipe, called the *sinking pipe*, or *eduction pipe*, going from the cylinder to the *hot well* H; it is turned up at the end, and has a valve opening upwards; this carries away the water thrown in by the cold water pipe, or injection pipe.

t is the *feeding pipe*, going from the hot well to the boiler, to supply it with water, by a cock opening at pleasure. *i*, *i* are two *gauge pipes*, with cocks, one reaching a little under the surface of the water in the boiler, the other a little short of it. By opening these cocks, it is known when there is water enough in the boiler; for one cock will give steam, and the other water; they stand in a plate, which may be opened, for a man to go into the boiler, to clean or mend it. *m* is the *puppet clack*; from this a wire comes through a small hole, to which is fixed a thread going over a pulley, with a small weight at it; the weight on the clack *m* is about a pound for every square inch. YZ the *steam pipe* going from the clack, out of the house. When the steam in the boiler is too strong, it lifts up the puppet clack *m*, and goes into the steam pipe YZ, by which it is conveyed away; otherwise the boiler would burst. KK a pipe carrying water from the cistern S, into the cylinder, to cover the piston to a good depth. I a cock opening to any wideness, that the water may run in a due quantity; M a hole to let it out, through a pipe, into the hot-well H, when there is too much. VVV a *force pump*, with a bucket, and clack, and two valves opening upwards. This pump is close at the top R, and being wrought by the lever LL, it brings water out of a pit, into the cistern S. Q the pit where water is to be raised. X, X, the *spears* which work in wooden pumps within the pit. The cylinder is supported by strong beams, as 7, 8, going through the engine house; 6, 6 is the first floor; 7, 7 the upper floor. At O, in the end of the beam LL, there are two pins, which strike against two springs of wood, fixed to two timbers, lying on each side the greater lever LL; these pins serve to stop the beam, and hinder the piston coming too low in the cylinder.

When the engine is to be set to work, the water in the boiler must be made to boil so long, till the steam is strong enough; which is known by opening the cocks *i*, *i*. Then the hole 2 is opened, by moving the spanner *h*3 by hand; then the steam is let into the cylinder, which lets that end of the beam LC rise up; this raises the working beam AA, moves the eff *prg*, which moves *gf*, and opens the cold water cock *f*: at the same time is moved the wye *xyl*, which draws *lh*, and shuts the hole 2. The cock *f* being open, the cold water rushing into the cylinder, is thrown up against the piston, and descending in small drops, condenses the hot rarified steam, and makes a vacuum under the piston. Consequently, the weight of the atmosphere, pressing upon the piston, brings down the end LC, which raises the other end LQ, which works the pumps X, X. As the end LC descends, the working plug AA descends, and moving the eff, *prgf*, and the wye, *xysth*, shuts

the cold water cock *f*, and opens the hole 2, and the steam gets into the cylinder, which takes off the pressure of the atmosphere; and the end LQ descends by the weight of the spears X, X; and the end LC ascends as before, which opens *f*, and shuts 2. So, by the motion of the beam AA up and down, the cock *f*, and hole 2, shut and open alternately: and by this means of condensing and rarifying the steam by turns, within the cylinder, the lever or beam LL constantly moves up and down; by which motion, the water is drawn up by the pumps, and delivered into troughs within the pit, and carried away by drifts or levels. At the same time, the motion of the beam LL works the pump VRV, and raises water into the cistern S.

When the engine is to cease working, the pins *n*, *o* are taken out, and the cold water cock is kept close shut, while the end LC is up.

The diameter of the pumps within the pit, is about 8 or 9 inches; and the bores of the pumps where the spears X, X, work, should be made wide at the top; for if they be strait, more time is required to make a stroke, and the barrels are in danger of bursting. Likewise, if water is to be raised from a great depth at one lift, the pumps will be in danger of bursting; therefore it is better to make 2 or 3 lifts, placing cisterns to receive the water.

The spears or rods, that work in the pumps, consisting of several lengths, are joined thus: each piece has a stud *a*, (*Fig.* 1. *Pl.* LVIII) and a hole *b*; which are made to fit; and the studs of one being put close into the holes of the other, and the iron collar *g* drove upon them to the middle *d*, they are firmly fixed together.

There is never made a perfect vacuum in the cylinder; for as soon as the elastic force of the steam within is sufficiently diminished, the piston begins to descend, the vacuum is such, that about 8 lb. presses upon every square inch of the piston, or, in some engines, not above 6 lb. This engine will make 13 or 14 strokes in a minute, and makes a 6 foot stroke; but the larger the boiler is, the faster she will work. A cubic inch of water in this engine will make 13340 cubic inches of steam; which, therefore, is 15 times rarer than common air. But its elastic force within the boiler is never $\frac{1}{10}$ stronger or weaker than common air; if stronger, it would force the water out of the feeding pipe.

This engine will deliver 300 hogsheads of water in an hour, to the height of 60 fathom, and cousumes about 30 bushels of coals in 12 hours.

In some engines there is a different contrivance to open and shut the regulator, which is performed thus: (*Fig.* 1. *Pl.* LIX.) as the beam QQ ascends, it raises G5, turns the wye 5GCED4 about the axis AB; and the weight C falling towards B, the end E strikes

a smart blow on the pin L, and drives the fork FL towards L; which draws the spanner PO towards L, and shuts the regulator. And when the beam QQ descends, a pin in it puts down the end 4, and turns the axis AB, and the weight C descending towards 5, throws the end D of the wye against L, which moves PO, and opens the regulator: the spanner PO sliding upon the horizontal piece O. There is a cord *ron*, fixed at *r*, *n*, and the top of the wye O, to hinder it from going too far on each side.

Likewise, for opening and shuttingt he injection cock; instead of the pieces *rg*, *gf* of the eff (*Fig.* 1, *Pl.* XLIX.) some engines have quadrants of 2 wheels H, I, (*Fig.* 1. *Pl.* LIX.) with teeth, which moving one the other, opens or shuts the cock *f*, of the injection pipe K.

In some engines there is a catch, held by a chain fixed to the great beam; and this catch holds the eff from falling back, and opening the cold water cock; till the rising of the beam pulls the catch up by the chain, and then the eff falls.

A calculation of the cylinder and pumps of the fire engine.

If it be required to make an engine to draw any given number of hogsheads of water in an hour, from f fathom deep, to make any number of strokes in a minute, by a 6 foot stroke; find the ale gallons to be drawn at 1 stroke, which is easily found from the number of strokes being given.

Let $g =$ *number of ale gallons to be drawn at* 1 *stroke.*

$p =$ *pump's diameter.*
$c =$ *cylinder's diameter.* } in *inches.*

Then $p = \sqrt{5g}$,

And supposing the pressure of the atmosphere on an inch of the piston, to be 7 lb.

$$\text{Then } c = p\sqrt{\frac{2.614f}{7}} = \sqrt{\frac{13.07fg}{7}}.$$

Note, if, instead of 7, you suppose the pressure of the atmosphere to be l pounds; and instead of a 6 foot stroke to make an r feet stroke; then $p = \sqrt{\frac{6}{l} \times 5g}$, and $c = \sqrt{\frac{13.07fg}{l}}$.

EXAMPLE LXXXV. (*Fig.* 1. *Pl.* L.)

AB is the *water engine* to quench fire. D, E are two *pumps* 5 inches diameter, having each a clack a, b opening upwards. CO a large copper *air vessel* 9 inches diameter. This vessel stands upon a strong plate *kw*, 5 or 6 inches above the bottom of the chest NM. SY is a brass pipe coming through the end of the chest at S; and at Y it divides into two cavities going

under the copper pot CO, to the two pumps E, D. The cavity YW leads to the pump E. And directly above this cavity at W, there is another cavity *y*, communicating with the pump E. And above the cavity *y*, is placed the valve *r*, opening upwards into the copper pot CO, from this cavity. There are the like cavities belonging to the pump D; the first going to the valve *a*; the other from the pump to the valve *s* of the copper pot. These cavities are made of hollow pieces of brass screwed fast together. Z is a cock, through which are two passages, one along the pipe SY, and another at the side of the pipe into the chest NM. This cock, by turning the handle *ce*, opens one passage and shuts the other, as there is occasion. X*x* a leather pipe to be screwed on the end S, to draw water out of a well or river.

PR is the *conduit pipe* reaching near the bottom of the vessel CO, and soldered close into the top of it. At R and Q are screws, so that the pipe may be turned in any direction by the man that holds it. And at V a copper pipe must be screwed on, or else a long leather one, which, being flexible, is carried into rooms and entries. HI an iron axis, to which the iron levers FG, LK are fixed. This axis moves in sockets about H, I, which are screwed hard down. FK, GL two wooden handles fixed to the levers to work them by. *gh*, *pl* are two arches of iron fixed on the axis HI. *fd*, *mn*, the shanks of the pistons, being two strong rods of iron. *fg*, *ht*, *lq*, *mp* four iron chains fixed at *f*, *g*; and *h*, *t*; and *l*, *q*; and at *m*, *p*. At *f* and *m* are screws to screw the chains tight: these chains work the pumps. For when FK is put down, the chain *fg* pulls down the rod of the piston *fd*. And when FK is raised, the chain *th* pulls it up again. And the same way the chains *lq*, *mp* raise and depress the piston *mn*. In some engines there are two arches, like *hg*, *tp*, fixed near the end I of the axis, and chains at them; from the ends of which, as also from *t* and *g*, two boards are suspended. These boards serve for treadles for men to stand upon, to help to work the engine.

The vessel CO and two pumps are inclosed in a chest AN, and the whole machine moveable on wheels. The fore axletree turning on a bolt in the middle, for the conveniency of turning to either side. But there are a great many forms of these engines. In some, the lever lies cross over; in others, lengthways; in some, there is no chain work, but only pins for the pistons to move upon.

When the engine is to play, if it is by the water in a river, &c. the pipe X*x* must be screwed on at S. and the end *x* put into the water. But if water is to be fetched, it must be poured into the chest M, which runs through the holes T, into the body of the engine N. Then turning the cock *ce*Z to open

the proper communication, the handles FK, GL, must moved up and down by men; by which means water is draw into the pumps E, D, and forced into the vessel CO, and o of the pipe PR. For the piston *mm* being raised, the water drawn along the cavity ZYW, through the valve *b*, into t pump E; and when *mn* is depressed, the valve *b* shuts, and t water is forced into the cavity *y*, through the valve *r*, and in the pot CO; which cannot return for the shutting of the val *r*, when the piston *mn* rises again. And the like for the oth pump D. Since the piston of one pump goes down whilst t other goes up, the water is forced by turns into the vessel C by these two pumps; so that there is always water going i And the air confined at top of the vessel at C being condense will press the water up the pipe PRQV, and make it flow wi a continual stream. If the water in C be compressed into h the space, it will force the water to 30 feet high.

In some engines, there is another pipe, as PR, coming throu the copper pot, and through the side of the engine, and the two pipes may play both at once, if there is occasion. And not, the end of one is screwed up.

Example LXXXVI. (*Fig.* 1. *Pl.* LI.)

A ship. This is the noblest machine that ever was invente It is so compounded, and consists of so many parts, that it wou require a whole volume to describe it. Some of the princip parts are these.

A the hull.
B the bow.
C the forecastle.
D the main deck.
E the stern.
F the ancient staff and ensign.

V stays.
V*v* main-stay, &c.
W shrouds.
X main-top-mast back-stay &
Y the crane-line.
Z the anchor to which th

acts perpendicularly upon the plane of any sail, and urges the ship in direction of that perpendicular. And by the help of the rudder H, she is made to keep any direction required. For if the rudder be put about to any side, the water (as the ship moves along) will act violently against it, and drive the stern the contrary way, or her head the same way, as the rudder. A ship with a fair brisk wind will sail 8 or 10 miles an hour.

That any one sail may have the greatest force to move a ship forward, it must be so placed between the point of the wind and the ship's way, that the *tangent* of the angle it makes with the wind, may be twice the *tangent* of the angle it makes with the ship's way.

When the rudder is set to an angle of 54¾ degrees with the keel, it has the greatest force to turn the ship, and make her answer the helm.

Because the figure of a ship is the cause of her going well or ill, and of making more or less way through the water; I shall here give the construction of the fore part of a vessel, that will move through the water with the least possible resistance.

Let DdAcC (*Fig.* 1. *Pl.* LXV.) be the *water line*, or horizontal section of the water and the hull of a ship, AB 30 feet, CD the greatest breadth 20 feet, BC 10 feeet. AcE the stem and part of the keel. Then the following table shows the length of every ordinate, as *bc*, taken at the distance A*b*, or 1, 2, 3, &c. feet from A; by which the curve AcC is determined.

Length of AB in feet.	Length of *bc* in feet.	Length of AB in feet.	Length of *bc* in feet.
1	0.90	16	6.36
2	1.48	17	6.64
3	1.96	18	6.92
4	2.39	19	7.19
5	2.79	20	7.46
6	3.17	21	7.73
7	3.54	22	7.99
8	3.89	23	8.25
9	4.22	24	8.51
10	4.35	25	8.76
11	4.87	26	9.01
12	5.18	27	9.26
13	5.48	28	9.51
14	5.78	29	9.76
15	6.07	30	10.00

The practice is thus: having made AB=30 feet, and, accordingly, divided it into 30 equal parts; at the several points of division, erect perpendiculars to AB, equal to the lengths given in the second column of the table. The curve A*c*C drawn through the ends of all the ordinates, is the figure of the ship on each side.

The curve A*e*E, which the stem and keel make, must be the same curve as A*c*C; if the depth BE is supposed equal to BC, and the ordinates *be*, BE must all be drawn perpendicular to AB. But if the depth BE be taken greater or less than BC, then the ordinates must be taken greater or less in proportion.

Again, if CDE be the section of the ship, made perpendicular to the axis AB, or horizontal plane CAD, and *ced* be any other section parallel to it; then, whatever the curve CED is, all the sections *ced* must be similar to it.

If a ship is required to be built either greater or less than this, then it is only taking a greater or less length, instead of a foot, and dividing it decimally, and using it, instead of a foot, to measure off the lengths, as in the table.

Likewise, if it was required to have the breadth to be greater or less than is here assigned, whilst the length remains the same, then it is only taking a proportionally greater or less line, instead of a foot, and setting off the ordinates *bc* by that. And thus the requisites may be altered at pleasure, still retaining the general construction.

If any ship carpenter thinks fit to build a ship according to this model, it will be found to move faster through the water, than any other ship of the same length, breadth, and depth, and of a different form. The form of the curve is truly represented by the curve A*c*C.

But it must be observed, that the curve at C, the broadest part, is not perpendicular to the ordinate BC, but makes an angle of about 76 degrees: to avoid this, it will be proper to produce AB a little further, and turn the side AC, at C, round in a curve, as quick as possible. Or else make the 2 or 3 last perpendicular ordinates, something less than in the table, that the part of the curve at C may be in a parallelism with AB, as it ought; because C is the broadest part.

But though the form here given is the most proper for sailing fast; yet, perhaps, it may not be so commodious as the common form, upon other accounts, as for the stowage of goods, &c. Yet privateers and ships of war made to pursue the enemy, ought to be built as near this form as they can conveniently. For it is a matter of great moment, either to have it in our power to come up with a ship we are able to take, or else to fly, and escape from one of superior force.

That a ship may steer well, the water ought to come freely

and directly to the rudder; and, therefore, she must not be too short from the midship to the stern: and towards the stern she must rise well, and be built very thin below, lessening gradually to the stern-post. Likewise, she must draw considerably more water abaft than afore. To carry a good sail, and also to avoid rolling, she must be made pretty deep in the hold. As to the dead work, or upper part of the ship, that may be left to the fancy of the builder, or contrived to answer such conveniences as may be wanted.

EXAMPLE LXXXVII. (*Fig.* 1. *Pl.* LII.)

AT an *air pump*. C, D two brass cylinders, 2 or three inches diameter, and a foot high, having two valves at the bottom opening upwards. *t*, *t* two pistons working in the cylinders, having two valves also opening upwards. FF a handle going upon the axis of the wheel or lantern E, which wheel, by the teeth, moves the racks G, G; and by them the pistons, within the cylinders or pumps. AB a table or plate supported by the pillars I, I, I, I. H the receiver of glass, which, by the hollow pipe of brass *nooo*, called the swan's neck, communicates with the cylinders by means of a hollow brass pipe PQ, into which the swan's neck passes. *rrs* a mercurial gage, being a glass tube standing in the bucket of mercury *s*, and communicating with the pipe *no*. K a cock under the table AB to let in air into the pipe *no*, and so into the receiver, when there is occasion.

When the air is to be drawn out of the receiver H, a wet leather is placed over the plate, and upon that the receiver H. Then raising the right hand F, the piston *t* of the barrel D is raised, which takes off the weight of the atmosphere; consequently, the air passes out of the receiver H, through the swan's neck *no*, and through the hollow brass PQ, through the valve into the cylinder D. Then the right hand F put down, the valve at the bottom of the cylinder D shuts, and the air passes through the valve at *t*: at the same time that the left hand being raised, draws the air from the receiver, through *no*P, through the valve into the cylinder C. So that by the motion of the handle FF up and down, the air is at length drawn out of the receiver H, by the pumps C, D; and the rarity of the air within the receiver, is known by the height of the mercury in the tube *rs*, which is known by the graduated frame. An absolute vacuum can never be perfectly made. For when the spring of the air is so weak, as not to be able to lift up the valves at the bottom of the cylinders, no more air can be drawn out.

The handle F (*Fig.* 1. *Pl.* LIII.) is lately made to turn always

one way; thus A is a crank turned by the handle F. NN the leader or sword going over the pin I, in the wheel E. Whilst the crank A is rising, it raises the side S of the wheel, and when the crank descends, it thrusts down the same side S of the wheel E. So the racks are alternately raised and depressed as the crank goes about.

There are several sorts of glasses made use of for the air pump. As A (*Fig.* 2. *Pl.* LIII.) a receiver open at top, covered with a brass plate and oiled leather at D, and kept down by the cross piece EF, screwed down upon the pillars BC, which pillars are screwed into the table AB of the air pump.

H (*Fig.* 3. *Pl.* LIII.) a receiver open at top, with a plate and collar of wet leathers K, through which goes the slip wire GI, so tight as to let no air in. This serves to lift any thing up by the hook I.

MP is a transferrer. N is a plate and leather, on which stands the receiver M. NP a hollow tube going through the plate. O a cock to open or shut the passage. The cock O being open, and the air exhausted by the pump, and then the cock being shut, the receiver and pipe may be taken away from the air pump, the vacuum remaining in M.

L a receiver close at the top; with infinite others of like sort.

Example LXXXVIII. (*Fig.* 1. *Pl.* LIV.)

London-bridge water-works. AB the axis of the water-wheel CD; which wheel is 20 feet diameter, and the axis 3 feet, and 19 feet long. E, E 26 floats 1½ foot broad. G a spur wheel fixed to the axis AB, 8 feet diameter, 44 cogs of iron; this moves the trundle H 4½ feet diameter, and 20 rounds; HI its iron axis. IK a quadruple crank of cast iron 6 inches square, each crank being a foot from the axis. The crank is fastened to the axis at I, by help of a wedge going through both, which causes the crank to turn. L, L four iron spears belonging to the cranks, and fixed to the 4 levers MN, 3 feet from the ends; which levers are 24 feet long, moving on centres in a frame of wood. P, P four force pumps of cast iron, wrought by four pistons or rods, NP. These pumps are 7 or 8 inches diameter, having valves opening upward. O a hollow trunk of cast iron, to which the pumps are close fixed. Q a sucking pipe going into the water. R, R four hollow pipes, 7 inches diameter, and close fixed to the lower part of the pumps; these pipes are close screwed to the hollow iron trunk S, into which 4 valves open. T a pipe communicating with the trunk S, through which the water is forced to any height. There are also four forcers placed at the ends M, M of the levers M, N, and working in four

more pumps, to which belong other trunks and pipes, O, Q, R, S. At B the other end of the axis, there is placed exactly the same work as at A; so that the great wheel CD works 16 pumps. There is also a machine made of cog-wheels and trundles, contrived to raise the great wheel as the tide rises. The great wheel will go at any depth of water; and as the tide turns, the wheels go the same way with it; but stand still at high and low water.

As the great wheel is carried about by the tide, it carries round the spur wheel G, which carries the trundle H with the cranks IK, which, by the swords L, move the levers MN. When the end M is pulled down, N is raised with the piston NP in the pump P, by which means the water is drawn out of the river through the pipe Q, into the pump P; and when NP descends, the valve shuts, and the water is forced through the pipe R, through the trunk S, and along the pipe T into the town. And when N rises up again by the motion of the cranks, the valve in S shuts, and that in the pump opens, and more water rises through the pipe Q into the pump P. And as the cranks stand every way, there is always water rising in some of the pumps; and some always forcing through R, S, T. When the tide is strongest, the great wheel goes 6 times round in a minute. This engine is said to raise 30 or 40 thousand hogsheads of water in a day.

Example LXXXIX. (*Fig.* 1. *Pl.* LV.)

The pile-engine for Westminster-bridge. A the great cog-wheel fixed to the great shaft D. MO, a trundle and fly turned by the cog-wheel; this is to prevent the horses from falling when the ram is discharged. B the drum or barrel on which the great rope is wound. C a less barrel on which the rope L is wound, carrying the weight N. The use of this is to hinder the follower from falling too fast. The barrels BC are moveable about the great axis D. The cog-wheel and barrel B are fixed together by the bolt F, going through the cog-wheel into the barrel. EI is a lever moveable about 1, going through the great shaft D; this lifts up the bolt F, the end E being made heavier by a weight; by which means it locks the barrel B to the great wheel A. KI the forcing bar going into the hollow axis of the great shaft D: this rests upon the lever EI. XY a great lever moveable about 3, the end X being heavier, which, with the end Y, presses down the bar KI, thrusts down the end of the lever at F, and lets the bolt F descend, to unlock the barrel C. Z a rope fixed at X, and going up through the guides at R. GK a crooked lever moveable about 2, the roller at the end G being pressed with the great rope, forces the end K against the catch

at K, and hinders the bar KI from ascending. When the rope H slackens, the spring 7 forces the end K from the catch, and the bar KI ascends. H the great rope going round the barrel B, over the pulley P, up to the top and over the pulley Q, then down to the follower, where it is fixed. T the ram that drives the piles. S the follower, in which is fixed the tongs W, moveable about the centre. VV the guides between which the ram falls. At the inside of the guides at R, where they are fastened together, there are two inclined planes. At the bottom of the follower is a slit, to receive the handle 6 of the ram T, to be taken up by the tongs W. *a, b, c, d* timbers for horses to draw at, in direction *abcd*.

As the horses go round, the great rope H is wound about the barrel B; and the follower S, and ram T are drawn up, till the tongs come between the inclined planes, which squeezing the ends 4, 4 together, opens the end 5, and lets the ram fall down. Then the follower S taking hold of the rope Z, raises the end X, and depresses the end Y of the lever XY, which thrusts down the bar KI, which thrusts down the end FI of the lever EI, with the bolt F, and unlocks the barrel B, which turning about the axis, the follower S descends by its weight, till it comes to the ram T; and the end 5 of the tongs slips over the handle 6 of the ram. Then the rope H slackens, and the spring 7 forces the end K from off the catch at top of the bar KI, and lets the bar rise, and the weight E raises the bolt F, and locks the barrel B to the wheel A; and the horses still going about, the end 5 of the tongs takes hold of the handle 6, and the ram T is taken up as before.

All this machine is placed upon a boat, which swims upon the water; and so is easily conveyed to any place desired.

Example XC. (*Fig.* 1. *Pl.* LVI.)

GH a *blowing wheel.* AB, CD an iron cross; to this is fixed the circle of iron EF. To these are fixed 12 leaves I, I, I, which reach no nearer the centre than the iron circle. 1, 1, 1 are holes, through which the air passes into the cavities between the leaves. There is the same cross and iron circle on the other side, but without any hole. Through the centre of both sides is put an iron axis and fixed there, and on the further end a handle is put to turn it by. This wheel is inclosed in a case, which just touches the edges of all the leaves. But the rim or out edge KK is at a small distance from the ends of the leaves. On this side or flat of the case, there is a hole left against the holes 1, 1, to let the air through; the other flat is close. LM is the sucking pipe, being a tube fixed upon the case, so as to communicate with the cavities, by the holes 1, 1. G is

the blowing pipe, and is another wooden tube communicating with the inside of the case. The axis turns upon two concave pieces of metal fixed to the case, the handle being on the back side of the figure. *abcd* is a thin ring of wood fastening the leaves all together; and the like on the other side. On these rings are put two circles of blanketing to go close to the case, and also upon the iron circle EF.

The frame being fixed, and the handle turned about in the order BCAD, the motion of the leaves moves the air very swiftly to the outside, which being confined by the rim, is forced in a tangent along the tube G; whilst new air ascends along the sucking pipe LM, passes through the hole in the frame, and through the holes 1, 1, into the cavities between the leaves; and so thrown out of the wheel, through the blowing pipe G.

If the pipe LM be continued to the place where any foul air is, it may soon be thrown out through the tube G, and dispersed abroad. Or if the tube LM communicate with the fresh air, and G with any close room, fresh air may presently be injected into the room.

Example XCI. (*Fig.* 2. *Pl.* LVI.)

AB an *artificial fountain* to play with either end up. A and B two cavities; FO, KB two open pipes, fixed to the basins at K and O. GHI and CDE two curve tubes open at both ends. When the fountain stands on the end A, pour water in at O. Then turning the fountain like an hour glass upon the end B, the water will descend through the pipe CDE, and spout out at E. The air passing up the pipe OF to give it liberty. The water falling down upon the basin EK, runs through the pipe KB, into the cavity B. And the fountain being turned, the water will descend through GHI, and spout out at I, as before. And so being turned, it will play a-fresh as often as you will.

Note, while the jet E is playing, if the end O of the pipe FO be stopped with the finger, the jet will cease playing; which being taken off, it will begin again; and so may be made to play or stop at pleasure.

Example XCII. (*Fig.* 3. *Pl.* LVI.)

AF a *water barometer*. AD is a small tube open at both ends, cemented in the neck of the bottle CE. Then the bottle being a little warmed to drive out some of the air; the end A is immerged in water tinged wtth cochineal, which goes into the bottle as it cools. Then it is set upright; and the water may be made to stand at any point B, by sucking or blowing at A. This is a very sensible barometer; for if it be removed to any higher place, a very small decrease in the air's gravity, will make

the water rise sensibly in the tube. This may be made use of to find the level of places. But it is subject to this inconvenience, that it is a thermometer as well as a barometer, the least alteration of heat raising the water in the tube. To prevent which, it must be enclosed in a vessel of sand; and then the air included in the bottle, will retain the same degree of heat, at least for a small time.

Example CXII. (*Fig.* 1. *Pl.* LVII.)

ADOF is a *jet d'eau*. AB the reservoir where the water is kept. CDIO the pipe of conduct, which conveys water from the reservoir. O the cock, or adjutage, being a small hole in a thin horizontal plate, fixed upon the end of the pipe, through which the water flows. OF the jet of water, spouting up through the hole O, which descends again in the streams FE and FH. OF the height of the jet. AG the horizontal height of the water in the reservoir. If the part LIO of the pipe of conduct be buried under the surface of the water KH, and be invisible, the jet will seem to rise out of the water KH, as in many artificial fountains.

The adjutage is sometimes made conical, but the best sort for spouting highest, is a thin plate with a hole in it. The bore of the adjutage ought to increase with the height of the reservoir, and the larger the adjutage, the higher the jet will go, provided the pipe of conduct be large enough to supply it with water. Pipes of conduct ought not to be made with elbows, but to turn off gradually in a curve as DIO The diameters of pipes of conduct ought at least to be 5 or 6 times the diameters of the adjutage, or else it will not spout so high. If a reservoir be 50 feet high, and the adjutage half an inch, the pipe of conduct should, at least, be 3 inches; or if the adjutage be an inch, which is better, the pipe of conduct must be 6 inches. And in these cases the jet will rise to the greatest height it can have. In general the diameter of the adjutage ought to be nearly as the square root of the height of the reservoir. And if you would have the velocity in the pipe of conduct to be the same at all heights of the reservoir, that the friction may not increase too much; then the square of the diameter of the pipe of conduct must be as the cube of the diameter of the adjutage. When water is carried a great way through pipes, the friction of the pipes will diminish its velocity, and the jet will not rise so high.

A jet never rises to the full height of the reservoir. If the height be 5 feet 1 inch, the jet will only rise to 5 feet; thus the jet OF wants the space FG of the height of the reservoir. And the defect FG is as the square of the height of the reservoir OG. But smaller jets fall short more than in that proportion,

being more retarded by the resistance of the air. The greatest jets never rise 300 feet high; for the velocity is so great, that the water is dissipated into small drops, by the resistance of the air. If a ball of cork or light wood be laid at F, it will be suspended by the pillar of water, and play there without falling.

Example XCIV. (*Fig.* 2. *Pl.* LVII.)

AGE is a *compound steelyard*, for weighing vast weights. IG, CK two levers moveable about B and E. LE, MB two fixed pieces. AC a cross bar supporting the end C, and moveable about the pins A and C. The centre of gravity of IG and AC is in B; and of CK and the hook DN, in E. H the weight to be weighed; F the counterpoise moveable along the graduated lever BG. The machine is hung upon the hooks at L, M. Here the power F is to the weight H, as AB × DE to CE × BF.

Example XCV. (*Fig.* 2. *Pl.* LVIII.)

ABC is a *horse mill* to grind corn; C the *spur wheel* having 72 cogs; B the *lanthern* of 7 rounds; A the *hopper*, E the *shoe*; F, G the two *mill-stones*. H a lever or *arm* 8 feet long, going into the axis D of the great wheel; I the traces to which one or two horses are yoked. As the horses goes about in the path 1 2 3, he draws the arm H, which turns the great wheel C, and this drives the trundle B with the upper stone F, which grinds the corn; the corn is put into the hopper A, and falling into the shoe E, runs through a hole at top of the stone F, and in between the stones where it is ground. KL is the upper floor. The whole is within a house.

Example XCVI. (*Fig.* 2. *Pl.* LIX.)

AB a *lifting stock*, set perpendicular; its use is to raise a great weight. LO is a slit going through it, in which there moves the lever CD. II, KK two sets of holes; into which go the pins G, H. When the weight W is to be raised, it is hung on the hook and chain at the end D of the lever. And the pin G being put into the first hole I, and the end C being put down, the other end with the weight is raised, and then the pin H is put into the second hole K, under the lever; then the end C being raised to E, the pin G is put into the second hole I. Then E being put down to C, and the end F raised, the pin H is put into the third hole K. Thus the lever and pins being thus shifted from hole to hole, the weight W is by degrees raised up.

Example XCVII. (*Fig.* 1. *Pl.* LX.)

A *bob gin*, for raising water. AB a large water-wheel carried

by the water W. C and D two cranks, upon the axis, on each side the wheel, lying contrary ways. EF, GH, two pieces of timber moving about on the cranks C and D, and also moveable at the joints F, H, upon two pins. FI, HK two beams, moving on the axes L and M. I, K two arches with chains fixed to them, by means of which the pumps O, N are wrought. When the water-wheel goes about, one crank as C pulls down the bar EF, together with the end F of the beam FI, and at the same time raises the end I, which draws the water up in the pump O. In the mean time the other crank D is raising the end H, and depressing the end K. When, by motion of the wheel, the crank C begins to ascend, the end I begins to descend, and the end K to ascend. So that one beam goes up whilst the other goes down, and there is always one pump working.

Example XCVIII. (*Fig.* 1. *Pl.* LXI.)

A *gunpowder-mill*. AP the water wheel; B its axis. RPS the water-course. E a spur-wheel carrying the two drums C, D, and the rollers CF, DH, to which they are fixed. *a*, *a*, &c. 10 or 12 pins, or cogs, fixed in either roller, equally on all sides. *b*, *b*, &c. 10 or 12 pestles, 10 feet long, and 4 or 5 inches broad, armed with iron at the bottom; in these pestles are pins fixed to answer the pins *a*, *a*; which lift them up as the rollers turn round. *m*, *m*, &c. are wooden mortars, into which the pestles fall; each mortar will hold about 20 lb. of paste. OQ, IK, LN are timbers through which the pestles work, and serve to keep them direct.

The materials being put into the mortars *m*, *m*; as the mill goes about, the pins in the rollers take up the pestles *b*, *b* by their pins, and when these pins go off, the pestles fall into the mortars *m*, *m*, and beat the ingredients to a paste. And as these cogs are placed on all sides the circumference of the rollers, there will be always some pestles rising, and others falling, in a regular order.

Example XCIX. (*Fig.* 1. *Pl.* LXII.)

A *crane* or engine to raise a great weight, and keep it in any position. AB a double wheel for a man to walk in; CD a spur-wheel upon the same axis. E, F, G are three wheels also fixed upon one axis, of which G is of wood, and E is moved by CD. HI is a catch, moving on the pin I; this falls in between the teeth of the wheel F. KLM a half ring of iron, in which is a groove, going upon the edge of the wooden wheel G. NM a piece of timber fixed to the ring at M, and to the lever PN, and is moveable about the pins M, N. The lever PN is moveable about the centre O. QR a wooden rod, reaching to the catch

III. PST a string fixed to the lever at P. VXW the rope going about the axis of the great wheel to raise the weight.

When the great wheel AB goes round, together with CD, the rope VXW raises the weight W. The wheel CD drives E, together with F and G; and the end of the catch III slides freely over the teeth of the wheel F; and the motion being stopped, the catch IH acting against the teeth of F, hinders the wheel F from turning back, and so keeps the weight W suspended. But if you pull at the string T, it raises the lever PO, and thrusting the rod QR against the catch, raises it out of the teeth of the wheel F, and lets the weight W descend. But lest it descend too fast, the lever PO is to be raised higher, by pulling at the string TS, and this depresses the end ON of the lever, and draws down the piece NM, together with the ring KLM, which ring being drawn close against the wheel G, stops the motion, or regulates it at pleasure.

Example C. (*Fig.* 1. *Pl.* LXIII.)

An engine for drawing water. A the cog-wheel, ten feet diameter; B its axis, running in the frame FFFF, and on the foot Z. C a trundle, three feet diameter. K its axis, fifteen or twenty feet long, running in the stocks G, G. D, D two cranks of iron on opposite sides of the axis, and two feet long. OP, QR two beams moving upon an axis in the frame SSSS. PD, RD two rods of wood or iron, reaching from the beams to the cranks, moveable about R and P; and turning round on the cranks D, D. I, I two rods of iron, fixed to two chains that go over the arches O, Q; and to two pistons that work in the pumps *x*, *y*. E the tiller to which the horses are yoked; 1 2 3 4 the path in which the horses go round. H, H, the surface of the earth. The wheel A, and trundle C are in a pit; the axis K under ground; and the cranks D, D, are in a pit.

When the horses, walking in the ring 1 2 3 4, draw about the cog-wheel, by the tiller E; this turns the trundle C, with the cranks D, D; and the rod PD being drawn down, pulls down the end of the beam P; and raises the other end O, with the rod I*x*; and draws the water out of the pump *x*. In the meantime, the other crank raises the rod DR, with the end R of the beam; and the other end Q, with the rod I descends, and the piston goes down into the pump *y*. But as the wheel A goes about, the rod RD is pulled down, and QI rises up, and draws water out of the pump *y*, whilst OI and the piston descends into the pump *x*. Thus, whilst one piston goes up, the other goes down, and there is always one pump discharging water.

Instead of two cranks, one may have three or four cranks, at equal distances round the axis, and these will move three or four

beams QR, and work three or four pumps. But beams of timber should be put between every two working beams, OP, QR, for the axles to run in.

Example CI. (*Fig.* 1. *Pl.* LXIV.)

AEL *a twisting mill to make thread or worsted*. B a cog-wheel 3 feet diameter, of 33 or 34 teeth. C the drum of 4, 6, or 8 rounds, going on the square end of the axis of the cog-wheel. D a spur-wheel, 2 feet diameter, and 30 or 32 teeth; this is fixed to the reel E. The reel consists of 4 long pieces of wood, 6 or 7 feet long, 3 of which are fixed in the cog-wheel D, and are also fixed to one another by cross bars going through the axis of the reel; the fourth long piece of wood, which composes the reel, is not fixed in the cog-wheel, but may be set nearer or further from the axis, by help of the pins 1, 1, 1. F a drum of 12 rounds, carried by the cog-wheel B; these rounds are fixed into the barrel G, of 1 foot and 6 or 8 inches diameter. MNOP a fixed frame 6 or 7 feet long, and 4 feet broad. 22, 22, &c. are whorles, carried round by the leathern belt IKLIH. These whorles run in iron sockets at the bottom of the frame, and are kept in their places by the snecks 3, 3, fixed to the upper side of the bottom part of the frame; upon the spindles of these whorles are put the bobbings, with the thread or worsted. The spindle and whorle is represented at *a*, the bobbing at *b*, the bobbing with the worsted on it at *c*. The length of the whorle and spindle is 10 or 11 inches, length of the bobbing 6 or 7 inches; diameter of the whorle where the belt runs about an inch; diameter of the bobbing at top 1½ inch, at the smallest part ⅜ of an inch; these are for worsted. The whorles may be taken out of the snecks at pleasure, and they are kept in these snecks by a feather put across the slit through two holes. The bobbings they use for thread are represented at *d*; *e* is a piece of lead which goes upon the top of the spindle to keep down the bobbing; *f*, *g* are two wires fixed in it, for the thread to run through, from the bobbing to the reel, the diameter of the whorle about half an inch. The number of snecks, spindles, and bobbings on one side of the engine is 20 or 24, that is 40 or 50 in all. 4, 4, &c. are wires in the upper part of the frame, for the thread to run through from the bobbings; the number of wires are equal to the number of spindles. Also, in the horizontal beam QR are the same number of wires, 5, 5, 5, &c. to direct the thread to the reel. *n*, *n* are rollers for the edge of the belt to move over. 6, 6 are two hanks upon the reel. When the belt grows slack by stretching, the frame MN is drove back, by means of a wedge S, and so kept at a greater distance from the roller G.

The trundle C may be taken off, and others of more or fewer

rungs put on, as occasion requires, by lifting the end T, of the axis of the keel, out of its socket; and the finer the thread, the fewer rungs it must have. The circumference of the reel DE for worsted is 4 feet, 4 inches; for thread is 5 feet, 5 inches.

When the thread or worsted is wound upon the bobbings, by the help of a wheel, they are put upon the spindles, as c, and then put within the belt IKL under the snecks 2, 2; then the handle A being turned, carries the cog-wheel B about, which drives the drum F, and the barrel G; the barrel G moves the belt in direction IKL about the frame MN, which resting on the whorles 2, 2, moves them, and the whorles and bobbings, very swiftly about. In the mean time, the drum C is turned round by the axis of the cog-wheel B, and C carries about the spur-wheel D and the reel, with a slow motion. So the threads being put through the wires 4, 5, and fixed to the reel, these threads will be wrapped about the reel, and make the hanks 6, 6, as many as there are bobbings. When the hanks are of sufficient bigness, they must be taken off the reel, which is done by pulling out the pins 1, 1, and then one side of the reel will fall in, and the hanks slacked, and may be taken off one after another, by lifting the end V of the axis out of its socket.

The double yarn, &c. is to be wound tapering on the bobbings, as at c, making it broadest at the low end, otherwise it will not come freely off the bobbings, without breaking.

The frame work consists of perpendicular beams, fixed in others, lying horizontal, as described in the figure, the breadth from A to V being 9 or 10 feet. The lower part of the frame MN consists of two elliptical pieces, cut out of boards, and set at about a hand's breadth distance one above the other, with pieces of wood between. In the lower (which is broader than the other) are the sockets, in which the bottom part of the spindle of the whorles move: in the upper, the snecks are fixed. The part OP, in which are the wires, is an elliptical piece like the under ones, and fixed thereto by 4 perpendicular pieces or pillars of wood. All the rest will be plain from the figure.

Example CII. (*Fig.* 1. *Pl.* LXVII.)

AEKF *is a clock.* The different forms and constructions of clocks are almost as various as the faces of those that make them. The following is a common 8 days' clock. KF is the moving part; AE the striking part.

The work contained between 2 brass plates is as follows: F the *first* or *great wheel* of 96 teeth; G the *second wheel* of 60 teeth, its pinion *g* of 8 leaves; H the *third wheel* of 56 teeth, its pinion *h* of 8 leaves; I the *balance wheel* of 30 teeth, its pinion *i* of 7 leaves; and K the *balance*. Likewise, A the *great wheel* of

78 teeth; B the *pin wheel* of 48 teeth, *b* its pinion of 8 leaves; C the *hoop wheel* of 48 teeth, *c* its pinion of 6 leaves; D the *warning wheel* of 48 teeth, and *c* its pinion *d* of 6 leaves; E the *fly*, *e* its pinion of 6 leaves.

The ends R, R, of the *arbors* of the wheels A, F come through the face of the clock, and these arbors are fixed in the *barrels* P, P, of 6 or 7 inches circumference; and on these barrels the therm *strings* T*t* are wound, which go round two *pullies* with the weights, that carry the wheels about. These two barrels are moveable round about within the wheels, but are kept from turning back, by the *catch* S and its *spring*, and the *racket wheels* Q fixed to the barrel. The weights are wound up by help of the *winch* or handle 11. In the rim of the wheel B are 8 *pins*, which, as the wheel goes round, thrust back the end 5 of the *hammer* O, and when it goes off the pin, the *spring* 7 makes the hammer O strike against the *bell* N.

The wheel C has a *hoop* upon its rim, which is cut away in one place, to let the end 2 of the *detent* fall in. In the rim of the wheel D, there is a pin which stops against the end *x* of the arm *wx*, and hinders the wheels turning about.

On the axis *op* are fixed the two *pieces vs*, and the detent 1 2. On the axis *qr* is fixed two *pieces wx* and the *lifter* 3; and on the end *r* of that axis, which comes through the fore plate, is put the *lifter* 10, (*Fig.* 1. *Pl.* LXVI.) and pinned fast on.

The *arbor* of the wheel G comes through the *face*; upon this arbor, between the face and *fore plate*, is put the wheel *z* (*Fig.* 1. *Pl.* LXVI.) of 20 teeth, its arbor being hollow, and under the wheel is put the brass spring *l*, with the concave side upward, this spring having a square hole in it, to fit the shoulder of the arbor of G, (*Fig.* 1. *Pl.* LXVII.) The wheel Z (*Fig.* 1. *Pl.* LXVI.) of 40 teeth turns upon a fixed pin or axis, and is driven by the wheel *z*. The dial wheel *f* of 48 teeth, is put with its hollow *socket* upon the arbor or *socket* of *z*; then the face being put on, their ends comes through it, and the *hour hand k* is put upon the square end of *f*, and the *minute pointer* W, (*Fig.* 1. *Pl.* LXVII.) upon the end *z*, (*Fig.* 1. *Pl.* LXVI.) the wheel *z* being thrust down to bend the spring, and then a pin put in to keep it there; the pinion of Z, called the *pinion of report*, has 8 teeth, and drives the wheel *f* and the hour hand. Now, the spring *l* keeps the wheel *z* pretty tight upon the axis of G, so that G will carry it about along with it. And if the minute pointer be thrust about, it will force about the wheel *z*, and also Z, and likewise *f* with the hour pointer.

The arbor of the wheel A (*Fig.* 1. *Pl.* LXVII.) goes through the *back plate*; upon it, behind the plate, is put the wheel V (*Fig.* 1. *Pl.* LXVI.) or *pinion of report*, of 28 teeth, and pinned

there. The double wheel XY is carried by V, and turns upon a pin fixed on the back of the plate. The wheel X has also 28 teeth, and the *count wheel* Y is divided into 11 parts of unequal lengths, according to the strokes the clock is to strike at every hour; part of this wheel is represented at *s*, (*Fig.* 1. *Pl.* LXVII.) A slender spring is put on with this wheel to keep it tight. This part may be made more simple, by leaving out the wheels V, X, and putting Y upon the axis of A instead of V; but it must be put on the contrary way.

The arbor of the balance wheel I (*Fig.* 1. *Pl.* LXVII.) comes through the fore plate, almost to the face; and through a hole in the face is put the hollow socket of the *second pointer* 12; and this shows the seconds by a small circle divided into 60 parts. And the face is also divided into two circles, showing hours and minutes.

The *pendulum* hangs on the fixed piece of brass M, by a button at top, and a thin piece of brass going into a slit at M, and a flat piece of brass goes into the *fork* L, so that if the pendulum moves, it must move the rod KL, and balance K along with it.

The *pallats* 8, 9 of the balance K, are so formed, that the under side of 8, and upper side of 9, where the teeth of the wheel I act, are polished planes, and made sloping, so that a tooth sliding along the under side of the pallat 8, will force the balance K to the left hand; and a tooth sliding along the upper side of the pallat 9, will force it to the right.

The work is put together, by setting the teeth together that are marked in the wheel B and in the pinion *c*, and likewise in the wheel C and pinion *d*. Then the minute pointer is put on the arbor of *z*, mark to mark; and the hour pointer the same way on the arbor of *f*. And the wheels *z*, Z, *f* are set to one another, according to their marks.

The weights hanging upon the wheels A, F, and the pendulum made to vibrate, the wheel F drives G, which drives H, which drives I; then, whilst the pendulum vibrates to the right, a tooth slips off the pallat 9, and in its return to the left, a tooth slips off the pallat 8, then on the right another goes off 9, and so on alternately; and the weight causing the teeth to act against the pallats of the balance, keeps the pendulum in motion; and the wheel I goes round in a minute.

As the wheel G goes round, it carries about *z*, with the minute pointer once round in an hour; *z* drives Z, which drives *f* once round in 12 hours. Whilst the wheel *z* goes round, the pin *m* raises the lifter 10, which lifts up the piece 3, and the arm *wx*; the piece 3 raises the detent 1 2, together with *vs*; the end 2 of the detent being raised above the hoop, the wheel C moves about,

and by the oblique figure the end of the detent 2, it raises the end of the detent higher, and also raises *s* out of the notch of the count wheel. Then the wheel D turns round, till the pin in the rim stops at the end *x*, which hinders the motion. But as the wheel *z* goes further about, the lifter 10 falls down off the pin, together with the piece *wx*, and latch 3, which suffers the wheel D and the rest to turn round; and the pin-wheel causes the hammer to strike so often, till the end *s* falls into a notch of the count wheel, and then the detent 2 falls into the vacancy in the hoop, and locks the work; which continues so till the next hour, that the piece 10 is raised again, and then she strikes as before; the wheel C goes round every stroke of the clock; but she strikes 1 more every succeeding hour, because the teeth between the notches are made longer and longer in the count wheel; and it turns round once in 12 hours.

General rules in all clocks.

In the striking part, the pin wheel being divided by the pinion of the hoop wheel, the quotient shews the number of pins in the pin wheel.

If 78 be divided by the number of pins, the quotient shews the revolutions that the pin wheel makes for one revolution of the count wheel.

The hoop wheel, divided by the pinion of the warning wheel, must be a whole number.

In the moving part, the *train* is the number of beats the clock makes in an hour; which is 3600, if she beats seconds: in this case, the balance wheel must have 30 teeth.

If G turns round once in an hour and shews minutes, then the quotient of G divided by the pinion of H, multiplied by the quotient of H divided by the pinion of I, and that multiplied by twice the teeth in I, must be equal to the train. And if she beats seconds, then the product of the two quotients must be 60.

If, also, G shows the hours, then the quotient of *f* divided by the pinion of Z, multiplied by the quotient of Z divided by *z*, must be 12.

From the great wheel to the balance, the wheels drive the pinions; but to the dial wheel, the pinions drive the wheels; the former quickens, the latter lessens the motion.

Any wheel being divided by the pinion that works in it, shows how many turns that pinion hath to one turn of the wheel. As if the pinion be 5 and the wheel 60, it is set down thus,

5) 60 (12 times. Or thus $\frac{60}{5} = 12$ times.

The teeth of several wheels and pinions, that work in one another, are set down thus,

$$\begin{array}{l} \qquad\qquad\quad 4)\ \overline{36}\ (9 \text{ times} \\ \qquad\quad 8)\ \overline{80}\ (10 \text{ times} \\ \qquad 6)\ \overline{54}\ (9 \text{ times} \\ 5)\ 40\ (8 \text{ times} \end{array}$$

Or thus $\frac{36}{4} \times \frac{80}{8} \times \frac{54}{6} \times \frac{40}{5}$.

In the former way, the number on the left hand of any wheel is the pinion that it drives; and the number over it is the pinion on the same axis. In the latter way the several fractional quantities represent the quotient.

Any wheel and the pinion it drives, will have the same motion with another wheel and pinion, when their quotients are equal. Thus, a wheel of 36 drives a pinion of 4, all the same as a wheel of 45 does a pinion of 5; or a wheel of 90 a pinion of 10.

In any motion you may use one wheel and one pinion, or else several wheels and several pinions, provided they all give the same motion. Therefore, when a number is too big to be cut in one wheel, you may divide it into two or more quotients.

In a wheel and pinion that work in one another, their diameters must be as the number of teeth in each. And the diameter must be measured, not to the extremity, but to the middle of the tooth, or where they act.

The excellency of clock-work consists in forming the teeth truly, and to fit the notches exactly without shaking, and to play freely; the teeth must be cut into the form of cycloids, which resembles the shape of a bay leaf.

A clock goes exacter as the pendulum is longer, and the bob pretty heavy, and to make but small vibrations; and for more exactness, to play between two cycloidal cheeks; and the longer the arms K8, K9, the easier the clock goes. The length of a second pendulum is $39\frac{2}{10}$ inches. See the theory of pendulums in Prop. XL., XLI., LVIII.

The pallats 8, 9, are here formed after the common way: but there is another way of forming them. From the centre of motion θ, (*Fig.* 1. *Pl.* LXVII.) describe two small arches $\alpha\beta$, and $\delta\epsilon$. These small lines or planes $\alpha\beta$, and $\delta\epsilon$, and also the working side of the tooth $\lambda\mu$, must all range to κ the centre of the balance wheel. And the ends of the pallats $\alpha\gamma$, and $\delta\eta$, must range a little to the right hand of the centre θ. Then the teeth of the balance wheel will fall alternately on the sides $\alpha\beta$, and $\delta\epsilon$,

And any tooth, whilst it acts against $\alpha\beta$, or $\delta\epsilon$, will have no effect in moving the pendulum; but lies *dead*, till it makes its *escape* off the angle α or δ; and then in moving along the plane $\alpha\gamma$, or $\delta\eta$, it forces the pendulum to the right or left.

But the construction will be better thus; take $\theta\tau$, $\tau\phi$ each equal to $\frac{1}{3}$ $\alpha\delta$. From the centre τ describe the arches $\alpha\beta$, $\delta\epsilon$; and let the end $\alpha\gamma$, $\delta\eta$, range to ϕ. Or, perhaps, it may answer the end as well, to describe $\alpha\beta$, and $\delta\epsilon$, from the centre θ; and let the acting side of the tooth range (not to κ, but) to υ the outside of the circle described with the radius $\kappa\upsilon = \frac{1}{3}$ $\kappa\mu$.

The inconvenience of any of these constructions, is, that the pallats are too thick, and can hardly find room to fall in between the teeth of the balance wheel.

Example CIII. (*Fig.* 1. *Pl.* LXVIII.)

ABC is a *cutting engine* to cut the teeth of clock wheels. AC an iron plate 2½ feet long, and 3 or 4 inches broad. EE another plate fixed 4 or 5 inches lower. G a *slider*, sliding along a groove in the end C: this is made of several plates of iron fixed to one another with screws, and fitting closely to the edges of the plate, and to the sides of the groove, and likewise to the upper and under side of the plate; this is to cause it to move truly along the groove when forced forward or backward, by the screw at I and its handle; for this screws through C, and turns round in a collar in the end G. The end of this slider turns up perpendicular: to this is fixed the part F by a pin K, which goes square into this part, and through a round hole in F; so that the part F can turn about the screw pin K, and may be fixed by turning the nut 2 with the key 9, (*Fig.* 1. *Pl.* LXIX.) which nut screws upon the end of the pin.

B is a brass wheel of 96 teeth, carrying the pinion D of 12 leaves; these move between the cheeks LL, MM; which are joined by the cross bars N and P; these cheeks and their machinery turn round on the axis LM, in the part F. *f* is the cutting-wheel, whose edge is nothing but a file to cut the teeth, as it goes about; this goes upon the arbor of the pinion D. There are a great number of these cutting wheels, of different shape and bigness, which may be taken off the arbor, and others put on; these parts are described at *a*, *b*, *c*, *d*; (*Fig.* 1. *Pl.* LXIX.) *a* the pinion on its arbor, *b* the cutting wheel going upon the arbor which is octagonal, and fits it exactly, having the sides marked that are put to each other, *c* a hollow piece which goes on the same axis; and the nut *d* screws on the end of the arbor, to keep *c* and the wheel *b* fast on. The ends of the arbor are hardened steel, and pointed; and this arbor runs between the cheeks LM, through which cheeks there goes 2 screws, with holes to

receive the points of the arbor; and these screws are set to a proper distance, by screwing them in or out by help of a key *q*, going on square upon the end, and then the screws are locked there, by the nuts O, O. *rs* (*Fig*. 1. *Pl*. LXVIII.) is a spring, fixed with one end to the under-side of the cross-bar N, and the other end *s* lying upon the plate AI, and this spring raises the part LLMM, when the notch is cut. *tu* is a screw pin going through the bar P; its end *u* rests upon the plate AI, and hinders the wheels from descending lower. I, I, are two screw pins, which screw through LM, by help of the key *q*, (*Fig*. 1. *Pl*. LXIX.) and go with their points into F, (*Fig*. 1. *Pl*. LXVIII.) which has two holes to receive them; these pins are locked to L, M, by turning the two nuts, which also screw upon the pins. These screw pins, nuts, and cheeks, all turn round together in the holes in F.

H is the *dividing plate*, being a brass circular plate 15 or 16 inches diameter. This plate is fixed to a hollow brass axle Q, an inch in diameter; and this axle goes through the two plates AI, EE; and both the wheel and its axle turn about together; the lower plate cannot be seen, (but is represented at R, *Fig*. 1, *Pl*. LXIX.) Near the edge of this plate, there are 24 concentric circles, each divided by points into a certain number of equal parts, *viz*. 366, 365, 360, 118, 100, 96, 92, 90, 88, 84, 80, 78, 76, 27, 70, 68, 64, 62, 60, 58, 56, 54, 52, 48. The use of these is to divide a revolution into any number of equal parts, according to these different circles.

ei (*Fig*. 1. *Pl*. LXIX.) is an arbor going through the hollow axle QR, (*Fig*. 1. *Pl*. LXVIII.) with the shoulder *h* against the top of that axis; then the nut *n* is screwed upon the end *i*, to keep it fast. *m* is the wheel to be cut into teeth; there is a hole made in the centre, just to fit the part *ge*, which being put on, and the piece *l* above it, they are then screwed hard down with the nut *k* going on the end *e*. Then if the wheel H be turned round, it carries about with it the wheel *m*. There are several arbors *ei*, for fitting different wheels *m*.

wy (*Fig*. 1. *Pl*. LXVIII.) is a moveable index; it turns about a nail, as a centre in the end *w*, there being a slit in it, to let the bottom of the screw *x* pass through as it moves. *y* is moveable back and forward, and may be fixed any way by the two screws. *z* is a steel point, which moves along the circumference of any circle you require, from one point to another. T is the winch to turn the wheel B; S is the handle to pull down the machinery near the plate.

To use this machine. An arbor *ei* proper for the wheel *m*, which is to be cut, being put through the axis Q, and screwed fast, as it appears at R; and then the wheel *m* and the parts *k*, *l* put on above Q, and screwed fast. Loosen the screw *x*; and, moving the index *wy* till the steel point *z* fall in the circle,

containing the same number of parts, the wheel *m* is to be divided into, there screw it fast with the screw *x*. Then putting on the cutting wheel *f* proper for the work, turn the handle and screw I, and drive the machinery with the wheel *f* towards Q, till the edge of *f* lie just over the edge of the wheel *m* to be cut; there fix it by the handle V; and turn the wheel H till *z* falls into some point of the circle; then take hold of the handle S, and pull it down, till *f* falls against the edge of *m*; then holding it there with one hand, turn the winch T with the other; which carries about B, and this drives D with the cutter *f*, and this motion cuts a notch in the edge of *m*, and when it is deep enough, the pin *tu* (properly set) stops at the plate AC, and hinders it from going further. Then let go S, and the spring *rs* raises up the wheels, &c. This done, pull about the wheel H, till *z* fall in the next point of division; then draw down S, and turn the machine as before, till you have made another cut deep enough. And thus you must proceed, till *z* has gone through all the points of division in the circle, and then your wheel is cut into its proper number of teeth.

When the number of teeth wanted to be cut answers to none of the circles, take such a circle as can be divided by your number, and if the quotient be 2, 3, 4, &c. then you must set *z* to every 2d, 3d, 4th point, &c. skipping the rest. As if you want 21 teeth, take the circle 84, which divided by 21 gives 4; so that you must set *z* to every 4th tooth only, and so cut it.

A crown wheel may be cut the same way; but then the centre of the wheel *f* must be brought over the edge of the wheel to be cut, and there fixed. Also oblique teeth may be cut in a wheel after the same manner; but you must first ease the screw K, and then turn the cutting frame about K as an axis, till the cutter *f* have a proper degree of obliquity, and there screw fast the pin K, by the nut 2, and proceed as before.

After the teeth are cut with this engine, they are still to be wrought into their proper form, with files suitable for the business; and this the workman must do by hand.

Example CIV. (*Fig.* 1. *Pl.* LXIX.)

EH is a *glazier's vice*, for drawing window lead. PG, QH two axles, running in the frame KL, ML. C, D two wheels of iron case-hardened, $1\frac{1}{2}$ inch broad, and of the thickness of a pane of glass; these wheels are fixed to the axles, and run very near one another, not being above $\frac{1}{12}$ of an inch distant; across their edges are several nicks cut, the better to draw the lead through. E, F, two pinions, of 12 leaves each, turning one another, and going upon the ends of the axles, which are square,

and kept fast there by the nuts P, Q, which are screwed fast on with a key. A, B two cheeks of iron, case-hardened, and fixed on each side to the frame with screws; these are cut with an opening where the two wheels meet, and set so near the wheels, as to leave a space equal to the thickness of the lead; so that between the wheels and the cheeks there is left a hole, of the form represented at N, which is the shape of the lead when cut through. The frame KLML is held together by cross bars going through the sides, and screwed on: and a cover is put over the machine to keep out dust; and it is screwed fast down to a bench, by screw nails LL.

When it is used, the lead to be drawn is first cast in moulds, into pieces a foot long, with a gutter on each side. Take one of these pieces, and sharpen one end a little with a knife, and put it into the hole between the wheels; then turning the handle I, the lead will be drawn through, of the form designed.

Example CV. (*Fig.* 1. *Pl.* LXX.)

AC a *water-mill* for grinding corn, without either trundle or cog-wheel. BC is the arbor, or axis of the mill; this is a cylindrical piece of wood, about two feet diameter; GHIKLMN is a leaf or wing of wood, whose breadth is about the radius of the arbor; this runs spiral-wise round the arbor from bottom to top, ascending in an angle of about 35 degrees; it must every where stand upright on the surface of the arbor. Instead of one you may use two of these spiral leaves, especially if they be narrow. This arbor and its spiral leaf, turns round upon a pivot P at the bottom; and at the top B, it has a spindle, which goes through a plank, and is fixed to the upper mill-stone D, which turns round with it; so that the arbor has little or no friction. QRST is a hollow cylinder, made of stone or brick, to enclose the arbor and its leaf; and whose inside is walled as near as possible, just to suffer the leaf to turn round without touching; so that no water can escape between the leaf and the wall; and, consequently, it can only run down the declivity of the leaf; its top is represented by the circle QBTV. RWS is an arch to let the water out at the bottom, to run away; and big enough to go through to repair the engine. F is the trough that brings the water; D, E the two mill-stones. A the hopper and shoe. The arbor and its leaf may be cut altogether out of the solid trunk of a tree; or else the leaf may be made of pieces of boards, nailed to several supporters of wood, which are to be let every where into holes made in the body of the arbor, so as they may stand perpendicular to its surface; and all set in a spiral. And the spiral is made on this consideration; that for every 10 inches in the circumference of the axis,

you must rise 7 inches in length. But at the top G, it will be better to rise faster, so as to have its surface almost perpendicular to the stream.

When the mill is to go, the corn is put into the hopper at A, which runs down the shoe, through the mill-stone D. And the spout F being opened, the water falls upon the oblique leaf GHIK, and by its force turns the axis BC about, and with it the stone D, and grinds the corn.

Example CVI. (*Fig.* 1. *Pl.* LXXI.)

DBF is the *arch of a bridge*, which shall sustain itself, and all the parts of it, in equilibrio. Such an arch will be stronger than any other, because an arch that can sustain itself, will more easily sustain an additional weight, than an arch that cannot sustain itself, but only by the cohesion of the mortar. This arch DBF is a semicircle, whose centre is R, and vertex B; and the wall AT*ta* must be so built, that the height AT, in any place A, must be as the cube of the secant of the arch BA, which will cause it to run upwards towards D, in the form of the curve *t*ST. But as this form is not commodious for a bridge, the construction may be performed thus. In any place of the arch, as A, let the superincumbent part AT be built of heavier materials than at B, in proportion of the cube of the secant of the arch BA, for the parts near B, but in something less proportion in the parts towards A and D. And the right line GS*g* being drawn, will nearly terminate the top of the wall. But as materials cannot well be procured for this purpose, the following way may be used.

With the radius BR, (*Fig.* 2. *Pl.* LXXI.) describe the arch DB*d*, of 90 degrees; DB, B*d* being each 45. And if BR consists of 100 parts, make BS, 16. Draw the right line GS*g*, perpendicular to SBR; and the arch DB*d* shall support the wall DG*gd* in equilibrio in all its parts. If the arches DB and B*d* be made each 60 degrees, and the height BS 7 parts, and the right line GS*g* drawn, then the arch DB*d* will equally support the wall DG*gd* in all parts: but then the materials made use of about the places *a*, *a*, ought to be only about half the weight of those at B and D. And these are the principal cases in which a circle is serviceable, for making an arch stand in equilibrio.

Another equilibrial arch is from the catenary. Make the *latus rectum* BS, (*Fig.* 3. *Pl.* LXXI.) 100 equal parts; BR, AR, RF each 159; describe the catenary ABF, through the points A, B, and F. Then ABF will be an arch which will support the wall AG*gf* in equilibrio, in every point of it. The fault of this arch is, that by reason of the height BS, there

is too much weight of wall upon it, which will endanger the sinking the piers, except the foundation be very good; and it likewise raises the bridge too high.

Another arch of equilibration is this; make SB, BR, AR (*Fig.* 4. *Pl.* LXXI.) of any lengths at pleasure; draw the right line GS parallel to the horizon; and to the assymtote GS, draw the logarithmic curve AB: which may be done thus; draw AG perpendicular to GS; divide SG into any number of equal parts; and as many points of division as you have, find so many mean proportionals between SB and GA; set these from the respective points in SG downwards, in lines drawn through these points parallel to SR; and these will give so many points, through which the curve BA is to be drawn; and the curve *b*F is drawn the same way: between these, the pier BD is placed with a tower upon it. The only fault this arch has, is, that the water-way is diminished by the pier BD; and as many arches, so many supernumerary piers there will be.

I shall now shew how to describe an arch clear of all these inconveniencies. Make BR, AR, RF, (*Fig.* 5. *Pl.* LXXI.) each equal to 30 feet; BS, 3½ feet. Draw AG, F*g* parallel to RS. Divide SG, S*g* into 30 equal parts, or 30 feet; through all the points of division, draw lines parallel to SR., as TC. Then, upon each of these lines, set off from SG downwards the number of feet you find in the following table, respectively, as TC; then C will be in the arch. Do the same for the side S*g*. Then the curve FBCA, drawn through all these points C, will be the arch required. The curve is easily drawn through these points, by help of a bow held to every three points; or rather to four or five points at once; which may easily be done by two or three persons holding it.

Value of ST in feet.	Value of TC in feet and dec. parts.	Value of ST.	Value of TC.	Value of ST.	Value of TC.
0	3.500				
1	3.517	11	5.754	21	14.014
2	3.568	12	6.231	22	15.417
3	3.653	13	6.769	23	16.970
4	3.774	14	7.372	24	18.687
5	3.931	15	8.047	25	20.585
6	4.127	16	8.799	26	22.682
7	4.362	17	9.636	27	24.999
8	4.639	18	10.567	28	27.557
9	4.961	19	11.600	29	30.381
10	5.332	20	12.745	30	33.500

If the thickness of an arch at top, BS, be supposed to be 3 feet, 4 feet, 5 feet, &c. it will require a different curve to be constructed: but this seems to be strong enough for the bigness of the arch, especially if built of strong sound stone. Here 3½ feet is assigned for the thickness of the arch; but it must be made 2 or 3 inches less, on account of the parapet wall, for this adds weight to the whole. Also, if the top GS is not exactly horizontal, but is 2 or 3 feet lower at G than at S, the thickness BS ought to be 2 or 3 inches less upon that account; or if higher at G, two or three inches more: but these niceties make no sensible difference in practice.

This curve differs from the catenary (in *fig.* 3.) For at the vertex B it is less curve than the catenary; and towards A it is more curve. The curvature at B in this arch is very near that of a circle whose radius is BR. And the curvature increases from the vertex B, and is at least about C.

At the points A, F, where the arch springs, it rises at an angle of 73° : 1′, above the horizon.

If an arch is required to be either greater or less than this, it is no more than taking any other equal parts instead of feet; and setting off all the lines by these equal parts.

In this scheme, I have drawn a circle to shew the difference between that and this arch. The like I have done in figures 3 and 4. Whence it appears, that a circle circumscribes all these arches of equilibration; and, consequently, a circle is too curve at the lower parts, or at the haunch of the arch.

If any architects or builders of churches or bridges, shall please to make use of this curve here constructed (*fig.* 5.) for the form of an arch, they will find it the strongest arch possible to be made for these given dimensions. And where many thousand pounds are laid out in building a single bridge, it is certainly worth the pains to seek after the form of an arch, which shall be the strongest possible, for supporting so great a weight. And it is very surprising that no body has attempted it. Instead of that, all people have contented themselves with constructing circular arches; not knowing that different pressures against the arch, in different places, require different curvatures, which does not answer in a circle where the curvature is all alike. A circle, it is true, is very easily described, and that may be one reason for making use of it: but, surely, the description of the curve here given, is very easy, by the foregoing table, and can create no difficulty at all. If there be any difficulty in the practice, it is only in cutting the stones of a true curvature, to fit the arch exactly in all places; but this is easily managed with a little care, by taking proper dimensions; observing, that every joint must be perpendicular to the curve in that point.

A circle, or any other curve, where the curvature is not properly adapted to the weight sustained, is not capable of sustaining so vast a weight, but must, in time, give way, and fall to ruin, except the mortar happen to be so strong as to keep it together. On the contrary, the arch here described, sustaining every where a quantity of pressure proportional to its strength, will never give way, so long as the piers, which are its bases, stand good; but, by virtue of its figure, will stand firm and unshaken, as long as the materials the arch is made of will last.

As to the piers, their thickness may be $\frac{1}{5}$, $\frac{1}{4}$, or $\frac{1}{3}$ the width of the arch, according to the firmness of the ground they are to stand on. They must be considerably broader than the bridge, reaching out on each side into the water, being built with sharp edges to divide the stream. At the bottom, they must be well fenced with sterlings for their security. The outermost pier must be built far backwards, to sustain the oblique pressure of the arch, which has nothing else to butt against; otherwise the pier or buttress will yield to the pressure of the arch, and the arch will break.

Another construction.

In the former construction, I made the height BS to be only $3\frac{1}{2}$ feet. But as that may be reckoned too weak for an arch of 60 feet wide, like Westminster bridge, where the height above the arches is 8 or 10 feet; therefore I have here given a new table for constructing the arch of a strong bridge, calculated upon the same principles as the former, being 7 feet, all the other dimensions remaining the same. This arch rises from the pier at an angle of 70° 20′. In the former, I set off all the points of the curve from the line SG; in this, I set them off from the line BEE, which is a tangent to the top of the arch at B.

The construction is thus. Having drawn the line EBH through the top B of the arch parallel to the base AF, take from the table, col. 1, any length, and set it from B in the line BE, as to L, and from R to I, and draw the line LI; then, from col. 2. of the table, take the correspondent length of LC, and set it from L to C, in the line LI; then C is a point in the curve. And thus all the other points of the arch must be found; and then a curve drawn regularly through them all, gives the form of the arch.

THE TABLE.

Value of BL in feet.	Value of LC in feet.	BL	LC	BL	LC
1	0.021	16	6.251	25½	19.252
2	.086	17	7.173	26	20.263
3	.194	18	8.183	26½	21.316
4	.346	19	9.285	27	22.412
5	.543	20	10.488	27½	23.553
6	.787	20½	11.129	28	24.741
7	1.078	21	11.798	28½	25.978
8	1.419	21½	12.495	29	27.266
9	1.811	22	13.223	29½	28.605
10	2.258	22½	13.981	30	30.000
11	2.761	23	14.772		
12	3.324	23½	15.596		
13	3.951	24	16.455		
14	4.644	24½	17.349		
15	5.409	25	18.281		

EXAMPLE CVII. (*Fig.* 1. *Pl.* LXXII.)

QCF is the *weighing engine* at the turnpikes, for weighing road waggons. CD is a strong beam of wood, moving about the centre I. EF a steel-yard, moveable about the centre H, and suspended at D, by the iron hook DH. PA several iron chains suspended at a hook, moveable about the centre P; PH is about 3 or 4 inches, HF about 10 or 12 feet. The 4 chains at A are to put round the waggon. F a leaden weight fixed at the end F, whose weight is about 1⅝ hundred weight. G a moveable weight of ⅝ of a hundred weight; this is moved along the graduated beam HF, at pleasure. KNL a scaffold to walk on. CS is a chain hanging at C, and fixed to the brass pulley S. Round this pulley goes the rope MSR, whose end M is fixed to the cross bar QQ of the frame QQTT. In this frame, the wheels and axles 1, 2, 3 move round; being turned by the handle B, fixed to an iron wheel or fly. These wheels and trundles are iron: the trundles contain 11 teeth, the wheels about 60. The rope R is wound about the wooden axle 3, being 5 or 6 inches diameter. At the end of the axle, opposite to B, is another handle to be used upon occasion. The frame QT is fixed fast in the ground, that it may not be pulled up. The beam CD and steel-yard EF move between the cheeks KZ and NX, which serve to guide them, and likewise strengthen the frame they move in, which frame is tied together with several braces, as NO, VO, &c.

When any waggon is to be weighed, the 4 chains A are hooked round it, and a man turns the handle B; which, by turning the wheels, winds the rope about the axis 3, which pulls down the end C, which raises the end D of the lever CD. The end D raises the steel-yard EF, with the chains A, and the waggon; and a man upon the scaffold NL, moves the weight G till it be in equilibrio; and the divisions of the beam shew how much the waggon is above 60 hundred weight.

In some engines, the beam CD is wanting; instead of which, there are two blocks and pullies, the upper one fastened to a cross beam ZX, the lower is hooked to the piece DH, and the rope goes from the top block to the axis 3; but, in this case, the axis of the wheels are parallel to the side of the machine, and not perpendicular, as they are in the former; and then there is but one wheel and pinion, each of iron, the wheel of 110 or 120 teeth, and pinion 11.

Lastly, in some machines, that likewise want the beam CD, the wheels and axles 1, 2, 3 are placed in the top of the machine, above ZX; where, being turned round, they raise the beam EF, either by a rope going from it, or by blocks and pullies.

Example CVIII. (*Fig.* 1. *Pl.* LXXIII.)

HRK is a *large organ*. HIH the *sound board;* this is composed of two boards, the *upper board*, or *cover* HIH, and the *under one* III, which is far thicker than the upper one. Each of these is made of several planks laid edge-ways together, and joined very close. In the under side of the under board, there are several channels made, running in direction LL, MM, &c. continued so far as is the number of stops in the organ; and coming almost to the edge HK. These channels are covered over very close, with leather or parchment, all the way, except a hole which is commonly at the fore end next HK, upon which a valve or puff is placed. These channels are called *partitions*. When this flap or valve is shut, it keeps out the air, and admits it when open. On the upper side of the under board there are, likewise, cut several broad square gutters, or channels, lying cross the former, but not so deep as to reach them; these lie in direction LN, PQ, &c. And to fit these channels, there are as many wooden *sliders* or *registers*, *f*, *f*, *f*, &c. running the whole length; and these may be drawn in or out at pleasure. The number of these is the same as the number of stops in the organ.

IKKK the *wind chest:* this is a square box, fixed close to the under side of the under board, and made air tight, so that

no air can get out, but what goes through the valves, along the partitions.

V, V are the *valves* or *puffs* which open into the wind chest; and are all enclosed into it, and may be placed in any part of it, as occasion requires. One of these valves, with the spring that shuts it, and wire that opens it, is represented apart, on the left hand.

C, D, E, F, &c. are the *keys* on which the fingers are laid, when the organ plays. These keys lie over the horizontal bar of wood W, in which are stuck as many wire pins *z*, *z*, on which the keys are put; and the keys move up and down upon this bar as a centre. 3 is another bar, against which the keys fall when put down; on this, also, are several wires, going through the keys to guide them; and on this bar a list is fastened, to hinder the knocking of the keys against it.

Now, the keys are made to communicate with the valves several ways, as I shall now describe. *s*, *s*, *s*, are the *key rollers*, moving on the pivots *t*, *t*; these rollers lie horizontally one above another, and one at the end of another, of such a length, as to reach from the valve to the key. *a*, *a*, *a*, arms or levers fixed to the key rollers; *w*, *w* the *valve wires* fixed to the arms *a*, *a*, and to the valves V, and going through the holes *h*, *h*, in the bottom of the wind chest. *b*, *b*, *b* arms fixed likewise to the key rollers. *d*, *d*, *d* the *key wires*, fixed to the arms *b*, *b*, and to the keys C, D, E. Now, putting down the end of any of the keys C, D, E, it pulls down the arm *b*, by the wire *d*, which turns the roller *s* about, with the arm *a*, which pulls down the wire *w*, which opens the valve, which is shut by the spring, as soon as the key is let go. In this construction, there must be a worm spring fastened to the key, and to the bar W, on the further side, to keep the end 5 of the key down.

Another method of opening the valves is this. *xy*, *xy*, are slender levers moveable upon the centres 1, 1. 5*x*, 5*x*, are wires going from the far ends of the keys, to the ends *x* of the levers. *y*V, *y*V, other wires reaching from the ends *y* of the levers, through the holes *h*, to the valves V. So that putting down the key C, D, &c. raises the end 5, which thrusts up the end *x* of the lever, by the wire 5*x*; this depresses the end *y* of the lever; which pulls down the wire *y*V, and opens the valve V.

A third way of opening the valves is this. At the end of the key 6 is a lever 8, 9, moving upon the centre 7. This, with the key, makes a compound lever. From the end 9, there is a wire goes to the valve. Now, putting down the end 6 of the key, raises the end 8, which depresses the end 9 of the lever

8 9, and pulls down the wire, and opens the valve. I have only drawn one of these in the scheme, and but a few of the others, to avoid confusion.

R, R are the *rollers* to move the sliders, by help of the arms *cf*, *cf*, which are fixed horizontally in these rollers. *ke*, *ke* are levers also fixed in the rollers. *le*, *le* are the handles, which lie horizontally, and pass through the holes *l*, *l*, and are fastened to the lever *ke*, being moveable about a joint at *e*.

Now, any handle *lp* being drawn out, pulls the end *e* towards *l*, which turns R*k* about, along with the arm *cf*; and the end *f* pulls out the slider *fg*. And when *p* is thrust in, the arm *cf* likewise thrusts in the slider *fg*.

Upon all the several rows of holes which appear on the top of the upper board, are set upright so many rows of pipes: X is a *flute pipe* of wood, Z a *flute pipe* of metal, Y a *trumpet pipe* of metal. The pipes pass through holes made in boards, placed above the upper board, to keep them from falling.

The pipes are made to communicate with the wind chest, after this manner. When any slider *fg* is drawn out, holes are bored through the upper board, through the slider, and through the under board, into the partition below: so that any pipes, placed upon these holes, will then communicate with the partition; which, by its valve, communicates with the wind chest. But when the slider is thrust in, the holes in the slider do not stand against the holes, in the upper and under boards; and the communication is stopped, so that no wind can get to the pipe.

*q*T, *q*T are the *bellows*, which must be two at least. *q*, *q* the *wings*; O, O the *handles*, moving upon the fixed axes *nn*, *nn*. Each of these bellows consists of two boards; the under board is fixed immoveable. In this there is a *valve r* opening inwards, and a tube leading to it, called the *conveying tube*. There is also a hole in this under board, from which a tube leads to the *port-vent*, which is a square tube 24, rising upwards, and is inserted into the under side of the wind chest at 2. And in the tube leading to the port-vent, there is a valve which opens towards the port-vent; which suffers the air to go up the port-vent, but none to return. All the bellows are constructed after the same manner. Now, the handle O being put down, raises the upper board T, and the air enters through the valve *r*; and when the handle is let go, the weight of the upper board T, (which carries 3 or 4 lb. to every square foot,) continually descending, drives the air through the port-vent to the sound-board. And as one pair of bellows, at least, is always descending, since they work alternately, there will be a constant blast through the port-vent.

In chamber organs there is but one pair of bellows, which con-

sists of three boards, in nature of a smith's bellows; and so has a continual blast.

All the inner work is hid from sight, by the face of the instrument standing upon 36.

As many partitions LL, MM, &c. as there are in the soundboard, so many valves V, V, rollers *s*, *s*, or else so many levers *xy*, or 89, and their wires, and that is just as many as there are keys A, B, C, D, &c. And there are generally 61 keys, with flats and sharps, reaching from G to G, the compass of 5 octaves. But the scheme could not contain them all. Likewise, there are as many handles *l*, *l*, &c. rollers R, R, &c. sliders *f*, *f*, &c. as there are different stops upon the organ. And it must be observed, that, between the sliders *f*, *f*, &c. there are as many sliders on the right hand; and the same number of handles and rollers, which cannot be expressed in this scheme. And other rows of pipes placed between LN, PQ, &c. And towards the middle of the organ, the least pipes are placed, and the least partitions; the greatest being on the outside.

There are many stops in some organs, but generally 10 or 12 on each hand; these are some of them;—diapason, principal, fifteenth, twelfth, tierce, cornet, trumpet, French horn, vox-humana, flute, bassoon, cremona, &c. and a contrivance to swell the notes of some of the stops.

When this noble instrument is to be played upon, put down the handle O of the bellows, this raises the upper board T, and causes the air to enter in at the valve *r*. Then that handle being let go, the other handle O is put down; during this time, the board T of the first, descending, and shutting the valve *r*, drives the air through the other valve, up the port-vent into the wind chest. Then drawing out any handle, as the flute stop *pl*, this draws out the slider *fg*, and all the pipes in the set LN are ready to play, as soon as the keys C, D, E, &c. are put down. Therefore, putting down the key D, by laying the finger upon it, opens the correspondent valve *m*V, and the air enters through it, into the pipe X, and makes it sound. In the same manner, any other pipe, in the set LN, will sound, when its key is put down. But no pipe in any other set PQ will speak, (because the communication is stopped) till its slider *f* is drawn out by the corresponding handle *l*.

Pipes are made either of wood or metal; some have mouths like flutes, others have reeds. The smallest pipes are made of tin, or of tin and lead. The sound of wooden and leaden pipes is soft. Short pipes are open, and the long ones are stopped; the mouths of large square wooden pipes are stopped with valves of leather. Metal pipes have a little ear on each side of the the mouth, to tune them, by bending it a little in or out. Whatever note any open pipe sounds, when the mouth is stopped it

will sound an octave lower: and a pipe of twice its capacity will sound an octave lower.

(*Fig.* 2. *Pl.* LXXIII.) It will not, I think, be foreign to my design, if I give a short account of the method of tuning organs or harpsicords. But I must first premise something concerning the scale of music. It is known, from undoubted experiments, that if AL be a string of a musical instrument, and if the string be stopped successively at Y, *s*, *q*, *p*, *o*; and $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$ of it be made to vibrate, it will sound an eighth, a fifth, a fourth, a greater third, and a lesser third, respectively.

Now, as the difference between the fourth and fifth is accounted a whole tone, whereof there are 6 to make up the octave, therefore, we shall have $\frac{3}{4} - \frac{2}{3}$, or $\frac{1}{12}$ for the difference of the strings, that are to sound a note one above the other, whereof the greater is $\frac{3}{4}$. Consequently, if the string is 1, that difference would be $\frac{1}{9}$. Therefore, $\frac{8}{9}$ of any string will sound a note higher; and $\frac{8}{9}$ of this second would sound 2 notes higher than the first; and $\frac{8}{9}$ of this third would sound 3 notes higher than the first, &c. and this being 6 times repeated to make up an octave, we shall have $\frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} = \frac{1}{2}$, as it would be if the note was exact, but that product is less, being but .4933; and, therefore, $\frac{8}{9}$ is too small, and $\frac{1}{9}$ too great, for a tone. And 6 of these notes do not exactly make up an octave.

After the same way, if we take the difference between the fourth and lesser third for a whole tone ($\frac{5}{6} - \frac{3}{4}$), we shall get $\frac{1}{10}$ of the string for a whole tone, but this will be found to be too great, being .5314, instead of .5; therefore $\frac{1}{10}$ is too small for a note.

If we try by half notes, we shall still be no better. The lesser third, the greater third, and the fourth differ by half a note; of which there ought to be twelve in the octave. In the former case we get $\frac{24}{25}$ for the length of the string, in the latter $\frac{15}{16}$; and $\frac{1}{25}$ or $\frac{1}{16}$ for the length of half a note. The first is far too little, and the latter as much too big.

As none of these notes or half notes will make up an octave, so neither will any number of thirds, fourths, or fifths, make one or more octaves. A lesser third contains 3, a greater third 4, a fourth 5, and a fifth 7 half notes. Therefore, 4 lesser thirds, or 3 greater thirds, make an octave; and 12 fourths should make 5 octaves; and 12 fifths 7 octaves. But if this was so, then we should have $\left(\frac{5}{6}\right)^4 = \frac{1}{2}$, $\left(\frac{4}{5}\right)^3 = \frac{1}{2}$, and $\left(\frac{3}{4}\right)^{12} = \left(\frac{1}{2}\right)^5$ (that is $\left(\frac{3}{4}\right)^{\frac{12}{5}} = \frac{1}{2}$), and $\left(\frac{2}{3}\right)^{\frac{12}{7}} = \frac{1}{2}$. But never a one of these is so. And hence we may conclude, that no scale, made up of these notes, or half notes, or any combinations of them, or of thirds or fifths, &c. can be perfectly exact.

Now, to contrive a scale to answer as near as possible all the

requisites; let AL, A*m*, A*n*, A*o*, &c. to AY or $\frac{1}{2}$ AL, be 13 geometrical proportionals; then these strings AL, A*m*, A*n*, &c. will sound all the half notes in the octave, gradually ascending. Therefore, if AL be put $= 1$, AY $= \frac{1}{2}$; A*m*, A*n*, A*o*, &c. being so many mean proportionals between 1 and $\frac{1}{2}$ will be found, as set down in the following table.

Cords.	Strings.	Notes equally ascending.	Pure Concords.	Errors.
ground	*AL*	1.00000	1.00000	0
♭ *second*	*Am*	.94387		
♯ *second*	*An*	.89090		
les. third	*Ao*	.84090	.83333	♭ $\frac{1}{13}$
gr. third	*Ap*	.79370	.80000	♯ $\frac{1}{15}$
fourth.	*Aq*	.74915	.75000	♯ $\frac{1}{100}$
♯ *fourth*	*Ar*	.70711		
fifth	*As*	.66742	.66666	♭ $\frac{1}{100}$
les. sixth	*At*	.62996	.62500	♭ $\frac{1}{15}$
gr. sixth.	*Au*	.59460	.60000	♯ $\frac{1}{13}$
♭ *seventh*	*Aw*	.56123		
♯ *seventh*	*Ax*	.52973		
eight	*AY*	.50000	.50000	0

In this table, the 3d column shews the lengths of the vibrating string, when the scale ascends by equal degrees of sound, or when all the half notes are equal.

The 4th col. shews the length of the string to sound the pure concords.

The last col. relates only to the concords: and shews the error of the 3d col. expressing what part of a whole tone it is; and whether it is below (expressed by ♭) or above (by ♯). By this col. we can judge how the scale in the 3d col. will perform. And these errors are found by comparing the 3d and 4th col. together. As suppose you would know what it is in the fifth, we shall have $66742-66666=76$, and $70711-62996=7715$, which represents a tone in that place. Then $\frac{76}{7715}$ or $\frac{1}{100}$ is the error, which is but the hundredth part of a whole tone. And as the number in the 3d col. is greater, it shews, that, by this scale, the fifth is flatter than it ought to be. And so are the rest of the errors found out, and examined.

Now it is evident, that the error in a fifth or a fourth is quite insensible in practice; but the thirds and sixths suffer the most,

being in some but the 13th part of a note, which, perhaps, may be sensible to a good ear; but then it will not be so perceptible in a third as it would be in a fifth; because a third is less perfect than a fifth. And the sweeter the cord, the more easily is an imperfection discovered. Now, although these trifling errors will take away something from the sweetness of the harmony, and will hinder the scale from being absolutely perfect, yet there is no remedy in it, but what is worse than the disease.

As to the tuning this instrument, it is plain that the notes ought not to be tuned by perfect fifths, for the upper note will always be the hundredth part of a note too high. And since one must take 12 fifths, before he can come at the same note again, whence he set off, there will, at last, be an error of $\frac{12}{100}$ or $\frac{1}{8}$ of a note, which is very discoverable in a fifth. The method therefore to be taken, is, to make the upper note a very little flatter than a perfect fifth, by first tuning it perfect, and then lowering it a small matter, but not so much as to offend the ear. And after you have thus gone through the octave, if you find the last note either too high or too low, begin anew, and alter them all a little, according to your judgment, till the last does agree: but this judgment is to be attained principally by practice. Upon the first octave being rightly tuned, all the rest depend; and, therefore, one ought to be very exact in it; for, in all the rest, there is nothing to do but to take the eighths above and below. And you ought to begin to tune about the middle of the instrument.

Those that tune by thirds, ought to take the upper note of the greater third, as sharp as the ear will bear. And lesser thirds should be taken as flat as they may.

Most people, in tuning, take some of the fifths perfect, and leave others imperfect; which they call bearing notes. But this is attended with great inconvenience: for the music ought to be so set, that no fifth ought to fall on any of these bearing notes, which, instead of being a perfect concord, will be no better than a discord, since the error in these bearing notes is very great. For if there is but one in an octave, its error is $\frac{1}{8}$ of a note; if two of them, and both alike, $\frac{1}{16}$ of a note. Now, if these people be so nice as to distinguish $\frac{1}{100}$ part of a note, much more would they be offended at $\frac{1}{8}$ or $\frac{1}{16}$. And always to avoid taking the fifths upon these notes, when they come naturally in the way, would be cramping, and even spoiling the music. And another disadvantage would arise, that a piece of music could not be transposed upon any key at pleasure, whatever need there might be for it; but must be tied down to a very few. And if this method could cure, in any measure, the errors of the fifths, yet it would not at all mend those of the thirds, which are far greater. And if any one third should happen to be bettered, it is certain that

others will be made as much worse, and will be turned into discords.

Some that like not the *equiharmonic* or *isotonic scale,* above described, would compose a scale of several sorts of tones and semitones, as $\frac{1}{8}$, $\frac{1}{10}$, and $\frac{1}{16}$, $\frac{1}{17}$, $\frac{1}{18}$, $\frac{1}{19}$, $\frac{1}{20}$, &c. but what end can this answer? it is very easy to shew, that if some cords may be taken perfect, others will be miserably bad, and degenerate into discords. But this scheme seems to be built only upon the consideration of abstract numbers; for it regards only this, how to make several fractional quantities resolve themselves into others more simple, by multiplication; which is a thing of no manner of use in music.

Others, to avoid the badness of the cords which fall in some places, have invented quarter notes; which makes the music extremely hard to play; and is, besides, far from answering the end proposed. Therefore, upon the whole, I cannot but think the scale above described to be the best for practice. For so small an error as $\frac{1}{150}$ of a note in a fifth, and every where the same, cannot be sensible, or do much hurt to the music. And I will venture to say, that the very alteration of the weather, in 24 hours' time, by heat or cold, drought or moisture, will have such an effect upon either strings or pipes, as to cause a greater difference than this amounts to. There are imperfections in every thing, which we cannot quite take away; all we can do, is to make them as little as possible.

More examples of the constructions of engines might here be added. But as there is such an infinite variety in the world, it would be an endless task to describe all the kinds of them. Therefore I think it needless to produce any more, especially, since their construction and use depend upon the same principles as these already described. And if the reader does but thoroughly understand the powers and forces of these before mentioned, he cannot be at a loss to find out the powers, forces, or motions, of any other machine, though never so compounded.

AN

ALPHABETICAL INDEX

OF THE TERMS USED IN MECHANICS.

A.

AIR PUMP, a machine to draw the air out of a glass. *Fig.* 1. *Pl.* LII.

Ajutage, the spout for a *jet d'eau* in a fountain.

Amplitude, the distance a ball is shot to.

Anemoscope, a machine for shewing the point of the wind. See *Fig* 2. *Pl.* XL.

Angle of application, is the angle which the line of direction of a power makes with the lever it acts upon.

—— *of inclination*, is the angle an inclined plane makes with the horizon.

—— *of traction*, the angle which the direction of a power makes with an inclined plane.

Aqueduct, an artificial river, or tube to convey water.

Arbor, the axle or spindle of a wheel. *Fig.* 4, *cf.* *Pl.* XVIII.

S

Arch, a hollow wall made of a circular form, to support : building.

Areometer, an instrument to measure the weight of liquors.

Arm, any piece of timber or metal, that projects horizontally fi some part of a machine.

Axle, or *Axis*, the line or spindle about which a wheel turns rou *Fig*. 4. *Pl*. XVIII.

Axis in peritrochio, a machine for raising weights, consisting o wheel, fixed upon a cylinder for its axis. *Fig*. 3. *Pl*. III.

B.

Balance, a machine to weigh bodies in: one of the mecha powers. *Fig*. 6. *Pl*. XVIII.

Balance wheel, the fly or pendulum of a watch.

Barometer, a machine to shew the weight of the air or atmosphe See *Fig*. 4. *Pl*. XXVI.

Baroscope, the same as barometer: a weather-glass.

Barrel of a wheel, is the axle, or cylindrical body, about wh the rope goes.

——— *of a pump*, is the hollow part of the pump where piston works.

Barrs, straight pieces of timber or metal, that run cross from c part of a machine to another.

Base, the foot of a pillar.

Basil, that angle the edge of a tool is ground to.

Batten, a piece of timber three or four inches broad, and an ir thick.

To *Batter*, to lean backward.

Bauk, a long piece of timber.

Beak, the crooked end of a piece of iron, to hold any thing fas

Beam, a large piece of timber lying across any place.

Beetle, a wooden instrument, or mallet, for driving piles.

Butments, those supports on which the feet of arches stand.

Buttress, a piece of strong wall that stands on the outside of another wall to support it.

C.

Capstan, a machine on board a ship, to hoist the masts, or raise any weight. *Fig*. 2. *Pl*. XXXVII.

Cascade, a fall of water.

Cataract, is a precipice, or violent fall of water in a river, through high rocks, causing the water to fall with a great noise and force.

Catch, some small part of a machine; which, in its motion, hooks or lays hold of some other part to stop it.

Centre of motion, the point about which a body moves.

——— *of gravity*, the point upon which a body being suspended, it will rest in any position.

——— *of magnitude*, a point equidistant from the opposite extremes of a body. The middle.

——— *of percussion*, the point of a vibrating body that gives the greatest stroke.

Centre pin, a pin about which, as a centre, a body moves.

Chain pump, a pump having several buckets fixed to an endless chain, which goes through it, and is moved round upon an axle. *Fig*. 3. *Pl*. XL.

Chaps, two sides of a machine which take hold of any thing.

Cheeks, two upright parts of a machine, answering to one another in position and use.

Chronoscope, a pendulum to measure time.

Clack, a sort of valve which is flat, like a board; serving to stop a fluid from running out. *Fig*. 2. *Pl*. XLVII. T. a flap.

Clampt, when the edges of two pieces of boards are joined together, so as the grain of one may lie cross the grain of the other.

Clasp, a sort of buckle to fasten any thing.

Clasp nails, those with little heads to sink into the wood.

Claws, slender crooked pieces of metal in a machine, which serve to move or hold any thing. Long teeth.

To *Clench* or *clinch*, to double back the end of a nail, that it may not draw out again. To rivet.

Clench nails, nails that may be clinched.

Cock, a brass spout to let a fluid run out, or stop it by turning.

Cogs, the wooden teeth of a great wheel. *Fig*. 1. *Pl*. XLII. E.

Cog-wheel, a large wheel made of timber, where the teeth stand perpendicular to the plane of the wheel. *Fig*. 1. *Pl*. XLII. E.

Collar, a ring of metal that goes about any thing, near the top, in which it turns round.

Column, the shaft or trunk of a pillar.

Contrate-wheel, a wheel in a clock, where the teeth are parallel to the axis, and stand on the under side of the rim, and the same as shewn at *Fig.* 2. *Pl.* XX.

Corbel, a piece of timber, or stone, set under another piece, to discharge the weight.

Crab, a small capstan with three claws, to be placed on the ground, moveable from one plane to another. This is called a flying capstan.

Crane, a machine for hoisting goods out of a ship, or for raising timber or stone. *Fig.* 2. *Pl.* XXIX.

Crank, that part of an iron axis which is turned square with an elbow. *Fig.* 2. *Pl.* XXXII. I I.

Cross-tree, a horizontal beam fixed across another.

Crow, a strong square bar of iron, forked at the end, to remove heavy timber, &c. by using it with the hands.

Crown-wheel, in a clock or watch, is that next the balance; its teeth stand in the upper side of the rim, and not in the edge. *Fig.* 3. *Pl.* XVI. FG.

Cupola, a hollow arched tower, in form of a hemisphere, or of a bowl turned upside down.

D.

Density, is a greater or less quantity of matter contained in a given space.

Detents, are those stops, which, being lifted up, the clock strikes; and falling down, she stops.

Dog nails, nails used for fastening hinges.

Dome, a round vaulted roof or tower. A cupola.

Dormant, a great beam lying cross a house. A summer.

Dormer, a window in the roof of a house.

Dove-tailing, letting one piece of timber into another, with a joint in form of a dove's tail, being broader at the end, that it may not draw out again.

Drum, the lantern or trundle, which is carried by a great wheel. *Fig.* 1. *Pl.* XLVI. EF.

Drum head, a timber head, or lump of timber, in form of a drum.

E.

Edging, the outside or border.

Endless chain, a chain with the ends joined together; by which any part of a machine is wrought.

Endless screw, a screw working in the teeth of a wheel; which may be turned about for ever. *Fig.* 1. *Pl.* XX. E.

Engine, a mechanical instrument composed of wheels, levers, screws, &c.

Eolipyle, a hollow globe of metal, filled with water, and put in the fire; the heat and vapour rushes out at a small hole, with a great noise. *Fig.* 3. *Pl.* XLV.

Equilibrium, the equality of weight, of two or more bodies, &c. keeping one another at rest.

Eye, a hole in some part of a machine, through which any thing is put.

F.

Face, the foot or foreside of a machine, or of some principal part of it.

Fang, some small piece of metal like a long tooth, that by its motion moves some other part.

Fellies, pieces of wood on the outside of a wheel, which make the rim.

Ferril, a sort of hoop.

Floats, the flat boards set perpendicular on the edge of a water-wheel; by which the water drives the wheel about. *Fig.* 1. *Pl.* XXXII. D, D.

Fly, that part in a clock, &c. that regulates the motion, and makes it uniform. *Figures* 5, 6, 7. *Pl.* XVI.

Force, any thing that acts upon a body to put it in motion.

Force pump, a pump that discharges water by pressing it upwards. *Figures* 1 and 2. *Pl.* XLVII.

Frame, the outwork of any machine, or what holds all the rest together.

Free, clear of all impediment.

Friction, the resistance that bodies have by rubbing against one another.

Fulcrum, that which supports a lever in moving any heavy body.

G.

Gain, the levelling shoulder of a joist or other timber.

Gin, a machine to raise great weights. *Fig.* 1. *Pl.* XLII.

Gravity, the weight of bodies.

——— *specific*, by this one body weighs more or less than another of the same bulk.

——— *relative*, is the weight of a body in a fluid, or on an inclined plane.

Groove, a channel cut in wood or stone.

Gudgeons, the eyes in the stern of a ship, on which the rudder hangs. The centre pins of an axle.

Gyration, a whirling round.

H.

Hand, an index or pointer.

Handle, the part of an instrument to take hold on with the han

Hand-spike, a wooden lever to be used with the hand, in mov: any thing.

Head, the top part of any thing.

Hem, the edge of some cloth turned down and sewed.

Hinge, an iron joint on which a door turns, &c.

To *Hitch*, to catch hold on, with a hook or turn of a rope.

To *Hoise*, or *hoist*, to heave up, or raise by force.

Hook pins, taper iron pins with a hook head, by which they : struck out again. They serve to pin the frame of a roof floor together, till wrought off.

Hoop, a circular ring to put about any thing, to keep it fast.

Hydraulics, the art of making engines for water works.

Hydrometer, an instrument to measure the density of fluids. *Fig* *Pl.* XLVII.

Hydrostatics, a science teaching the weights, pressures, motio and properties of fluids.

Hydrostatical balance, an instrument for finding the specific g vity of bodies.

Hygrometer, } an instrument for measuring the moisture and d:
Hygroscope, } ness of the air. *Fig.* 4. *Pl.* XLV.

I.

Jack, an engine to lift up a loaded cart, or the roof of a house, & See *Fig.* 1. *Pl.* XXXVIII. Also an engine to roast me *Fig.* 1. *Pl.* XLIII.

Jack pump, a chain pump.

Jaums, door posts, or window posts, &c.

Jet d'eau, the pipe of a fountain, which spouts up water in the air. *Fig.* 1. *Pl.* LVII.

Impetus, any blow or force wherewith one body strikes or imp

Latch, that which fastens a door, &c. A *sneck*.
Leaver, or *lever*, a bar of iron or wood to raise a weight: one of the mechanic powers.
Leaves, the teeth of a pinion. *Fig*. 4. *Pl*. XVIII. *c. c. c.*
Ledge, a flat border, or plain, adjoining to a thing.
Level, an instrument to place any thing horizontal.
Linch pin, a pin that keeps a wheel from coming off its axle.
Lip, a thin edge turned hollow.
Loop, a piece of metal having a hole in the end, which goes over something. A noose in a rope that will slip.

M.

Machine, a mechanical instrument for moving bodies.
Mechanics, a science that teaches the principles of motion, and construction of engines, to move great weights.
Mechanic powers, are these six; the balance, lever, wheel, pulley, screw, and wedge: and, according to some, the inclined plane.
Mitre, an angle of 45 degrees, or half a right one. An *half mitre* is a quarter of a right angle.
Momentum, quantity of motion; or the force or power a body in motion has to move another.
Mortise, a square hole cut in a piece of stuff, to receive the tennant.
Motion, is the successive change of place of a body; or its passing from one place to another.
Moving force, any active force or power that moves a body.
Mouth, the part or parts of a machine, that take hold of any thing. The entrance into any cavity.

N.

Nave, or *Naff*, the block in the middle of a wheel.
Neck, a part near the end, cut small.
Notch, a dent, nick, or slit, made in any thing.
Nut, the pinion of a wheel. *Fig*. 4. *Pl*. XVIII. AB. A small piece of metal going upon the end of a screw nail.

O.

Oscillation, the vibration or swinging of a pendulum.

P.

Paddles, a sort of oars. The laddle boards on the edge of a water-wheel.
Pedestal, the base or bottom of a pillar.
Peers, or *Piers*, a sort of buttresses, for support and strength.
Peg, a pin to go into a hole.

Pendulum, a weight hung by a string or wire, swinging back a forward, to measure time.

Penstock, the sluice or door, that opens or shuts the passage of t water to a water-wheel.

Percussion, the striking of one body against another.

Pestle, a long piece of wood or metal, which rises up and fa down again to beat or bruise something.

Pevets, or *Pivots*, the ends of the spindle of the wheel in a cl or any machine, which play in the

Pevet holes, the holes in which the ends of a spindle or axle o wheel turn. *Fig.* 4. *Pl.* XVIII. *e, f.*

Picket, a stake pointed with iron, to drive into the ground.

Pillar, a perpendicular column, supporting one end of an ar &c.

Pinion, a little wheel at one end of the spindle, consisting but a few leaves or teeth. *Fig.* 4. *Pl.* XVIII. AB.

Piston, a round piece of wood, moving up and down within t body of a pump, to draw up the water.

Plate, a piece of timber, on which some heavy work is framed, wall-plate, &c. A flat piece of metal.

Pneumatics, a science teaching the properties of the air.

Pole, a long staff, or slender piece of wood.

Post, a perpendicular or upright beam of wood.

Power, the force applied to an engine to raise any weight. any force acting upon a body to move it.

To *Project*, to jet out or hang over.

Projectiles, balls or any heavy body thrown into the air.

Prop, a stay or support for any thing, to bear it up.

Pulley, a small wheel with a channel in the edge of it, movi about an axis fixed in a block; the channel is to receive rope or chain that goes over it. One of the mechanic powe

Ribs, slender pieces of timber, serving for strength and support.
Riglets, little flat, thin, square pieces of wood.
Rim, the circular part or outside of a wheel.
To *Rivet*, to batter down the end of a nail, that it draw not out again.
Rod, a long slender piece of wood or metal.
Roll, or *roller*, an engine turned by a handle, to raise weights. *Fig*. 1. *Pl*. XXXVI.
Rounds, the staves or spindles in a lantern, against which the teeth of a great wheel work. *Fig*. 1. *Pl*. XLVI. *c*, *c*. The steps in a ladder, &c.
Ruler, a thin straight piece of wood, metal, or ivory, for drawing straight lines.
Rungs, spindles or rounds. *Fig*. 1. *Pl*. XLVI. *c*, *c*.
Runner, a flat circular ring, between the nave and linpin of a wheel, called also a washer. Also a sort of rope on board a ship, to hoist with.

S.

Sails, large pieces of canvas, by which ships, windmills, &c. are carried, by help of the wind.
Scantlin, stuff cut to a proper size.
Screw, one of the mechanic powers. The tap with the thread is the *male screw*, the hollow that receives it is the *female screw*.
Scribing, drawing an irregular line upon one piece of stuff parallel to the irregular side of another, with a pair of compasses opened to a due distance, and carried along the side of it. Then the wood in the first piece being cut away, these two pieces will fit each other.
To *Seaze*, to bind or fasten a rope, &c.
Shaft or *shank*, any long part of an instrument, especially that which is held with the hands.
Sheers, two poles set up an end sloping, and tied together at top, and secured by a rope from falling. Their use is to raise any weight by help of a block and tackle at top.
Sheevers or *shieves*, pullies, the little wheels that run in blocks, by a rope going over them.
Shelf, a board, &c. fixed horizontally.
Shoulder, a part of timber or metal, cut thicker than the rest, in order to support something.
Shrouds, the ledges on the edge of a guttered wheel.
Sills, *sells*, or *groundsils*, pieces of timber that lie on the ground, into which others are fixed. Sole trees.
Siphon, a crooked glass or metal tube, for drawing off liquors. *Fig*. 2. *Pl*. XXVI.

Sleepers, pieces of timber laid as a foundation and support others that are to lie upon them.

Slings, these are made of a rope spliced with an eye at eitl end, to go over a cask, or some heavy thing, which is to hoisted.

Snatch block, a great block with a sheever in it, and a notch through one of the cheeks of it, to hitch the rope into t pulley, for readiness.

Socket, a hollow piece of metal, in which any thing moves.

Sole, the bottom of the gutter or channel, in a guttered wheel.

Sole tree, the lowest piece of timber which lies flat on the grour into which the upper works are framed. The groundsel.

Spanish burton, a sort of tackle to hoist goods, like *Fig.* *Pl.* XXI.

Spear, a long pointed iron, or piece of timber.

Specific gravity, is that whereby one body weighs more or less th another of the same magnitude.

Spike, a pointed iron, or piece of wood.

Spindle, the axle of a wheel. *Fig.* 4. *Pl.* XVIII. *ef.*

Spires, the turns of a rope about a cylinder or roller.

To *Splice*, to join two ropes together by working the strands ir one another.

Spokes, pieces of wood running from the centre of a wheel to t circumference, like rays.

Spring, an instrument made of steel, that being bent, it con nually exerts a great force, that it may unbend itself agai Springing plates are sometimes made of brass.

Spur, a sort of prop, set aslope to thrust.

Spurs, long wooden teeth standing in the edge of a large timt wheel. *Fig.* 2. *Pl.* XIX. *a, a, a.*

Spur wheel, a wooden wheel where the teeth stand in the edge the rim. *Fig.* 2. *Pl.* XIX. CD.

Stroaks, or *straiks*, the iron going round the circumference of carriage wheels, also called the tire.

Stud, a knob, or little button. A solid piece of metal fixed to a plate.

Stuff, any wood that joiners work upon.

Swivel, a metal ring that turns about any way.

Syphon, the same as siphon. *A crane*. *Fig*. 2. *Pl*. XXVI.

Syringe, an instrument for injecting liquors into any place.

T.

Tackles, blocks with pullies and ropes in them, to heave up goods. *Figures* 2, 3, 4. *Pl*. XXI. The rope we pull by is called the *Tackle-fall*.

Tenon, the square end of a piece of wood, made to fit into a mortise hole.

Thermometer, } an instrument to shew the degrees of heat and
Thermoscope, } cold. *Fig*. 4. *Pl*. XLVII.

Thread, the spiral ridge that goes winding round a screw.

Thrust, the action against a body to push it forward.

Tight, stiff, close.

Tongue, a thin slender piece of metal or wood in a machine.

Tool, an instrument to work with.

Tooth, the indented part on the edge of a wheel that moves some other wheel. Or what serves to cut, or take hold on.

Transom, an overthwart beam in a building.

Triangle, an engine standing on three legs, to raise weights with. *Fig*. 1. *Pl*. XXI.

Trundle, the part which is carried about by the teeth of a wooden wheel. The lantern or drum. *Fig*. 1. *Pl*. XLVI. EF.

Trunk, a hollow tube or box.

Tumbler, a part in a machine that rolls about upon an axis, and plays back and forward.

Tumbrel, a roller, or cylindrical beam of wood.

Tympanum, a kind of wheel placed on an axle, and has staves or rounds instead of teeth, and is carried about by a great wooden wheel. A trundle or drum. *Fig*. 1. *Pl*. XLVI. EF.

V.

Valve, a piece of wood, &c. so fitted into a hole, that it opens and lets a fluid pass through one way; and shuts and stops it the other. *Fig*. 2. *Pl*. XLVII. V. W. *A sucking valve*, is that where the water follows the piston. *A forcing valve* when it is driven through before it.

Vane, a sail, or fan, generally to show the point of the wind.

Velocity, an affection of motion, and is that by which a body passes over a certain space in a given time. Swiftness, or celerity.

Vibration, the moving or swinging of a pendulum back and forwa

Vis inertiæ, a property of body, by which it resists any impress force, and endeavours to continue in the same state.

W.

Wallower, a trundle upon a horizontal axis. *Fig.* 1. *Pl.* XLII.

Waterpoise, an instrument to try the strength of liquors. A h drometer.

Web, the thin broad part of an instrument, as the web of a key, &

Wedge, an instrument to cleave wood. One of the mechai powers.

Weight, the tendency of bodies downward. The matter raised an engine.

Wheel, a machine consisting of an axis and a circular rim, wi teeth in it, and then it is called a toothed wheel.

——— *smooth*, a wheel without teeth, turned by a rope.

Wheel and axle, a machine to raise weights. One of the mechai powers. *Fig.* 3. *Pl.* III.

Winch, an instrument with a crooked handle, to turn any thi about with.

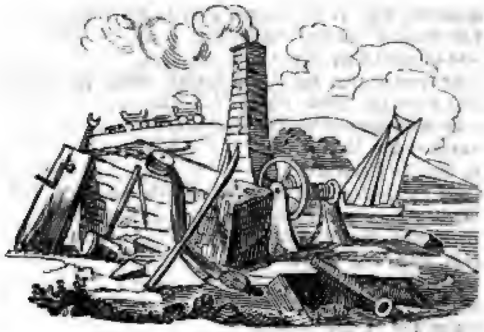
Winder, a winch or handle to wind about.

Windlass, a machine to raise great weights. On board a ship, serves to hoist the anchor. It is an horizontal roller, turn round by handspikes.

Windmill, a mill to grind corn, moved by the wind. *Fig.* 1. *P* XLVI.

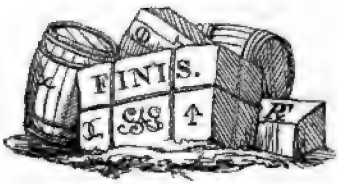
Wing, a thin broad part that covers something, or hangs over Also what helps to give due motion to any thing, as the han in a water wheel, a part of a sail, &c.

Worm, a spiral thread running round a cylinder, forming a sort screw.



A LIST

OF THE PRINCIPAL MACHINES DESCRIBED IN THIS BOOK.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APPENDIX.

NOTES, ILLUSTRATIONS, AND OBSERVATIONS,

EXPLANATORY AND FAMILIAR.

Definition 8. (*page* 2,) *Quantity of Motion, or Momentum.*—Thus, we say that a hammer strikes with a certain force, which force evidently depends on the weight of the hammer, and the velocity with which we strike the blow; for a light hammer will not strike so hard as a heavy one, nor a slow stroke be as effective as a quick one. Thus the quantity of motion (or the momentum) must depend upon the quantity of matter, (that is, the weight of the hammer,) and the velocity or quickness with which we strike, jointly; that is, in estimating the force of a blow, we must take into account both the weight of the body and quickness of the stroke.

Def. 9. (*page* 2,) *Vis Inertiæ.*—Thus, a block of wood or stone laid on the ground, would remain there without motion for ever,

were it not moved by force, which is applied by some power not seated in the body itself; or, if a cannc were shot in any given direction, it would for ever move direction, were it not affected by the gravity of the earth, w continually endeavouring to draw it downwards; thus, v term the vis inertiæ of bodies to be a propensity in all either to remain at rest, if not acted on by any other bod acted on by any force, to obey that force impressed on it; a certain sluggish quality of bodies, which implicitly ob force impressed on them; or, if no force is applied, they wo ever remain at rest.

Def. 22. *Mechanical powers.*—The writers on mechanic to differ with regard to the number of mechanical powers reducing them to three, while others number them at se follows:—the *lever*, the *wheel* and *axis*, the *pulley*, the *scr* *inclined plane*, the *wedge*, and the *funicular machine*, (or co tion of ropes,) which hereafter I will take an opportunity scribe, and to which our author adds another power, by thi nition, viz. the *balance*, which is evidently an oversight, balance is nothing but a lever, whose brachia or arms are o length. Now, we may, with great propriety, I think, redu number of simple mechanical powers to three, viz. the *lev* *pulley*, and the *inclined plane*, for all the properties of the a wheel depend upon the principle of the lever, and the c ference of the wheel and axis is but a continued series of In like manner, the screw is but an inclined plane, wound a cylinder, and the wedge but two inclined planes joined to the properties of which may be all demonstrated by the pro of the inclined plane itself; in fact, the screw is a compour gine, made up of the lever and inclined plane, for the force is employed to turn the screw is evidently a lever, while the

anvil, for it will re-bound from the stroke, shewing that a re-action has taken place, and in a contrary direction to the stroke impressed; and if the anvil and hammer were perfectly elastic bodies, the re-bound or re-action would be with a force equal to the stroke of the hammer. In the theory of mechanics, all bodies are supposed to be perfectly elastic, or perfectly solid, or non-elastic, when we are estimating their forces, or calculating their effects on each other.

Axiom 2.—Thus, if two men pull the ends of a rope with equal force, the men will both remain immoveable, or the strength of one man is counteracted by the strength of the other.

Prop. 3. (*page* 6,) *Cor.*—It may, perhaps, be not amiss to explain the term *reciprocally*, which may be thus shortly and familiarly illustrated. Suppose a carriage travels during the time of one hour, for instance; the distance or space it has moved will be according to the velocity of its motion; thus, it will pass over a great space with a quick velocity in the given time, but through a small space, with a slow velocity in the same time: thus, if the space is large, the velocity must be quick, and if the space is small, the velocity must be slow in proportion. Thus, if a carriage travel with a velocity of six miles per hour, for the space of 60 miles, the time it will be performing the whole journey will be 10 hours, that is, as $6 : 1 :: 60 : 10$ or $60 = 10 \times 6$, or $10 = \frac{60}{6}$ that is, the time is as the space divided by the velocity.

Prop. 4. *Cor.* (*page* 6.)—That is, if S represent the space, F the force, T the time, and M the quantity of matter, we shall have S is as $\frac{F \times T}{M}$. See the third equation, Scholium, Prop. 4.

Prop. 7. (*page* 10, *latter part of Case* 2.) *The n^{th} power.*—As some of my mechanical readers may not be sufficiently acquainted with algebraic notation, to comprehend the exact meaning of this term, I will endeavour to explain it as shortly as possible. The power of any number or letter, representing any number or quantity, is algebraically represented by the index or little figure, set to the right of the letter, a little above it, and shows the power or number of times the letter (representing quantity or number) is multiplied into itself: thus, A^2 represents the square, or, as it is called, the second power of the quantity A. A^3 represents the cube or third power of the letter A; and so on to any power required: but the expression A^n shews generally any power of the letter A, which the letter *n* may be made to represent, that it is a general expression shewing any power you may please. And thus we see by the expression $1 : t^n :: AB : Ab$; the index *n* shews that whatever power you raise *t* to, the proportion will always be the same; thus generalizing the proposition.

Prop. 8. (*page* 11, *Cor.* 2.)—As it is absolutely necessary the reader should be acquainted with the terms sine, tangent, &c. the accompanying diagram is introduced, to explain, as familiarly as possible, these terms. Draw any circle FDB, (*Fig.* 1. *Pl.* A.) and from the centre A draw any line, as ADC. and from B draw BC to meet ADC and perpendicular to AB; also from D draw DE perpendicular to AB, and draw also AG perpendicular to AB, and GI and DH perpendicular to GA, and we shall have a diagram explanatory of all the terms regarding sines, tangents, &c. viz. DB is an arc of the circle which measures the angle DAB; DE is the sine of that angle, CB the tangent, AB the radius, AC the secant, DH the cosine, IG the cotangent, IA the cosecant, and EB the versed sine of the angle DAB. GD is called the compliment of the arc BD, and DF the suppliment to the same arc.

Prop. 9. *Cor.* 1 *and* 2, (*page* 13,) *angle of incidence and angle of reflection.*—These are terms derived from optical principles; which are foreign to mechanical principles, and, with due deference to higher authority, I would wish superseded by the terms *angle of direction* and *angle of re-bound*, as, in my opinion, expressing more mechanically their properties; for what is the angle ABD but the angle or direction the body takes when it approaches the line DB (*Fig.* 4, or GF *Fig.* 5, *Pl.* I.); and what is the angle DBF but the angle which the body A re-bounds with after striking the plane GBF, and, in my humble estimation, explaining more clearly the signification, than the terms *incidence* and *reflection*.

Prop. 9. *Cor.* 3. (*page* 13.)—The term *impinges*, perhaps, well expresses what is here meant; that is, whether it falls by its own weight, or is impelled by any particular force on the body; that is, whether it strikes the body by any projectile force, or merely by its own gravity in the given direction. And by the term *magnitude* of the stroke, we are to consider the force of the stroke, that is, the absolute effect it has upon the body which it impinges or strikes against.

Section 2.—It may be scarcely necessary here to mention what is meant by the motion of projectiles in free space; but that no ambiguity or misconception may arise in the minds of those entering on the study of mechanics, I shall here define what is understood to be mechanically a *projectile*. It is any body or quantity of matter in motion, however that motion is produced; whether by the means of percussion, pressure, or any other force; whether from the force of gunpowder, as a cannon shot; or by a stroke with a bat, as a ball; by the effect of a spring, or otherwise; that when a body is moved from a state of rest to that of motion, it then becomes a projectile in the mechanical acceptation of the term. Also, by *free space* we mean any space that offers no resistance to the motion: thus, we argue upon the motion of pro-

jectiles theoretically, as if no resistance was offered to the motion of bodies; and then we come to the practice, that resistance is calculated and allowed for, whether proceeding from the density of the fluid in which the body moves, such as air or water, or from friction, gravity, or any other force which resists the motion of the projectile.

Prop. 13. *Cor.* 1. (*page* 19.)—In order to make this corollary appear plain, let us suppose a body at A (*Fig.* 2. *Pl.* A.) falling by its own weight in the direction of the perpendicular line AE, and let us divide AE into any number of equal parts, as B, C, D, E; then the meaning of this corollary is, that if the whole time AE is divided into any equal portions of *time*, as AB, BC, CD, and DE, it will fall with a certain velocity from A to B, and will gain in falling from B to C an increased velocity, equal to the velocity it acquired in falling from A to B, and so on. We may here remark, that the divisions AB, BC, &c. are not supposed here to be equal portions of distance fallen, but equal portions of time; for if we suppose them equal distances of space fallen, then we shall find that the distance fallen BC must be greater than the distance AC, for the body has acquired a certain velocity in passing from A to B, and it commences the motion from B with a velocity equal to that it has acquired at B, and not that with which it first set out at A; therefore, the velocities acquired in falling from A to B and from B to C, in equal portions of *time*, will be represented (if we call the velocity V) by $V + 2V + 3V + 4V$, &c. at the several points, marking equal portions of *time*, that is, at B the velocity will be $= V$, at $C = 2V$, at $D = 3V$, &c.

Prop. 14. (*page* 19.)—That is, (referring to the last figure) if AB is any *distance* fallen, and we divide it into any number of equal parts, as AB, BC, &c. then equal portion of distance will be passed over by a body descending from A to E by the force of gravity, as the squares of the *times* occupied in passing over these equal *spaces* AB, BC, CD, &c. thus calling the equal distances S, and the time in falling from A to $B = T$; but the time is as the velocity, and at B the body has acquired a certain velocity V. Now, as equal velocities are gained in equal times, $T \times V$ is the distance passed over by the body in passing from A to B; hence, as T is always equal to V, at the point B, the force becomes T^2; but as the spaces described evidently depend upon the time and velocity of the motion, we shall have the spaces AB, BC, CD, &c. as T, T^2, T^3, &c. that is, as $S : T :: 2S : T^2 :: 3S : T^3$, &c.

Prop. 15. (*page* 21.)—It may not be here amiss to define what is meant by the expression, to describe a parabola. The reader will very readily perceive, that if a body is projected in a direction oblique to the horizon, its path during its flight will be a curve line; for if it was a straight one, it would for ever fly off at

greater and greater distances from the surface of the earth. But the power of attraction is continually drawing it out of that line; it, consequently, approaches the earth, or deviates from that strait line every moment of its flight, and falls ultimately to the earth at greater or less distances, according to the greater or less force with which it is projected. Now, this path or curve the projectile describes, is in this Prop. termed a parabola; that is, a parabolic line, which curve may be thus conceived:—let a cone be cut by a plane parallel to one of the slanting sides, then the boundaries of the section will be a curve line, which is termed a parabola, or the curve itself may be thus conceived to be drawn, without the help of the cone.

Let AB (*Fig.* 3. *Pl.* A.) be any right line, and CD perpendicular to AB; then if we draw any number of parallel lines to CD, as AF, a, b, and if from any point E, which we may assume at pleasure, we make EF $=$ AF or Eb $=$ ab, if a curve line is drawn through the points Fb, that curve is called a parabola or parabolic line, and is the line in which projectiles move, or would move, did not the projectile meet with a resistance from the atmosphere through which it passes.

We may here observe, as it will perhaps be of use in the investigation of the properties of projectiles, that the point E is called the focus of the parabola, the point G the vertex, that is, where CG $=$ GE on the line CD, also that CD is called the axis, and if from the point F we draw FH perpendicular to CD, FH is called an ordinate to the point of the curve at F, and so of any other perpendiculars, from any other point in the curve. GH is also called the abscissa to the point F, or ordinate FH, and thus every ordinate has its corresponding abscissa; and the comparison of these lines, or the proportion they bear to each other, determines the nature of the parabola, or an equation or algebraic expression is deduced from these lines, which expresses the nature of the curve; and this is what is meant by the expression (*by the conic sections the curve* AFG *is a parabola*,) that is, AK, AL, &c. (*Figures* 6 *and* 7. *Pl.* I.) are always as, or in, a certain proportion to KF^2, LG^2, &c.

Prop. 15. *Cor.* 1.—The definition of the term *latus rectum* and *parameter* is necessary to the understanding of this *Cor.* To those who are unacquained with the conic sections, I shall, therefore, give in plain terms their signification:—first, the *latus rectum* is that line in a parabola which passes through the focus perpendicular to the axis, and meets the curve both ways, as bEc is the latus rectum to the parabola, (*Fig.* 3. *Pl.* A.) and is always a quantity equal to four times the distance of the focus E from the vertex G of the curve; and the *parameter* may be thus explained: let any other line, as FK, be drawn parallel to the axis GD, that line is called a diameter of the parabola; now, the ordinates of

this diameter are lines drawn parallel to the tangent at the vertex or end of the diameter at F, and are called ordinates to that diameter; and if we conceive a line drawn parallel to the tangent, at the vertex F, and, consequently, to any ordinate to that diameter, and if this line passes through the focus or point E of the parabola, that line is called the *parameter* to the diameter FK, and it is always equal to a third proportional to the ordinate and abscissa of that diameter, or four times FE is the parameter to the diameter FK, where E is the focus, and EF always equal to AF.

Cor. 4. *Prop.* 15.—Hence, the velocity at any point of the curve, which a projectile describes, may be represented by a line drawn from that point to the focus of the parabola, or, more properly, by the *time* a body would be falling through the extent of this line, by the force of gravity.

Prop. 16. *Cor.* 3.—The meaning of this Cor. is that a body will be projected to the greatest distance which the force applied will carry it, when the elevation, or the line in which it is projected, makes an angle of 45 degrees, or the half of a right angle with the horizon, and that at elevations at equal distances from the angle of 45 degrees, the distance the body will go, or the point it will fall on the horizontal line, will be the same: for instance, the body is projected at an angle of 35 degrees, or of 55 degrees, being one 10 degrees less than 45, and the other 10 degrees more than 45.

Prop. 17. *Cor.* 3.—That is, the greatest elevation a projectile will rise to, is in proportion to the square of the times of its whole motion, before it again fall to the ground.

Prop. 19.—The bended lever is, by most authors, given as a fourth kind, though, in fact, it is a lever of the first, having the fulcrum at the point where it is bent, and the power applied at one end, and the weight at the other, as we see in real practice; that as no lever is without some thickness, so, as the fulcrum is at the underside of the bar or lever, and the power at the top, the line of direction in which the force acts, makes an angle with the line of direction joining the weight and fulcrum; for, let ABCD (*Fig.* 4. *Pl.* A.) be a lever or crow-bar, and let the fulcrum be at F, the power applied at B, and the weight W to be raised at the point D or A. Now, as the thickness of the lever in actual practice is BC or AD, draw FB or FA; then AFB must now practically be considered as the lever, which is evidently what is so often termed a bended lever; consequently, as in practice no lever is without thickness, so, according to the thickness of the lever, the lines AF and FB will always make an angle with each other, and the greater as the thickness is increased; and thus we see all bended levers may be reduced to the first form: and as it is a maxim in science to reduce every thing to a few first principles, so in that branch of it, on which we are treating, the less

compass we can compress our principles in, the less likely we shall be to form incorrect notions of the subject we have under investigation; for this reason, I would confine the species of levers to three only. Under the first class may be arranged those instruments called crow-bars, handspikes, and wrenches of different kinds; likewise scissars, pinchers, &c. are double levers of the first kind, where the joint is the fulcrum or support. The oars of a boat, the rudder of a ship, the sails of a windmill, and cutting-knives fixed at one end, may be arranged as levers of the second kind. Tongs, shears, the bones of animals, &c. may be classed as levers of the third kind. And though the action of a hammer, in drawing out a nail, is usually referred to that of a lever of the fourth class, yet its effects may evidently be calculated by considering it as a lever of the first, from what has been said above.

Prop. 23.—By way of illustration, we will here describe what is meant by the epicycloid or epicycloidal curve, as it is much commented on in this and several of the succeeding propositions; and generally we are to understand by an epicycloidal curve, a line mechanically formed by fixing any tracer or pencil in the circumference of a circle at any point, and supposing this circle to revolve about another circle, it (the point) will describe a curve which is called an epicycloid; and as this circle (which is called the generating circle) may revolve either on the convex or concave periffery of the other circle, the epicycloid is distinguished into two kinds, the *exterior* or *interior*, according as it is found by revolving on the convex or concave circumference. Thus, let ABC (*Fig*. 5. *Pl*. A.) be any circle, and AG any other; now, suppose AG to revolve round the circumference ABFC, and at any point A in AG a pencil or tracer affixed, and we will suppose, only for illustration, that the circumference of AG is half that of ABC; the point A will be found at C after it has revolved through ABC,

or generating circle; the arc ABC (*Fig.* 5. *Pl.* A.) or AC (*Fig.* 6. *Pl.* A.) is called the *base* of the epicycloid.

To avoid any misunderstanding, it may be necessary to observe that the generating circle may be in any proportion to the quiescent circle; and in that case, if the curve begin at A, it will meet the quiescent circle at greater or less distances, according to its periffery: thus, if the circumference of the circles are equal, the curve will be continued quite round the quiescent circle, and meet it again at A, or it will always meet the quiescent circle at a point where the quiescent circumference is equal to the circumference of the generating circle; this with regard to the exterior epicycloid; but in the interior epicycloid, if the circles are equal, the point of the generant will always be in the circumference of the quiescent circle; and if the circumference of the generant is half that of the quiescent circle, the point will trace the diameter of the quiescent AB (*Fig.* 6. *Pl.* A.)

There are may curious properties of the epicycloid which are worthy of the notice of the mechanic, and for which I shall only refer to *Hutton's Mathematical Dictionary*, which will amply repay the trouble of investigation to those who feel disposed to investigate the properties of this curious and useful curve line.

Cor. 3. *Prop.* 23. *Fig.* 25. *Plate* II.—The lines BM and KR are the *interior* epicycloids, generated by by the quiescent circle MRBK, and the generating circle BD, as mentioned in the foregoing note.

Cor. 4. *Prop.* 23.—*If* BC *be infinite*, or if we suppose the radius of the quiescent circle to be infinite, it then becomes a right line, and the revolution of the generating circle describes another curve, somewhat similar, in many of its properties, to the epicycloid, but is known by the name of the common cycloid; and as this curve is of great use in mechanics, particularly in demonstrating the properties and nature of the pendulum, it may not be uninteresting to the mechanical readers; we will, therefore, state a few of them, leaving the demonstration to the mathematical student. Therefore, let APCHB (*Fig.* 1. *Pl.* B.) be a common cycloid, generated or formed by the revolution of the circle EPE along the line AB, any point P tracing the curve during its revolution, and let CGD be the position of the generating circle, when it has revolved through half its circumference, or when AD = DB: then, if we draw DFC a diameter perpendicular to AB, and continue it indefinitely from C to I, and having drawn the chord CG, and parallel to which from H we draw HI, we shall have the following properties of the cycloid; and we may here observe, that the line FH is called an ordinate to the cycloid in the point H, and FC is called the abscissa to the same point H, and also that the relation the lines CF and FH bear to each other (or their propor-

tion to each other,) is termed the equation of the cycloid, which is an algebraic expression denoting the chief property of the curve. The following are the most generally useful properties of the cycloid.

1. The circular arch CG is always equal the line GH, or is equal the ordinate GH—the sine of the angle FCG making the abscissa the radius.

2. The semi-circumference CGD is equal the semi-base DB.

3. The cycloidal arch CH is equal the double of the chord CG.

4. The semi-cycloidal arch CB is equal the double of the diameter CD; and hence the whole cycloidal circumference ACB, is equal four times the diameter of the generating circle CD.

5. The area of the cycloid, that is, the surface bounded by the cycloidal curve ACB and the line AB, is always equal to three times the area of the generating circle CGD.

6. Any line, as IH, drawn parallel to the chord CG from H, is a tangent to the cycloid, that is, it touches it without cutting it in the point H.

We might here enumerate many more; but as these are a few of the most simple and generally applicable to the science of mechanics, we shall only give them as a specimen, referring the reader to books which treat on the nature and properties of curve lines, a list of which will be found in *Hutton's* or *Barlow's Mathematical Dictionary*.

Prop. 24.—There seems some little explanation of a character used in the fifth line of this Proposition. After the words, "therefore, the velocity of P to the velocity of W," we find this mark ∷, also in the two following lines; which, though a mark of proportion, is not, in my opinion, sufficiently intelligible; we will, therefore, explain the passage, that no ambiguity may appear. [redacted]

describe the right line FG, but will move in the line FH, which is the same parabolic curve it would describe in the air, if the velocity of the projectile in the air is in the same proportion to the velocity impressed on it on the plane, as the absolute gravity of the body (that is, its gravity falling perpendicularly) is to the relative gravity of the body on the plane, (that is, its gravity falling along a line parallel to the sides of the inclined plane,) and both are described with the same obliquity; that is, if we draw FI parallel to CB, and the angle IFG is equal the angle the direction of the projectile in the air makes with the horizon; and the same may be said if the body is projected upwards on the inclined plane, it will describe a parabola under the same circumstances. We may here note, that if the body is projected in the direction IF, that is, parallel to the sides of the inclined plane, it will then describe a straight line, in the same manner that a projectile impelled on a horizontal surface will; and the distances to which they will go, will be still in proportion to the absolute and relative gravity of the bodies.

Prop. 39.—As a proper understanding of terms often prevents misconception, as well as renders clear and intelligible what would otherwise seem obscure, it may be necessary to explain the meaning of the term *sub-duplicate ratio*, in the second line of this Prop., which may be simply thus shewn to be the ratio of the square: for if two quantities, A and B, be in proportion to each other, we say that the quantities A^2 and B^2 are in a sub-duplicate ratio to each other. Again, in line 7, page 46 of the same Prop. the words, "whence by composition," requires a familiar explanation, which may be thus explained: let there be four quantities, ABCD, in proportion to each other, that is, let $A : B :: C : D$, then, if B is added to A, and D to C, we shall have the quantities A, $A + B$, C and $C + D$; also in proportion for $A : A + B :: C : C + D$; and thus we say the quantities A, B, C, D, are by composition in proportion to each other.

Prop. 39. *Cor.* 3.—It may not be here amiss to explain the terms *reciprocal* and *reciprocally proportional*. Thus, the reciprocal of any quantity is unity divided by that quantity; thus $\frac{1}{A}$ is the reciprocal of the quantity denoted by A, and by reciprocal proportion we are to understand, that if we have any proportion, as, for instance, $A : B :: C : D$, we say that the quantities are reciprocally proportional when $A : B :: \frac{1}{C} : \frac{1}{D}$.

Prop. 40. *Cor.* 3.—It is scarcely necessary to remind the reader that the line VH is in this corollary supposed to be perfectly flexible, and that as the bob or ball of the pendulum vibrates along

the arch A, H, D, *a*, the line VH wraps itself round the cheeks ARV and V*ra*; but I take this opportunity of adverting to a term in frequent use in mechanical authors, when speaking of the nature of different curves, and which, in this instance, is somewhat curious, with regard to the nature of the pendulum vibrating between two cycloidal cheeks, that is, the curve ARD*a* is still the arc of a cycloid, and, therefore, describes large or small arches in equal times. The term I allude to is that when describing a curve line, formed by the unwrapping of a line laid on another curve; the end of the line so unwrapped describes another curve, which is called the *evolute* of the curve by which it is generated. Thus, a line wound round a circle; by being unwrapped, its end will describe another curve of a different nature to the circle; and thus of other curves: but, in this instance, the evolute curve being a cycloid, if we construct a pendulum, so that in its vibrations it shall play between two cycloidal cheeks, we shall have a pendulum possessing the property of vibrating in large or small arcs, in equal times, and this has been in practice tried; but, from the difficulty in making the cheeks of a true cycloidal form, has not much been used, though, in our improved state of operative mechanics, I cannot but think it of no very difficult attainment, and which I would recommend to the consideration of the mechanic, as being much calculated to improve the construction of our time keepers.

Prop. 41. *Cor.* 1.—The term *isocronal* is applied to the motion of pendulous bodies, when their vibrations are performed in equal times, whether the arcs they describe or move through are great or small; that is, if a pendulum of a given length performs a certain number of vibrations in a certain time, and if the arc it describes is augmented or diminished in extent, it still performs the same number of vibrations in the same time; these vibrations are called *isocronal*.

Prop. 41. *Cor.* 4. *Radius of Curvature.*—As a circle is a curve which is described by the revolution of a line about a point, called its centre, it is equally bent from a straight direction all round its circumference, and the line drawn from its centre is the *radius of curvature*: so all other curves, as the inflection or bending, varies in every point of its circumference, and the bending of every point must correspond to the bend of some circle; and hence the radius of this circle, that corresponds to the point of the curve, is called the *radius of curvature* to that point of the curve.

Prop. 41. *Scholium.*—As regards the regulating the vibration of a pendulum in a clock, the supposition here made, of screwing the bob or weight attached to the pendulum rod up or down, according to the theorem given, is under the idea that the weight

is considerable in comparison to the rod; for, if the weight is made of wood, as is the case in some common clocks; or of thin brass, as is also often the case; or, in compensation pendulums, where the rod is made with bars of different metals of considerable weight, in comparison to the bob of the pendulum; then this theorem will not hold good, but is meant only to apply to that case where the weight of the pendulum rod is trifling in comparison to that of the weight or bob attached to it.

Prop. 43.—By the term "an inflexible right line," is here meant an imaginary line, that possesses the property of resisting any effort or force to bend it, which, though not practically a line which has actual existence, still is necessary in the demonstration of this and all problems relating to mechanics; for though all bodies will bend or break by a force applied to them, yet it is necessary, in the investigation of the properties of the mechanical powers, to suppose all levers possessed of this property of resisting any effort to bend them, in order that all the force may be applied to the body acted on, though, in actual practice, allowance must be made for the elasticity of all bodies, or the strength of the material of which they are composed. Thus, it would be in vain for us to attempt to move a body, by the help of a lever composed of thin wire, whose weight is such as would cause the wire to bend before it had any effect on the body: and in actual practice, we must adapt the strength of the lever to the weight of the body to be moved, though, in calculating the power of it to overcome any obstacle, we are at liberty to suppose a line possessed of infinite strength to resist bending or breaking with any force exerted. Thus, in mechanics, we must suppose all bodies to be possessed of infinite resistance and perfect elasticity, as well as perfect solidity and perfect fluidity, when we demonstrate the properties of bodies, or are investigating the theory of machines, and afterwards allow, in actual practice, for the deviations observed in nature from these properties, ascertained by actual experiment. For, as in geometry, a right line is defined to be length without breadth, so, in mechanics, a lever is defined to be a line perfectly inflexible, though neither exist but in the imagination.

Prop. 44. *Cor.* 6.—The term, a *given quantity,* (in the fourth line of this Cor.) may, perhaps, need some illustration; at any rate, this curious property alluded to, of the centre of gravity of a system of bodies, merits our particular attention. I shall, therefore, here first explain what is meant by a *given quantity,* lest the inexperienced reader should imagine, that, in *any* system of bodies, the *quantity* here alluded to should be thought to mean a certain numerical undeviating sum, let the bodies be of any magnitude, or any how situated. I will, therefore, endeavour, by the help of a diagram, to shew what is, in the language of mathema-

tics, meant by a given quantity. Let ABC (*Fig.* 3. *Pl.* B.) be a semi-circle, ADC the diameter; now, if AC is divided in any point, as D, and DB drawn perpendicular to AC, and if we have the distances AD and DC of a given magnitude, the perpendicular DB is said to be a given quantity, as it is always a mean proportional to AD and DC, let the point D be any where situated on the diameter AC; hence, if we make this proportion as AD : BD :: BD : DC; and, as the product of the two middle terms is always equal to the product of the two extreme terms, we have $BD \times BD = AD \times DC$, or $BD^2 = AD \times DC$, or $BD = \sqrt{AD \times DC}$, that is, BD is always equal to the square root of the product of AD and DC; that is, BD will always be a quantity which is known or deducible from the quantities AD and DC; thus, in this Cor. the several distances of the bodies from the centre of gravity are known quantities, and the diameter of the circle is any known distance; the distances of the lines drawn from a point in the circumference of that circle to the several bodies, is said to be a *given quantity*, because it is deducible from these lines, (that is, their several distances from the centre of gravity,) and this is always the case, let the point taken be any where in the circumference of the circle drawn about the centre of gravity.

Prop. 46.—This Prop. does not seem sufficiently intelligible, at least not sufficiently clear, in the passage where it says, "Where any product laying the contrary way from R must be taken negative;" now, in adding algebraic quantities, we must take the sum of the difference of those quantities affected by the different signs minus and plus for the sum, that is, you subtract the sum of all the minusses from the sum of all the plusses for the whole sum of the quantities, or the contrary. Thus, in this Prop. the plane which is supposed to be acted on by the bodies ABCDE, extends only from N to R, and as the body situated at E, is beyond the extent of the plane RN, the plane will not be affected by the body at E; therefore, the bodies A and D are more than equal to maintain the equilibrium of the bodies B and C by a quantity E; and, therefore, to maintain the equilibrium, the body or quantity E must be taken from that of the sum of A and D: thus, as the Prop. states $A + D = B + C + E$; but as E has no effect on the plane, NR, acting in a perpendicular direction to it, we must, to make $A + D = B + C$, take the quantity E from it; then we have $A + D - E = B + C$; or if the distances aR, bR, cR, dR, and eR, are multiplied into A, B, C, D, and E, we shall have $aR \times A + dR \times D = bR \times B + cR \times C - eR \times E$, or, which is the same thing, $\dfrac{aR \times A + dR \times D}{eR \times E} = bR \times B + cR \times C.$

This is, therefore, the meaning of the eR *being taken negative*, (because its force upon NR is of no effect when E acts perpendicularly to NR); and, consequently, to preserve the equilibrium, the force expressed by $aR\times A+dR\times D$ must be divided by the force expressed by $eR\times E$ to maintain the equilibrium.

Section 6. *Prop*. 56.—As this Prop. seems involved in some obscurity, we will endeavour to give some illustration of it that will enable us to comprehend its true meaning. And, in order to which, it will be, perhaps, necessary to go through the whole Proposition, making our remarks as we proceed: and, first, we will state the Prop., as we conceive, in as intelligent a manner as the subject admits of. The Proposition, therefore, amounts to this, that if any number of bodies (*or a system of bodies*) revolve about a common centre, which, if we here consider as an axis, is as the axis of a wheel; and if in their revolution (caused by any force applied) they are made to revolve through a certain portion of the circumference they describe, in a given time, if we apply another force in a perpendicular direction to the axis, this last force will produce a motion in the system, which, if we call the original force f, and the force afterwards applied we call m, the whole motion of the system will now, in the same time, be represented by the expression $\frac{A\times SA+B\times SB+C\times SC}{A\times SA^2+B\times SB^2+C\times SC^2}\times SP_1+m$; but, by the concluding expressions in the demonstration, the whole motion seems to be equal to $\frac{A\times SA+B\times SB+C\times SC}{S}\times m$.

Now, it seems to those not well conversant in algebraic operations that there is a difficulty in, or rather an omission, in the author; in not shewing that these two expressions are equal to each other; for it is evident, this last is legitimately produced from the theory of mechanics: and as it does not seem to correspond with the original expression in the Prop. itself, we will endeavour to supply what certainly was only omitted under the consideration that it was not necessary to those conversant with algebraic expressions; but, in all elementary treatises, it is necessary that, at least, no ambiguity should appear. I will here shew that if the reader attentively observes the preamble of the demonstration, he will find there two expressions perfectly identical, and of course the Prop. is correct. Thus we find that we are desired to put the sum of the expressions $\frac{A\times SA^2}{SP}\times\frac{B\times SB^2}{SP}\times\frac{C\times SC}{SP}=S$; therefore the last expression above used will be $\frac{A\times AS+B\times BS+C\times CS}{\frac{A\times AS^2+B\times BS^2+C\times CS^2}{SP}}+m$,

where $\frac{A\times AS^2+B\times BS^2+C\times CS}{SP}$ is put instead of S. Now it is known, that to divide any quantity by a fraction, is the same as multiplying it by that fraction whose numerator and denominator are reversed; therefore (as every whole number may be considered as an improper fraction whose denominator is unity or 1,) we shall have to multiply the fraction $\frac{A\times AS+B\times BS+C\times CS}{1}$ and $\frac{SP}{A\times AS^2+B\times BS^2+C\times CS^2}$ which is evidently the same as $\frac{A\times AS+B\times BS+C\times CS}{A\times AS^2+B\times BS^2+C\times CS^2}\times SP$; and then being multiplied by m, we have the original expression $\frac{A\times AS+B\times BS+C\times CS}{A\times AS^2+B\times BS^2+C\times CS}$ $\times SP\times m$, equal or identical to the expression above shewn, viz. $\frac{A\times AS+B\times BS+C\times SC}{S}\times m$. Before we leave this, it may be necessary to explain what is meant by angular motion, or the angular motion of a system of bodies: thus, in any system of bodies, if any one of the bodies composing that system has, by its revolution, described any arc of a circle in a given time, the measure of that arc, or the angle, the lines drawn from the body to the centre of the system during any portion of its revolution, in a given time, is the angular motion of that body; and if the whole system describes the same arc in the given time, the angular motion of the system is the same as the angular motion of any body composing a part of the system; and the absolute motion in the system is measured by the angular motion: thus, if a system pass over a portion of its circumference equal to 15 degrees in one hour, we say the angular motion is at that rate; and that is the absolute motion of the system, without regard to the actual space passed over by each body composing the system: but if we consider the absolute motion, as measured by the actual space passed over, in feet, miles, &c. we must know the angular motion, and also the radius of the circle the body describes, and thence calculate the length of the arc itself, in miles, feet, &c. for the absolute motion of the body in space: thus, if in a system, the bodies which compose it are situated at different distances from the centre of the system, these bodies may have all the same angular motion; but these absolute motions in space will vary as they are at greater or less distances from the centre of the system.

Cor. 1. *Prop.* 56.—In this Corollary a supposition is made that

the bodies are all situated at O, that is, all at the same distance from the centre: now, we have seen, in the foregoing Prop., that, let the bodies be any how situated, the motion generated will be $\frac{A \times SA + B \times SB + C \times SC}{A \times SA^2 + B \times SB^2 + C \times SC^2} \times SP \times m$; but as we now substitute the distance SO in the places of SA, SB, SC, we shall have this expression from the motion generated, viz. $\frac{A \times SO + B \times SO + C \times SO}{A \times SO^2 + B \times SO^2 + C \times SO^2} \times SP \times m$; and as the expression is equal to the former, SO will be found equal to $\frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{A \times SA + B \times SB + C \times SC}$; which shews, that the absolute motion, or angular motion, will be still the same, whether we suppose the bodies to be all situated at the same or equal distances from the centre of the system, or at different distances, as we do not here take into account the weight of the bodies: but their angular velocities will be different, that is, the velocities with which the bodies describe or move through any given arc or part of the circumference will be different; for if a body describes a given arc, in a given time, at a certain distance from the centre, it is plain, that, to move through an equal arc at twice the distance from the centre, in the same time, it must move with a greater velocity, and the contrary; as it will have to move through a greater or less space according to the radius of the circle they describe in the same time, therefore must move with greater or less velocity, though the angular motion is the same.

Prop. 57. *Centre of Percussion.*—In order to well understand what is meant by the centre of percussion, to enable us to apply it in the construction of machines, we will endeavour to describe it in a familiar manner. Thus, in striking any body with a bar or lever, we know by experience, that if the blow is given at or near the extremity of the lever, it will jar or attempt to fly out of your hand; which, if suffered to do, the end you hold in your hand will strike the ground: on the contrary, if the blow is given with that part of the lever near the hand it will also jar, but in a contrary direction; that is, the end held in the hand will attempt to fly upwards: now, there evidently must be a point between these two, where, if a stroke is given, the full effect of the blow will be sensible, and the lever will not attempt, after the stroke, to fly upwards or downwards, but will remain at rest, without jarring the hand: and this point in the lever, or bar, is, in mechanics, called the centre of percussion, or the point in the striking body, where, if it strikes another, the effect will be the most powerful; and as the centre of gravity of a body is a point on which, if suspended, the body would be in equilibrio, so the centre of percussion is a point in which the whole momentum of the moving

body is placed to produce the greatest effect, or the force of p cussion communicated to the whole boby is centered, and, c sequently, balance each other when the stroke is given.

Prop. 57. *Cor.* 7. *Locus.*—As this term is in frequent use mathematical and mechanical speculations, as well in theory as practice, we would define it to be that line, whether straight curved, which is produced by the motion of another line, or ot lines, which always move either in a known proportion to e other, or in a constant ratio about one or more points, or along or more lines, whose positions are given. Thus, the locus of a given line, revolving about a fixed point, as a centre, will b circle; but if the line, instead of revolving about a point, alw moves parallel to itself, at any angle, along another straight li it will describe another straight line; and this line is called locus of the other, and of the line. Again, if two lines revo about two centres, crossing each other, and if the *sum* of the d tances, from the point of intersection to the two centres, is alw a given quantity, the locus (or place) of the point of intersecti will be an elipse: for the property of that curve is, that if fr any point in the circumference we draw two lines to the t centres (or foci), the sum of these lines is always a constant qu tity; and, therefore, (in the Cor.) the line OT will be the place which the centre of percussion will always be found, as bodies, or the system of bodies, ABC oscillate round S as centre; the meaning of which is, that let the line OT be of w length it may, if its end T strike an object, it is the same thing if the point O struck it.

Prop. 58. *Centre of Oscillation.*—In order that no misconc tion may arise from the definition given of this point, which that *if all the particles of the body were concentrated in a cert part of a vibrating or oscillating body, it would vibrate or oscillate the same time as the whole body*, let us suppose that a pendulum devoid of weight, except at the point, which, if it had weig

weight transferred, will make it vibrate in the same time as the latter; and this point is called the *centre of oscillation*; for when the bob of a pendulum is *very heavy*, in respect of the pendulum-rod itself, the centre of oscillation corresponds nearly to the centre of gravity of the bob, or ball, at the end of the pendulum-rod; and thus we say a pendulum is of a certain length, to vibrate in a certain time: we take for the length of the pendulum, the distance of the centre of oscillation from the centre of suspension; and then, if the pendulum is composed of a bar, or even an irregular figure, if we can find its centre of oscillation, and measure from that point to the centre of suspension, we can calculate the number of vibrations it will make in a given time.

Prop. 59. *Centre of Gyration.*—This point in a body, or system of bodies, is somewhat analogous to the centre of oscillation, and only differs in this respect, that this point is that in which *any* force applied will produce a certain effect, but that it is applied *only* to that point; whereas, in the latter case, the motion of the body is produced by the action of gravity on all the particles of the body: thus, in the centre of gyration, the body is put in motion by some other force acting only in one point; and if this point is such as to produce the same effect, with regard to the angular velocity of the system, as if the whole system were collected in that point, this point is called the centre of gyration.

Prop. 61. *Cor.* 1.—*Tension* is a term used in mechanics to denote that state of a cord, or string, when any force is applied to it, in order to stretch it beyond its natural length, and must not be confounded with the actual strength of the cord, &c. to resist a given force.

Prop. 66. *Scholium.*—In order to make this scholium perfectly intelligible, as well as to illustrate the term *reciprocally as*, we will endeavour to familiarize the expression, where it says, that the weight, or force, at any place, will be, to preserve the equilibrium reciprocally, as AG^2: now, the meaning of this is, that supposing any number of points in a flexible line, and a weight suspended from each point, these weights must, in order to balance each other, have a given proportion; which proportion is expressed by the square of the sine of the angle the weight or point from which it is suspended makes, with a line drawn from the centre of the two points of suspension, and either point of suspension. And as the sine increases as this angle increases, so the weight must diminish in the same proportion; and this is said to be reciprocally as the sine of the angle, or as AG^2: and, in like manner, as the secant of the angle decreases as the angle is augmented, so the weight diminishes in a certain ratio, viz. as the cube of the arc BA, that is, in direct proportion to the secant, or in reciprocal proportion as the sine of the arc, that is, in one case, the less the secant the less the weight; and this is direct pro-

portion; and, in the other, the less the sine, the greater the weight; and this is reciprocal proportion.

Prop. 69. *Cor.* 3.—That is, supposing two beams, of different lengths, for instance, 12 feet and 6 feet, and at the end of the one of twelve feet a weight is suspended of 50 pounds; now, as the beams are in length, to each other, as 12 is to 6, that is as 2 is to 1, then the weights must be in reciprocal proportion as these lengths; therefore a weight of 100 pounds hung at the extremity of the beam of 6 feet will strain that as much as a weight of 50 pounds hung at the extremity of the beam of 12 feet in length.

Prop. 73. *Cor.* 1.—In our observation on Prop. 15, we have explained the nature and construction of the parabola, that is, the parabolic curve, as found by the section of a cone; and we there see that there is always a certain relation between the abscissa and ordinate, and which relation, in the language of algebra, is expressed by $px=y^2$; that is, if we call p a constant given quantity, x the abscissa and y the ordinate, to any point in the curve, this expression denotes that the curve is the common parabola; now, if we have a curve given, if we mechanically measure the abscissa and ordinate corresponding to any number of points, we may, by comparing the relation which the abscissa and ordinate bear to each other, find the specie of curves to which it belongs; and if we find the abscissa vary as the square of the ordinate, we pronounce it a parabola; but if we find the abscissa varies as the cube of the ordinate instead of the square, it is called a cubic parabola, that is, if p be a given constant quantity, the equation or algebraic expression $px=y^3$ denotes it to be a cubic parabola.

Prop. 73. *Cor.* 2.—But if the nature of the curve is found to be such that the *square* of the abscissa always varies as the *cube* of the ordinate, the curve is said to be a semi-cubic parabola: thus, if p represent a given constant quantity, and if $px^2=y^3$, the curve is a semi-cubic parabola.

Prop. 73. *Cor.* 3.—The algebraic expression denoting the curve to be an elipse is $y^2=\frac{c^2\times\overline{tx-x^2}}{t^2}$, where y represents the ordinate, x the abscissa, c the conjugate or shortest diameter, and t the transverse, or longest diameter of the elipse; that is, in words, the square of any ordinate is always equal to the square of the conjugate diameter multiplied by the rectangle of the two abscissas, and the product divided by the square of the transverse diameter; hence, in the elipse, we have always the following proportion,—as the square of the transverse, or longest diameter, is to the square of the conjugate, or shortest diameter, so is the rectangle of the two abscissas, (that is, the two parts the transverse diameter are divided into by the point where the ordinate meets it,) to the square of the ordinate.

Prop. 79. *Cor.* 1.—The term *deflection* is here to be understood as distinct from the curvature the beam assumes, and is applied to the effect the force of gravity, or weight of the beam, exerts to make any point descend from the horizontal position it would otherwise remain in; and the quantity of deflection is measured by the perpendicular line drawn from any point in the curve to the horizontal line AB or AC, *Figures* 7 and 8. *Pl.* XI.

Prop. 79. *Cor.* 3.—The curvature of a beam, which, by its weight, bends from the horizontal line, is said to be given, because we have seen (in Prop. 66. Cor. 5.) that if several beams are connected together, and suspended from two points, the positions of the angular points where they meet are always to be found by the rule there given, and are, therefore, invariably alike, in the same or similar beams so connected; now if, instead of a number of beams, we divide one beam into a number of equal parts, we may call these parts separate beams; and the action of gravity on these several parts will cause it to bend, or deflect, from the horizontal line; and as these bendings, or deflections of any part, can, by the proposition, be calculated, we can always find their position: hence, if the parts are supposed to be infinite, they will form one continued curve line.

Page 102, *line* 21.—In reference to *Fig.* 14. *Plate* XI. the reader will perceive that the figure is wrongly represented, for it should not be drawn, as it is there, with the sides convex, but as *Fig.* 4. *Plate* B, with the sides concave towards the axis; and this error is corrected in the latter editions of this work, as well as noticed in the 8vo. edition of Emerson's *Elements of Mechanics*.

Page 102, *line* 3. *from the bottom.—Logarithmic curve.*—In order to have some idea of this curve, and its method of construction, let ABCDE (*Fig.* 5. *Plate* B,) be any indefinite right line, and let it be divided into any number of equal parts, and perpendiculars be drawn from all the points ABC, &c. and let their lengths AF, BG, CH, DI, and EK, be a series of geometrical proportions, that is, if AF $=$ 2, BG $=$ 4, CH $=$ 8, &c. and AB is an abscissa, and BG its corresponding ordinate, and so of the rest: thus the abscissas are the logarithms of the corresponding ordinates; for the abscissas are a series of arithmetical proportionals, while the ordinates are a series of geometrical proportionals, which are the properties of logarithms; hence, the name of the curve passing through the points FGH, &c. is called the logarithmetic curve, or logistic curve: and here it may not be amiss to explain what is meant by the term *assymptote to a curve*, it is that strait line which, if infinitely continued, approaches nearer and nearer the curve, but would never meet it: thus, in this curve, if BA is produced beyond A, and the curve GF is produced, it will continually approach AE. But as a series of

numbers, in geometrical proportion, may be continually creasing without ever becoming equal to O; so the ordinate the curve may continually decrease without ever coinciding w the axis of the curve: hence, in the logarithmetic curve, the a AE is an assymptote to the curve.

Prop. 81. *Cor.* 1.—We here see the reason why, in taking level for the construction of a canal, or other purposes, as conveying of water from one place to another in pipes, an allo ance must always be made for the curvature of the earth's surfa for, as the earth is a sphere, or nearly so, and the gravitat force situated at, or nearly in, its centre, the surface of any fl on it will take the form of the curvature of the earth's surfa and not that of a tangent to it: for let ABC (*Fig.* 6. *Pl.* B) b portion of the earth's surface, and it is required to constr a canal from the points B to C; now, if we take the optical, apparent level, it will be in the direction BD a tangent to earth's surface; but as its gravity will draw the water in the cu BC, we must make an allowance of such a nature, that, know the distance BC, we may make a proper allowance for the de ation of C from D, as the arc BC is the true level required, a not BD, which would be the level as taken with any instrum for levelling. Tables have, accordingly, been calculated to ma this allowance; of which the following will not, perhaps, be un ceptable; and here the distances are such as are measured on earth's surface.

Distance Measured in Yards.	Allowance in Inches.	Distance measured in Miles.	Allowance in Feet and Inches.
100	0.026	¼	0. 0½

[redacted]

This table will be found useful to the engineer and surveyor, in many instances: thus, 1. To find the height of the apparent level above the true. 2. To find from the distance given of a spring, or reservoir, from any house, the height the water will raise to a cistern in the given place, and many others; for which see Hutton and Barlow's *Mathematical Dictionaries.*

Prop. 83. *Case* 2. *Cor.* 1. *Page* 109.—In order to illustrate this Corollary,—for on it depends a well known, though seemingly contradictory property of fluids, as considered with respect to mechanical principles, and, hence, denominated the *hydrostatic paradox*, viz. that it is possible to make any quantity of fluid, however small, balance a weight, or quantity of fluid however large,—we will endeavour to explain the principle, as well as the problem itself, known under the title of the *hydrostatic paradox* and which is usually illustrated by what is called the *hydrostatic bellows*, mentioned in this work; and on this principle, the hydrostatic press, now generally known, is constructed, and which, as it is, in fact, a new mechanical power, or a power but lately applied to any useful purpose, a description of the machine, in its most simple form, may be acceptable to our readers: and, first, as respects the hydrostatic bellows, (which the reader will excuse me for again repeating,) let AB, CD (*Fig.* 1. *Pl.* C) be two boards joined together in the manner of a pair of bellows, but having no valves: in any part of AB, the upper board, let there be a small pipe, as EF, inserted: now, these boards will, in their natural state, rest on each other, and if we pour any fluid, as water, into the tube at E, it will, of course, raise the board AB, and fill the space included between AB and CD: now, if weights, as shewn at G, be placed on AB, they will, of course, force the water up in the tube DF: and, supposing the water included between ABCD to weigh one hundred weight, it is natural to suppose that if the weight laid on AB exceeded one hundred weight, the whole of the water would be expelled, and, of course, raise in the tube EF, which, if long enough, would contain the whole of it; or, if not, it would run out at the end E; and so it ought to do upon principles purely mechanical. But if we advert to the simple principle, that fluids press equally in all directions, which experiment fully justifies, and also that a fluid poured into a tube, bent into any form, will raise itself in each leg of the bent tube to equal heights, or that it will always find its level, therefore the weight on AB may be considered as the weight of a quantity of water equal to one hundred weight; and if a tube, as AB, (*Fig.* 2. *Pl.* C.) be joined to another, as CD, of a diameter equal to that of EF, (*Fig.* 1. *Pl.* C) the water in the two tubes will be at equal altitudes; or a hundred weight of water in AB will balance an ounce, or less, in CD, according to

the diameter of CD; so, in like manner, a hundred weight, AB, (*Fig.* 1. *Pl.* C) will balance an ounce, or less, of fluid EF, according to its diameter. Now, it is plain, that as wa always finds its level, if we suppose the tube CD of suck diameter that six inches in length will sustain one hund weight, we see that the six inches in CD exactly balance the inches in AB. In like manner, if the diameter of the cylinc formed by the boards ABCD, (in *Fig.* 1. *Pl.* C) is such as contain one hundred weight of water in six inches of height, a the diameter of the tube or pipe EF is such that six inches length shall contain one ounce of water, every hundred wei placed on AB will balance every ounce poured into the tu EF: hence, by a parity of reasoning, if the tube EF is minished, so that six inches in length shall contain but drachm of water, it will still balance one hundred weight pla in AB: thus, if the tube is of sufficient length, we may, a small quantity of water, raise any weight placed on the bo AB; and hence, we see that a small pressure on the fluid in tube EF will raise a weight on AB so much greater than its as the diameter of AB exceeds that of EF; and this natur leads us to the principle of the *hydrostatic press*, which we now describe.

Let AB (*Fig.* 3. *Pl.* C) be a pipe of small diameter, say inch; DE another of much larger, say 12 inches; and let th be connected together by the pipe BC; then, if we pour wa into one, as AB, it will raise to the same level in ED: now, piston, or a plug, is fitted into the tubes AB and ED, we fi upon the principles above shewn, that a small weight on piston at AB will balance a much greater one in the tube D thus, by a small pressure on the end of the piston FG, we enabled to raise the piston HI; so that, supposing the end I

valves that retain the water forced into ED, so that the press shall release the goods, or whatsoever has been subjected to its action. It is scarcely necessary here to remark, that the action of this press is directly the contrary of the common press,—that of exerting the force downwards, while this does the same in an upward direction.

Prop. 83. *Scholium*.—Though the action of capillary tubes are not the subject of hydrostatic laws, as the term has been introduced, and is several times mentioned in this work, it may be as well to say a few words on the subject, as well as to define the term.

Capillary Tubes are small pipes, or canals, whose bores are exceedingly small or narrow—their usual diameters being about one-twentieth or one-thirtieth part of an inch, and some much smaller than even that. Dr. Hook is said to have made some whose diameters were not more than that of a spider's thread. Some of the principal phenomena of these tubes I will now mention: take several of them, of different sizes, open at both ends, and immerse them a little way in water; you will immediately see the fluid raise up in them to different heights, according to their diameters, ascending the greatest height in the smallest tubes; and these heights have been ascertained to be in the reciprocal proportion of their diameters. The greatest height that water has been seen to ascend, is, according to the experiments of Dr. Hook, about twenty-one inches. These heights are, however, not the same for all fluids, for some elevating themselves considerably higher than others, whilst the mercury, instead of being elevated, falls lower than the surface in which it is immersed. Another extraordinary phenomenon is, that if a fluid is suspended in a capillary tube, and suffered to run out, it will only drop from it slowly; but if the tube is electrified, it will run out in a continued stream. These phenomena, though differing from the general principles of hydrostatics, have not yet been sufficiently investigated, though they seem to depend upon the general principle of attraction, or that principle which bodies have to adhere together, some in a greater and others in a less degree, while some seem endowed with a repelling propensity, and endeavour to fly from each other. These different qualities seem, in my opinion, to depend more upon the chemical than the mechanical properties of bodies; and, therefore, till some series of experiments have been accurately made, it will be in vain to attempt to lay down any general rules, or submit to calculation the forces of capillary attraction.

Prop. 84. *Homogeneous* is a term applied to all bodies that are of equal densities throughout, or are composed of a collection of particles of the same nature: thus—metals, stones, &c. are said to be homogenous bodies; and this term is used in opposition to heterogeneous, or that which is made up of a mixture of sub-

stances differing from each other in their nature and qualities; tl
a ball of lead is an homogeneous body; while one made up c
mixture of metal, wood, stone, &c. such as a building, or a sl
is called an *heterogeneous body*: and this is what is meant in C
3. of the same Proposition.

Prop. 84. *Cor*. 3.—This Cor. may be very aptly illustrated
the motion of an arrow, where, from the head being much heav
than the shaft, the centre of gravity is very near the head, and
centre of magnitude that point which is at equal distances fr
the two extremities of the arrow; hence, in its motion, the cen
of gravity, or that point near the head, goes foremost, while
centre of magnitude follows it as here stated.

Prop. 84. *Cor*. 4.—This Cor. seems to imply, that if the bo
which is placed in the fluid is of either greater or less specific g
vity, it will be always in motion; whereas, what is meant, is sim
that if the body is heavier than the fluid, it will continually sink i
till it reaches the bottom of the vessel which contains the fluid; o
it is lighter than the fluid, it will always raise towards the surface,
it be placed in any part of the fluid beneath that surface; but tl
if it is of the same specific gravity as the fluid, it will remain at re
if placed in any part, and will not have a tendency either to desce
or ascend.

Prop. 85.—On the principles shewn in this Proposition depe
the use and construction of the *hydrostatic balance*, or machine
ascertaining the specific gravities of bodies, and, consequently,
applied to the arts, the quantity of any alloy mixed with the gold
silver which we manufacture, either into coin, or different artic
of utility or ornament; the principle of which is as follows. Fir
we weigh the article in the air, in the common way; then, suspen
ing it from the scale by means of a horse hair or otherwise, we let
hang in a vessel of water, and balance it by weights in the oth
scale, and from the quantity of weight lost in weighing in the flu

drometer, or machine for ascertaining the specific gravity of fluids, mentioned in this work at page 204. Hence, we find the different strength of spirituous liquors by its means, as the stronger the spirit is, the deeper will the instrument sink in when immersed; and the contrary.

Prop. 105. *Cor.* 1. *Lee-way.*—Lee is a term used in navigation, and implies the quarter of the compass towards which the wind blows; thus, if the wind is in the west, or blowing from that quarter, we say that the east is leeward; and thus the lee-way is the deviation from the point of the compass on which we are steering towards that quarter to which the wind blows, or a drifting from the line of direction in which we wish to sail, caused by the wind blowing in an oblique direction, on the sails, hull, &c. of the vessel.

Prop. 105. *Cor.* 4.—For instance, if a ship is sailing with the force of the wind acting on her sails, and at the same time is driving with the force of a current in her stern, in the same direction, we must, in order to find the force with which the water acts on her stern, subtract the velocity produced by the force of the wind from the velocity produced by the effect of the pressure of water on the stern of the vessel; thus, calling the velocity produced by the wind x, and that by the current y, the whole absolute velocity of the ship would be $x+y$, that is, if we supposed $x=2$ miles per hour, and $y=4$ miles per hour, the velocity ought to be $2+4=6$ miles; but as the whole force of the current cannot be exerted to impel the vessel forward as it is in motion, the relative velocity of these two forces must be taken instead of the absolute velocity; that is, we must subtract the velocity x produced by the wind from the velocity y produced by the stream, that is, $y-x$ $=$the relative velocity of the stream, and from hence deduce the force of the water on the stern of the vessel by this Proposition and its Corollaries.

Prop. 111.—It is here necessary to say something respecting what is mentioned in reference to *Fig.* 11. *Pl.* XIV. as the original diagram given in the plate in the quarto edition is not sufficiently plain. We have, in our diagram, endeavoured to make the figure more so, by representing the cycloidal tooth EB, as well as the crooked tooth A*b*, and the bent tooth AG, somewhat plainer; but still we do not feel satisfied that it is sufficiently intelligible to the workman, and shall therefore, from the same principles, endeavour to shew what is here meant, and illustrate it by a diagram somewhat different in form, but more clear in the construction. Let A and B (*Fig.* 4. *Pl.* B) be two wheels, of any diameters, equal or unequal, and let the line CD represent the face of a tooth formed into an epicycloidal figure by the revolution of the wheel A about that of B, and let EF be a crooked tooth of any form, so that it

shall not touch the point D of the tooth DC; then if the wheel made to revolve on its axis, the tooth EF will, by pressing on cause the wheel B to revolve with an equable motion about axis, and by that motion the teeth will assume the position shewn at GH and KI, when the tooth GH will leave the poin of the tooth KI. Then, if we suppose other teeth in the circu ference of the wheels A and B so placed, that as soon as crooked tooth is leaving the epicycloidal tooth, another crool tooth shall have just touched another epicycloidal one, as shewn the figure, an equable motion will be produced, and the wh will revolve without being either accelerated or retarded dur their revolution about their axis.

Prop. 111. *Page* 145.—In reference to *Fig.* 2. *Pl.* XV, if notches in the machine DG are all right lines, the motion will uniform; but if parabolic curves, the motion will be accelerated.

Prop. 112.—In changing the direction of any motion by me of pullies, as here shewn, the line of direction must be nearly quite in the same plane; thus, the pullies B, C, D, &c., must be same plane, though the direction of the power may be varied in given direction in that plane; for if the pullies are not situated in same plane, or nearly so, the rope or cord going round them slip from the groove or channel of the pulley, or if not, will bear w unequal pressure on the axis of the pullies, and cause a great d of friction and a considerable loss of power; for suppose, in *Fig.* *Pl.* XV. the pullies B, C, and D, to be not in the same plane as direction of the power A B, it is evident that there will be not o a strain upon the axis of the pullies, but that the cord or rope goi round them will constantly rub on the sides of the grooves of pulley and endeavour to slip out. This in practice is endeavoured be remedied by suspending the pullies by ropes or hooks, so th their several axes shall accommodate themselves to any deviati

ABC lies in the same plane; now, in order that the pulley at B should move freely without the rope being liable to slip out of the groove, or its edge, or be strained by the rope bearing unequally against the sides of the groove, it is necessary that the axis of the pulley should be fixed in a perpendicular direction to the plane ABC. In like manner when the direction is changed from a vertical to an horizontal direction by means of another pulley C, it is necessary that the pulley C should be so fixed that its axis should be perpendicular to the plane BCD; in the same manner, if the motion is to be oblique, as shewn at BEF, the pulley E must have its axis perpendicular to the plane BEF, and so on for any number of pulleys or any change of direction; and thus we see, that to enable us to change the direction of a power by means of the pulley, it must be an invariable rule to fix the axis of the pulley in a plane in a perpendicular direction to that in which the rope that goes over it moves.

Prop. 112.—In changing the direction of motion by means of wheels, as shewn in *Fig.* 1. *Pl.* XVI. we have but a hint of the method now so much in use in all millwork, which is technically termed *bevel-geer*, the principles of which we will endeavour to lay down, as well as the methods in common use, to adapt the bevel of the teeth of such wheels to each other as they will move with the least friction, and press equally on all parts of the teeth during their revolution.

Let AB (*Fig.* 6. *Pl.* B.) be the direction of the axis or shaft of a wheel, and it is required to change the direction of the motion *(without regard to the velocity which we will here suppose, for example, the same,)* by a shaft situated in AC; let us suppose a cone AFD, placed on the shaft AB, and another of equal dimensions on the shaft AC, (the shaft passing through their centres,) such that FD =DE, it is evident that if grooves were made in both cones from their bases to their vertices, diminishing from their bases and slips, placed in those grooves, that by placing there these slips, which act as teeth between each other, if we make the cone FAD revolve on its axis, it will also make DAE revolve on its axis; thus producing a motion in the shaft AC at an angle BAC, equal to the direction in which we wish it to move; and if we suppose the parts of the cone AGB and ABI cut away, there will remain the bevel-wheels GFBD and BDEI, which is the principle of the bevel-geer. Now if we wish the velocity of the shaft AB to be greater or less than AC, we must augment the diameter of the wheel FD, or reduce it in the same ratio as FD is to DE, or the line AD must divide the angle BAC in the same ratio as we wish the velocity of the wheels FD and DE to move. Hence, we see that the bevel of the wheels C and D, (*Fig.* 1. *Pl.* XVI.) must be such, if we suppose their shafts produced, till they meet; a line drawn from that point

to the meeting of the circumference of the wheels will determine the angle or bevel of their edges, so as they will work freely and equally together.

Prop. 113.—The methods of regulating the motion of machines by means of pendulums are more especially adapted to clock works, and accordingly have been studied with great care to produce a uniform and steady motion in the hands of the clock, and are known by the technical terms of *scapements,* or *escapements. Fig.* 1. *Pl.* XVI. is the common vertical scapement, and which, though regulating the movement, causes the hand which is fixed on the axis FG, to point out seconds of time in common clocks; where the pendulum swings seconds, at the same time it causes a kind of oscilating movement in the hand, so that it appears to move not by direct impulses in one direction, but after proceeding forwards is made to go back a part of the space in which it has advanced.—*Fig.* 3. *Pl.* XVI. is the common horizontal escapement, and is liable to the same objections. Other escapements that cause the hand to proceed by regular impulses one way only, are called dead beat scapements, and are numerous, as the ingenuity of the artist or convenience of the construction dictates.

Prop. 119.—I would call the attention of the mechanic more particularly to this Proposition, as it contains such rules and observations in the application of theoretical mechanics to the construction of machines and engines that will, if properly attended to, enable us to apply with the greatest possible advantage any power we are in possession of, to produce the maximum of effect, and in such a manner, that we shall not encumber our engines with a multiplicity of parts altogether unnecessary to the purpose we have in view; thus enabling us to avoid a waste of material or an unneccessary expence in the manufacture. The eighteen rules here given embrace all the precautions that are absolutely necessary to ensure perfection, as well as many principles deduced from actual experiment, which will guide us in the application of machinery to the various purposes of life.

We will, therefore, for the sake of illustration, suppose a case, in which it is required to raise a weight of one ton by the strength of an individual, who is capable of exerting for a continuance a force of 30 pounds. Here we have the proportions of the power to the weight: now we have seen, that if a lever is supported at a point, which divides the whole length of the lever into two parts, in the same proportion as the power is to the weight, it will be balanced on the point of support or fulcrum, the longest arm of the lever being that to which the power is applied, and the shortest that to which the weight is suspended: now, if in our example we apply a lever so divided, in the proportion of 30 lb. to 1 ton, or 2240 lb. we shall be able to support (having a fulcrum

at that point) the weight required; and to raise this weight, we need only lengthen the longest arm of the lever, and we shall overcome the resistance of the weight; or the strength of one man with a lever so constructed will be sufficient to overcome the weight: but as the lever is calculated to raise bodies but a small distance, if we were required to elevate the body much above its original situation, we must have recourse to other mechanical powers, such as the pulley. We will now examine how that will answer our purpose. In the body of the work we have seen that a fixed pulley only changes the direction of the power, and does not increase it, and every moveable one doubles the power, and in a combination of pullies, as generally used, the power is increased according to the number of moveable pullies; or as many times as the rope goes round the lower pullies, so many times the power is increased by 2; that is, if the lower shieve, or block, contains 3 pullies, for instance, the power is equal to 6; now, in order to find, in our example, how many pullies there should be in the lower block to raise one ton, or 2240 pounds, with a power of 30 pounds, as the double of 30 is 60, which will be the weight we can balance with a moveable pulley; if we add another pulley also moveable, we shall have a power of 120, and so on as shewn in *Fig.* 3. *Pl.* XXI. We find, that with 7 moveable pullies we shall be able, with the strength of one man, to raise 3840 pounds; whereas 6 pullies will raise but 1920 pounds, insufficient for our present purpose; hence, we find that with the pulley we cannot effect our purpose in this case conveniently; and the common way, as at *Fig.* 7. *Pl.* IV. has still less power; therefore, let us try what we can do with a combination of wheels acting as perpetual levers; in order to effect which, let ABC (*Fig.* 1. *Pl.* C) be a lever, and let it be divided in B, so that AB : BC as the weight : the power; hence we find that the proposition of AB to BC is as $74\frac{2}{3}$ to 1: that is, if we have only two wheels in this proportion of their diameters, a proportion too great for actual practice (being nearly that of four inches to $23\frac{1}{3}$ feet,) if the power acts at the circumference of one wheel, and the weight at that of the other; or if we suppose the weight to be raised by means of the wheel and axle, and if the axle is 4 inches diameter, the wheel, in order to move the weight or produce an equilibrium, must be about $23\frac{1}{3}$ feet; we will therefore divide the weight, say into 10 parts, that is, we will suppose that we have, with a power of 30, to lift 2 hundred weight, or 224 pounds, instead of one ton, and then by the application of another wheel, (on whose axis is a handle,) applied to this, we shall be able to increase the power tenfold, which will enable us to overcome the weight proposed, viz. one ton. Let us see how this will now apply. Let AB (*Fig.* 2. *Pl.* C) be a wheel, on whose axis B a

cord is wound, which is connected with the weight W, which have supposed 224 pounds; then if the radius of the axle B is the radius of the wheel A as 30 is to 224, or if the radius of A something more than $7\frac{1}{2}$ times B, a power of 30 pounds applied the circumference A will balance the weight W. Now, if the wei W is increased ten times, we must apply a force at A ten tir greater than 30, that is 300 pounds, to balance it. We will the fore apply that force to A by means of another wheel, wh is turned by a winch or handle with the force of 30 pounds; have therefore to find the diameter of the second wheel, such that with a lever, DE, of two feet, (supposing that the length of handle,) we shall be able to exert a force of 300 pounds at circumference of the wheel C; we, therefore, say, as 300 poun the force to be overcome by DE, is to 30, the force applied at so is the length DE to the radius of the wheel C, which will found about $1\frac{1}{2}$ inches to balance the resistance; and if the len of the handle DE, or the radius of the wheel A, is increased b small quantity, we shall be able, with a force of 30 pounds appl at E, to raise a weight, as W, of a ton, or 2240 pounds; and thi the most simple form of the crane in common use; and note, t the size of the wheel A and axle B may be increased or diminish as convenient, but so that their proportions shall always be to ea other as here stated; and by attending to the rules laid down in t Proposition respecting friction, we shall always be able to inv a machine to overcome any resistance, and avoid giving m power, and consequently a waste of labour and material, than w be sufficient for the purpose to which we wish to apply our e gines.

Example 17. *Page* 172.—In considering this example, we a naturally led to inquire the length of an oar to row a boat w the greatest possible advantage; for our author has clearly she that long oars have the disadvantage of losing power, though, the same time, they may be made so short, that the power

against it. With regard to the second, it depends on the strength of the individual: and as we have seen, page 159, that a man, generally speaking, can draw horizontally about 70 or 80 pounds, which is about half his weight, and is the same thing as pulling the end of the oar; and as a man can exert a force of 30 pounds at the handle of a windlass by his muscular powers, so a boatman, having his feet firmly fixed against the thawts of the boat, will be enabled to add something by muscular power to the 70 or 80 pounds depending on his absolute weight; we can therefore suppose it, without much error, as a force nearly, if not quite, 100 pounds: and, in the third place, the distance of the handle of the oar from the pins in the side of the boat, (and this depends upon the width of the boat itself;) for if the man sits on the opposite side of the boat to that of where the oar goes in the water, it will allow him a leverage of the width of the boat itself nearly.

We will now apply this to the investigation of the following problem: *Given the resistance a boat meets with in going through the water, the strength of a man to pull at the oar, and the length of that part of the oar within the boat; it is required to ascertain the length of that part of the oar over the boat's side that shall be most advantageous.* We will examine this problem, considering the end of the oar in the water as resting against it as a fulcrum; the power as placed at the other end of the oar or lever; and the resistance as the part of the oar pressing on the pins on the side of the boat. Let, therefore, AEB (*Fig.* 2. *Pl.* C) be the side of the boat, CED the oar, ED the part in the boat, EC the part over its side; now, calling ED$=a$, the resistance opposed to the point F$=b$, and the force applied at D$=c$, we have to determine the proportion of ED to EC, so that the velocity of the point E may be a maximum, or the greatest possible. Now, without entering on any fluxional calculus, but merely from mechanical principles, it is plain that the velocities of the points E and D will be in the same proportion as CE is to CD; therefore it would seem that the nearer the point E is to D, the greater the velocity. But it is plain, that if E is very near D, the force applied at that point will be unable to overcome the resistance at the point E; therefore the point E must be so situated, that the force applied at D shall be able to overcome the resistance at E; and this is the point where, if we place E, the boat will be moved with the greatest velocity; that is, if CE is to ED in the same proportion as the force at D is to the resistance at E, the boat will be moved with the greatest velocity, or the power will be used to the greatest advantage to propel the vessel: thus we find that we are enabled, in the common wherries used on the Thames, to apply longer oars than in ship boats, as the former offer less resistance, from their sharp construction, than the latter,

whose bows are more round. Thus, knowing the resistance the water gives to the motion of the boat, with the force applied at the end of the oar, we can determine the length of the oar itself to produce the greatest possible effect in moving the boat.

Example 29. *Page* 179.—The latter part of this example needs some little illustration, where it is observed that when GD is greater than the radius OH+R, the chariot will move forward, &c. That is, if the force of the wind on the sails is sufficient to overcome the friction of the carriage in moving, together with the force of the wind acting against the body of the chariot, it will move against the wind: hence, we see, that unless that is the case the sails will have no effect, but the wind will exert itself on the chariot alone to produce a motion in it; or if these two forces, viz. that of the wind against the sail, and that against the chariot, are equal, no effect at all will be produced. This explanation may be necessary, as it seems, on the first view of it, to imply that the sails produce a progressive and a retrograde motion, according to the power the wind has over them, when compared with the resistance offered by the mast and vessel itself; and here it is evident that this construction of a sailing chariot will produce a progressive motion, let the wind blow from any point of the compass; for if the power of the sails is sufficient to produce motion, when the wind is directly contrary, much more will it produce motion when the wind is more favourable, as, in that case, the resistance of the wind against the chariot is taken away, and, if directly in favour, will augment the speed of it.

Example 32. *Page* 180.—With respect to the syphon, as used for drawing off liquors, &c. the following form is commonly adopted: ACB (*Fig.* 4. *Pl.* D) is a tube, or pipe, of copper, or metal, having the leg A about a foot or eighteen inches longer than B: at E is a stop-cock, CD a small tube soldered into the leg A, and opening into it at C, and having the end D open, and turned up, to apply the mouth to draw out the air after having immersed the end B in the liquor, and stopped the end A by turning the cock E; then, by opening the cock E, the liquid flows out at A; and when you have filled your vessel, you may turn the cock, and, at any time, draw off more liquor, without again extracting the air a second time. We may here observe, that the pipe DC is much smaller than AB; or otherwise, if the liquor did not flow faster from A than it passes by the end of the pipe at C, it would flow out at the end D, if, at any time, it should be lower than the surface of the cask, or vessel, which you are emptying; also, we must observe, that the point C (where the small tube is inserted into AB) must always be below the surface of the liquor, or the syphon will cease to flow.

Example 34. *Page* 182.—The rules here given for our judgment

of the weather by the barometer, are not absolutely to be depended on in all cases; but we should rather judge from the motion up and down of the quicksilver in the tube, than from the actual height at which it stands, and judge from the surface of the quicksilver whether it is concave or convex, whether it is rising or falling: for if it is rising, it will appear very convex; if falling, concave, as it adheres in some measure to the glass; indeed, though the barometer may shew, with correctness, the weight of a column of air pressing on the surface of the quicksilver in the basin, it will not always tell, or, more properly, foretell the state of the atmosphere, or whether we are likely to have foul or fair weather; and though many observations have been made, there still seems something wanting to enable us to judge with any degree of certainty in this respect; for in different latitudes, and under different circumstances, we find, often, such contrary indications, that in the present state of our knowledge respecting the prognostics of the weather, by this means, we are often led into many errors, which time alone, and experiment, can reconcile to the phenomena observed. Those who would wish to see more on the subject, and a multiplicity of observations, will find their curiosity amply gratified, by a reference to the article *Barometer*, in the Supplement to the *Encyclopedia Britannica*, or Rees' *Encyclopedia*, as well as in Barlow's and Hutton's *Mathematical Dictionary*.

Example 37. *Page* 183.—The application of the principle of Tantalus's cup to practical purposes is of great use, in many chemical operations; as, by its means, we are enabled, during the process of distillation, to separate the oil that comes over from the waters we are distilling: thus, in the process of making lavender water, what is distilled is mixed with a certain quantity of essential oil of lavender, which is separated by the following means: ABC (*Fig.* 5. *Pl.* D) is a vessel of glass, having the spout, or syphon, DE, communicating with it at E; then as the liquid drops from the still, it fills the vessel AB till it rises to *a b*, above the bend of the syphon, and it will run out at D till it falls from *a b* to *c d* level with D; it then fills again, and again empties itself, thus always leaving a portion of the liquid at the bottom of the vessel: now, as the oil which comes over with the distilled water will always float on its surface, and as E is situated at the bottom of the vessel, the oil will accumulate on its surface, while the water runs out at D, thus saving all the oil by itself in the vessel; and, at the end of the operation, we have but little trouble to separate the remaining distilled water from it; and this machine is applicable to a variety of similar operations connected with the arts and manufactures, but which, more properly, belong to the science of Chemistry than that of Mathematics; but, as the theory of the syphon is introduced as a part of hydrostatics, this

is, in some measure, connected with the general plan of the w
under notice.

Example 40. *Page* 185.—By the inspection of the figure she
in Plate 27, it may appear that the horse, in trotting, has two f
up on one side, while the opposite ones are down: now, that t
may not lead into error, we may here remark, that though the f
at F is represented as lifted up, we must suppose that, before
foot B is lifted, it is placed on the ground at F, and then the f
foot B taken off the ground; and this the attentive reader v
see is what is meant in this example.

Example 42. *Page* 187.—It has often struck me, in observ
the motion of a fish in water, that the air bladder has other p
poses than that of simply rendering it lighter and heavier than
surrounding water, so as to enable it to raise and sink itself
pleasure; for, frequently, the fish is in a perpendicular directi
either with its head or tail elevated: now, it is worth enqu
whether the fish has it not in its power to alter its centre of g
vity by means of the air bladder, so that its head, or tail, sl
preponderate, thus enabling it to steer itself in any oblique
even perpendicular direction in the fluid; for, by very attent
observation on some gold fish in a large reservoir, it appeared
me that the action of the tail and fins was insufficient for
purpose, and that they seemed only to use the fins as a means
steadying their position, or balancing themselves; for, if the
bladder itself is capable of but a small movement either towa
the head or tail, it must necessarily cause one end to prepondera
and thus, as it shifts the centre of gravity of the body of the fi
it must produce a corresponding ascent or descent obliquely
the head of it; and thus, by a very simple piece of mechanis
give the fish the means of advancing in any required direction.
am not aware that this idea has occurred to naturalists, l
I think, as all nature is subject to mechanical laws, it is wo

arch $c\ e$, (which must be cut oblique to the plane of the arch,) one revolution of the nut at D will move forward one tooth of the arch; and it is on this principle that the screw is applied to our astronomical instruments, to enable us to divide and subdivide the degrees and minutes engraven on their faces; for if, for instance, one tooth of a wheel represent a degree, we can, if we divide the nut of the screw at D into 60 equal parts, easily divide each degree into minutes, or sixtieth parts of degrees; for, if we move D one division forward, it will move the circle a minute, or its sixtieth part; and thus, by means of this contrivance, we can, with accuracy, divide either a circle, or straight line, into very minute portions; and, hence, the combination of the screw and wheel, as applied to delicate divisions of lines and circles, has obtained the name of the *micrometer screw*.

Example 54. *Page* 191.—There is an omission in the plate of the stays, S S S, as there should have been a line drawn to represent them, from the top of the tall posts, to which the pullies are fixed, to the short ones represented as S S on the ground; but the reader will easily see, from the figure itself, the use of these stays, and where they should be placed.

Example 61. *Page* 194.—The description of the jack, for raising great weights, here given, is, as far as it goes, correct, and, perhaps, at the time of our author's writing, was the only one in use; but, as there have been many later improvements in this most useful engine, I shall here describe them; and would first remark, that instead of the case KL being all of metal, as our author describes, it is of one solid piece of strong wood, having the wheels and rack let into it, and fixed in their places by iron straps, or plates; also, at the end B of the rack, AB, there is a strong forked piece turned up at right angles to the rack, AB: so that when we wish to raise any thing near the ground, as a block of stone, and the end B is very near the bottom of KL; then, if we place the end B, of the rack AB, under the stone, and turning the handle HI, we are enabled to apply the end B in the same manner that we applied the end A in the example. Another improvement is, in having a catch, or rack, fixed to the handle of the machine; so that after we have elevated the body to the height required, we are enabled to prevent the wheels, and, consequently, the rack, AB, from turning backwards, if we leave go the handle I; and thus prevent many accidents which would otherwise arise.

Another method of employing the power to raise AB, instead of the pinion EF, and common toothed wheel G, is, to have as the axis of the handle a perpetual screw, which works in the wheel G, whose teeth are cut beveling, to correspond with the screw, as shewn in *Fig.* 1. *Pl.* V.; this adds to the power of the machine,

and, at the same time, we can more easily retain the weight in any position. The third method is, that of employing a screw instead of the rack, AB, and for many purposes, is much more convenient, as well as more powerful, and has the advantage of maintaining itself, with the weight, in any required position; the most simple form of which is represented in *Fig.* 1. *Pl.* E, where ABCD is a solid block of wood, AB a nut in which the screw EFG moves; at F is a piece made larger than the screw, and pierced with holes to admit a lever to turn the screw; at E is a nut forked at the end, to be applied to the body we wish to elevate, and moveable about FG; hence, as we turn the screw, the nut E is elevated, and with it the weight; the instrument, in this form, is much used among builders for raising or supporting the walls, or part of buildings, when sunk, or undergoing alterations in the lower part, when we do not wish to disturb the superstructure.

I shall here, lastly, describe another form of this machine, which appears to combine all the requisites of strength and power: it is represented in *Fig.* 2. *Pl.* E, where A represents the stock; BB a screw, with a square thread, and a fork, F, at its end, fixed to it; D is a nut, on which is a wheel; C, whose edge is hollowed, and cut into teeth, to work in the axis of the handle; B, which is cut, with a screw to correspond, and which goes through two strong plates; *a*, *b*, screwed to the stock A: now, as we turn the handle E, it turns the wheel C, and, with it, the nut D, which raises the screw B; hence the end F, at the top, or the hook N, at the bottom of the screw, is raised, and, of course, the weight is supported in any position required.

Example 62. *Page* 194.—In this example, we must suppose a platform above the wheel *a* for the men, or horses, to work at the lever L, in order to move the machinery.

Example 65. *Page* 195. *Anemoscope.*—This term is sometimes applied to an instrument, by which we are enabled to foretell the changes of the weather, or from which quarter the wind is likely to blow, and thence judging of the probability of rain or fair weather; and, in this sense, we find it used by many writers. The instrument, now under our consideration, is more properly a *wind-dial*, as merely shewing the quarter from which the wind actually blows, and not enabling us to foretell, or consider, the different changes of the wind.

The term *Anemoscope* is frequently confounded with that of *Anemometer*, which is, properly, a machine to indicate the force of the wind, and not the direction in which it blows; the first being derived from two Greek words signifying *the wind*, and *I see*, or *I consider;* and the latter also from two Greek words signifying *the wind*, and *I measure:* and Martin, in his *Philosophia*

Britannia, describes the latter, viz. the *Anemometer*, under the title *Anemoscope;* though evidently erroneously, as they are two distinct instruments: I shall therefore describe the *Anemometer*. AB (*Fig.* 3. *Pl.* E) is a post sufficiently elevated, that the wind may act on the machine; CD is a frame, fixed on the top of AB, moving thereon on the pivot A; the axis G, on which the sails I I are fixed, is formed conical, as shewn at EF, and formed into a spiral groove to admit the cord EW, which is fixed at E, the end of the axis; and as the wind makes the sails I I revolve, the cord is wound round the cone EF, and the weight raised, or depressed, according to the force of the wind upon the sails; at H is a vane, or flat piece of board, or metal, for the purpose of keeping the sails, I I, always facing the wind; at the end of the cone at E is a racket wheel, in which a spring, K, catches, as the cone revolves; holding the weight at any height, the force of the wind may have raised it to: which enables us to ascertain, in our absence, the greatest force at which the wind has blown; this spring may, at any time, be detached, in order to see the present state of the wind. It is plain, from the consideration of the figure, that as the axis, is greater at F than at E, the wind must exert a greater force to maintain an equilibrium with the weight W, as it approaches the large end of the cone: now, if we put such a weight on the cord as will just balance the weakest wind, as the wind increases in force, the weight will be raised, and the cord wound round from E to F; and, if the diameter of the cone at F is such, as the force of the greatest wind will just sustain the weight at that end, then, in any intermediate wind, the cord will always be found somewhere between E and F, and, consequently, indicate the comparative force of the wind; but, if a scale is drawn on the cone, the point where the cord meets, it will tell the comparative force of the wind: for example, let the two ends of the cone be as 1 to 28, and if the weight W is one pound at the small end of the cone, and just balances the weakest wind exerted on the sails, if we suppose the wind to increase, and the cord to be wound round to the greater extremity of the cone, it must exert a force of 28 times the former; or the force required, to sustain the weight at the large end, will be 28 pounds: thus, if the axis, or side of the cone, is divided into 28 equal parts, each division will correspond to a pound of force exerted on the sails. Again, if the instrument is fixed on the outside of a building, and the weight descends into an apartment beneath, you may, by having a scale fixed against the wall, corresponding to the divisions on the cone EF, on inspection, see the force of the wind: in the same manner, the anemoscope tells its direction; or the two instruments may be very conveniently combned in one, as will appear plain to any mechanic, who has paid

attention to what has been said respecting the two instrumer

Example 72. *Page* 198.—In this example, where mention made of the lower stone being feathered or channelled, as th is no direction in what manner or form these channels are to cut, and that nothing may be omitted that the workman ought to acquainted with, I have subjoined the following figure, to sh the most general method in use; and here take occasion to serve, that, in actual practice, it is necessary to have these groo from time to time re-cut, that the mill may work to the great possible advantage. AB (*Fig.* 4. *Plate* E) represents one of stones of a mill; the circumference is divided into eight parts, a lines drawn to the centre; then begin by drawing lines at mo rate distances, parallel to every line drawn to the centre, as she in the figure; and if on these lines we make an angular groo as shewn at C, the stone will be prepared for being fixed in place.

Example 80. *Page* 204.—The thermometer here described the common spirit thermometer, which is now nearly, if wholly, superceded by the mercurial one, which only differs in th that the fluid inclosed in the tube is quicksilver, instead of spi of wine. I shall here, however, take the opportunity of shewi how the tubes are graduated, so that each thermometer, whate its length may be, will indicate the same degree of heat or cold.

Having filled the bulb, and part of the tube, with quicksil that has been previously heated, to expel the air it may contain, the end be hermetically sealed, that is, closed by melting it w a blow-pipe, and pinching it together while in a soft state; then we immerse the bulb in water just freezing, or snow just melti and mark the point on the tube at which the mercury stands, shall have the freezing point. Again, let the bulb be now i mersed in boiling water, and mark that point also for the boili point; then, if we take that distance, that is, the distance betw

and C for centigrade; and first, to convert Fahrenheit to centigrade, we have $\frac{F-32\times 5}{9}=C$; and, secondly, the converse $\frac{C\times 9}{5}+32=F$.

Whilst we are on the subject of thermometers, it may not be amiss to notice what is called the self-registering thermometer;—that is, one which indicates, in the absence of the observer, the greatest and least degrees of heat which have occurred in his absence. There are several contrivances for this purpose, but I shall only notice that most common at this time; it consists simply of two thermometers, the one a mercurial one, and the other filled with coloured alcohol, or highly rectified spirits of wine; as C and D, (*Fig.* 5. *Pl.* E) having their stems horizontal, and fixed in the same frame AB, the former has a small piece of magnetic steel wire enclosed in the tube, and the latter a fine piece or thread of glass, whose ends are made into minute balls by the flame of a lamp; now the magnetic wire being brought to the surface of the mercury by the application of a key or any piece of iron, as the mercury expands with heat it, is pushed forward; but left behind when the mercury falls by cold; thus shewing the extreme of heat: the latter is immersed in the spirit, and as the spirit sinks or retires, it carries the glass thread with it; but when it advances, it leaves it behind; thus shewing the greatest cold in the absence of the observer; and this index is set by inclining the instrument till one end corresponds to the surface of the spirit in the tube; the steel index is brought to the surface of the mercury by applying a magnet, if the magnetic power of it is insufficient to act on a piece of iron or steel applied to the tube.

Example 84. *Page* 207.—The fire engine here described is what is properly termed the atmospheric steam engine, in its original form, at least in that form to which it was applied to useful and general purposes; there is, however, since this invention, another machine, which depends solely on the elasticity or force of the steam itself, and which has, in a great measure, superseded the use of the former, whose powers are more limited, and of which we shall here attempt a short description; and, first, we will observe that the practice now adopted of placing the steam cylinder detached from the boiler, but communicating with it by a pipe, has quite superseded that wherein they are immediately connected: and it has this, among many other advantages, that the safety of the boiler is insured from any sudden jolts that the engine may be subject to. Again, the steam is not condensed in the cylinder by a jet within it, as formerly, but in a separate vessel, as the following figure will shew; (*Fig.* 1. *Pl.* F.) A is the pipe conveying the steam from the boiler to the cylinder, that is, to the upper side of the piston, by means of the pipe at B, and to the under side by means of the pipe at C; these pipes are furnished with valves, and are acted on

much in the same manner as in the common engines, which the figure will sufficiently shew. The piston rod D is a solid rod of iron, and connected with the beam, by a piece of mechanism called the parallel joint, at XY, which is so constructed that the piston rod is always in a perpendicular direction. There is also a communication from the top and bottom of the cylinder to a separate vessel at E, in which the steam is condensed after being drawn out of the cylinder by means of an air-pump F, which works by means of the above parallel joint at Z; and these two vessels or cylinders, viz. the condensing cylinder and the air-pump, are surrounded with cold water, and are constantly supplied by the pump G, worked by the other arm of the great beam: the air pump F draws off the water formed by the condensation of the steam, into the cistern H, and is pumped out of it by means of the pump I, and is thence conveyed by the pipe K back again into the boiler.

I shall now explain the action of this engine. The communication being opened from the boiler to the underside of the steam cylinder, its elasticity forces the piston up, and at the time the air-pump forms a vacuum, and the steam rushes into the condenser, a vacuum is formed beneath the piston; at which moment the steam enters above the piston, and is, in like manner, drawn off and condensed; and thus, alternately, the pressure of the steam acting on the under and upper sides of the piston, a reciprocating motion is produced in the beam, and a self-moving power is kept up as long as there is a supply of steam. Now, for the purposes of mechanical operation, this motion is converted into a rotatory one, by means of a rod L, from the end of the beam, which is connected with a crank (and fly wheel M) at its extremity, and thus a constant revolving force is applied. There are a number of contrivances to regulate the motion of the engine, and for ensuring the safety of the boiler, which the limits of this article will not admit of enumerating, and for which I would refer the reader, amongst other works, to Stewart's *Descriptive History of the Steam Engine*, Barlow's *Mathematical Dictionary*, or the *Encyclopedia Britannica*, and *Supplement*. I have, however, in the plate here given, introduced the most usual contrivance of two balls *o*, *o*, which are connected with the fly by means of a pulley, P; and as the motion increases, or diminishes, the balls are nearer, or farther apart, from the centrifugal force imparted to them: thus raising, or depressing, a lever Q, connected with a cock R, in the pipe AA, which comes from the boiler; and, consequently, regulating the quantity of steam admitted into the cylinder, and, by that means, the velocity of the whole engine: thereby producing a steady and uniform motion to all the machinery connected with it.

Example 94. *Page* 221. *Compound Steel-yard.*—On the subject of steel-yards, it may not be altogether foreign to our purpose if

we here mention two or three machines that are in use, for the purpose of weighing goods; and as the steel-yard is calculated to weigh any thing without a multiplicity of weights, these engines may, I think, very properly be mentioned in this place, as they are calculated to perform the same operations as the steel-yard, and have this convenience,—that not any weights are required, but simply shew the weight by an index, or graduated scale, and tell, at sight, the weight of any substance without even the trouble of shifting a weight, as in the common steel-yard. *Fig.* 1, *Pl.* G, represents a machine commonly known by the name of the *spring steel-yard*: AB is a hollow cylinder of iron, or brass, having a cap at the end A, with a square hole in it to admit the rod, or beam, DE, whose end E is formed into a round nut, and end E has a ring by which it may be held in the hand, or suspended from a hook: within the tube AB is a spiral spring, wound round the bar DE; and at the end B is a hook C, to which the weight, or substance to be weighed, is hung. The action of this instrument is as follows: Having suspended a weight from the hook C, lay hold of the ring D, then the nut E pressing on the spring, which is a spiral piece of steel, it causes the bar attached to the ring to draw out; and if we make a mark on the bar, corresponding to the weight hung from C, that point will always indicate the weight of any substance, equally ponderous, that may be suspended from the hook. In this manner, having suspended several weights, and made marks corresponding to them on the bar, we shall have a scale which will always indicate the number of pounds, &c., any body may weigh that is suspended from the hook at C: and we may here observe, that though, for common purposes, it will be found sufficiently accurate, yet it is subject to variations from the different degrees of temperature of the atmosphere, which lengthens the spring, or causes it to be more stiff at one time than at another, and, consequently, the divisions of the scale on the bar will not always be sufficiently accurate where any nicety is required.

The next instrument which I shall describe, and which is similar in its construction, being dependent on the action of a spring, is represented in *Fig.* 2, *Pl.* G, and is generally termed the *dial balance*. AB is a flat bar, as in the last construction; but instead of pressing against a spiral spring in a tube, it exerts its force by its end H (when drawn out by a weight attached to the hook C) on an eliptic or circular spring, GHL, enclosed in a circular box, DE, the end opposite H being fixed to the box. Now, as A is drawn out by the weight W, the two sides of the spring approach each other; then, if there is a rack, on the bar AB working in the wheel F, it will turn another wheel, K, on whose axis a hand, L, is fixed, which, coming through a plate

screwed on the front of the box DE, it will indicate the num of pounds hung on the hook C; and this plate is divided hanging weights of 1, 2, 3, &c. pounds in succession on C, making marks on the plate; then, at any time, we can see, inspection, the weight which is hung on the hook C. T instrument, though, perhaps, superior in many respects the first mentioned, is still subject to many of its defects; the strength of the spring not being at all times uniform, m errors must consequently arise; and however well it may executed, with respect to workmanship or accuracy, in divisions in the first instance, continual use must weaken force, or elasticity, of the spring, and thereby cause the hand point to divisions in the plate which do not perfectly corresp to the weight on the hook.

The last instrument I shall now notice is, what is usu termed the *bent lever balance*. This is represented at *Fig.* 3, G, where FED is a bent lever, supported on an axis at E, fi to a standard, ABC, to which is attached a graduated quadra GB. At D is a scale; and the end F is loaded with a wei that just keeps the scale H in equilibrium when F is at the co mencement of the scale, or division of the arc BG; then, if place a weight in the scale H, the end of the lever F ascen through the arch till it comes to an equilibrium with the scale a the weight in it. Now, if we suppose an horizontal line, I drawn through the line of suspension E; and if from F and we draw the perpendiculars FL and DK; and if the weight the scale H, and the weight at the end F, are reciprocally p portional to the distance KE and EI, we shall always have equilibrium; but as the weight at E, by ascending through arc BG, constantly makes the distance EI greater as it a proaches G, so the weight in the scale H must, of necessity, increased to produce an equilibrium. This form of the balance

particular purposes to which they are most applicable, but as that would be an almost endless task, and as the machine here described is sufficient for the workmen to understand the general principle on which this most useful engine is constructed, I shall content myself with a description of one, the principles of which are somewhat different; and which, though described by various authors, is, or has been, but little attended to by modern mechanics, but which, in many cases, may be used to very good purpose, being, in a great measure, free from that inconvenience all engines are more or less subjected to, that is—friction. I shall therefore give, with some little alteration, the form of the crane, as it is described, by M. Perrault, in rather a scarce work, published many years ago at Paris, under the title of *Recueil de Plusieurs Machines de nouvelle Invention.* *Fig.* 1, *Plate* H, is a perspective representation of the machine: A A a roller, on which is affixed a pulley B; this moves, or rolls, up and down between the parallel pieces OP, OP; the cords C C, are twisted round the roller A A, and affixed to O O, round the pulley B, in another cord E E, which is twisted round the capstern G G, and proceeds also round the roller I, to the hand at R, where it is held. Now, as the capstern is turned round by the levers K K, which capstern slides in a long hole made in the side pieces O O, the mode of its action will be best seen by considering *Fig.* 2, *Pl.* H, where, as there are the same letters of reference, a comparison will make it quite clear. A A is the cylinder, having the pulley B fixed to it; the ropes C C are fastened, as shewn in the *Figure;* the rope D sustains the weight W; the rope E going through F, (within which there is a contrivance hereafter shewn, to hold the rope in any position,) is twisted round the capstern G G, and roller I; the ropes *a a* are fixed to F, and partly wound round G G in the direction shewn, and here fixed; and the same by the rope *b b* fixed to H H, and partly wound round G G, and there fixed. These ropes, it will be seen, are wound round G G in contrary directions; therefore, in turning the lever K towards you, the capstern approaches F, by rolling along the groove between the side pieces.

We will now suppose the machine in action, and refer again to *Fig.* 1, *Pl.* H, and let the pulley B, with the roller A A, down towards R, and the weight W attached to the rope D, and the capstern G G near H H: now, by drawing or pulling the levers K K towards you, A will draw the rope attached to the pulley B, and make it roll upwards towards O O, as it is prevented from descending by the cords C C, which, as B ascends, are wound round A A. During this motion of the pulley the cord D is wound round A, and, consequently, elevated both by that and the pulley ascending along the side O D; also, after the pulley has ascended a little along the sides, the capstern G G is drawn up towards F.

The rope is now held at R, and the capstern turned back towards H, and the weight is prevented descending again by the rope E being secured, by the contrivance within F, as above mentioned. The same operation is repeated, till, at length, the body W is raised to the height required. The whole machine is supported on a perpendicular axis L L going through the braces R and M, and the standard Q, which is firmly fixed to the ground. Now, it will be readily seen, that as the roller A A and capstern G G do not move as an axis, but merely roll along the sides O D, the friction is very inconsiderable.

I shall now describe the contrivance within F, which holds the rope E in any position. In reference to *Fig.* 4, *Pl.* G, F represents a box, in which are affixed two clamps A A moveable about a centre A A; a plate B C connects them together; one part at B being fixed to one clamp, and the other part at C, having a slit or hole, with a peg or pin. The rope G H passes between the ends of these clamps, and is pressed or squeezed by a spring D, which holds it fast; and if we pull at H, the rope will slide between them, but is prevented moving in the direction towards G. There is also a cord E E passing through one clamp, which, by pulling, releases the cord G H when necessary, and which is shewn in *Fig.* 2, *Pl.* H, at SS, but omitted in *Fig.* 1, to avoid confusion.

I have introduced this description, not so much with a view of showing something which has peculiar advantages, but more to give a general idea of a principle which has been little attended to, and as something not generally known amongst mechanics; as I feel persuaded, that, in the hands of many of my readers, this principle may be applied with great advantage to many engines, and to many purposes hitherto not thought of: and as it is one that has the great advantage of removing, almost altogether, that great difficulty of lessening the friction of machinery, it might furnish, at least, some hints which the practical workman will, doubtless, improve on.

Example 102. *Page* 225.—*Clock.*—As the art of clock-making is a subject which, of itself, would fill several volumes to describe their various forms and constructions, I can only, on this head, recommend to my readers some works that will give any information required; and, first, I would recommend for a copious list of authors, both English and Foreign, Gregorie's *Mechanics*, vol. ii., page 140; besides which, I would recommend Martin's *Mechanical Institutions;* Ferguson's *Mechanical Exercises;* the *Encyclopedia Britannica,* and *Supplement;* Chamber's *Cyclopedia;* and Harrison's *Lexicon Technicum,* (as well as the article *Clepsydia* in the same,) and to vol. xxv. of the *Philosophical Transactions;* added to which, the reader will be amply gratified by referring to the Transactions of the Society of

Arts, for modern improvements, with respect to pendulum, escapements, &c.

Example 103. *Page* 230.—*Cutting Engine.*—The instrument here described has been greatly improved in its construction since the time of our author, both in respect to its several adjustments, and with respect to the cutter: but as the principle is still the same, I shall not here particularly describe the improvements, as every workman must, at once, see the defects; but I think it necessary to make some observations with respect to the cutting of teeth in wheels in general, and to the dividing plate of this instrument. In the first place, in any piece of clock-work, having first calculated the number of teeth in each wheel necessary to produce the motion required, it is, also, requisite to know the proportion of the diameters of the several wheels to each other, so that when they are cut into the required number of teeth, they shall work freely together; that is, for instance, having two wheels, in which we are required to cut any particular number of teeth, we must proportion the diameter of these wheels to each other, so that the teeth in each of them may be of the same size, or, that the teeth of one may fit accurately into the spaces of the other: and here I cannot, I think, do better than extract a passage from Mr. Fergusson's *Select Mechanical Exercises*, under the form of the following Proposition, mentioned at page 42, the third edition.

"*Supposing the distance between the centres of two wheels, one of which is to turn the other, be given; and, that the number of teeth in one of these wheels is different to the number of teeth in the other, and it is required to make the diameters of these wheels, in such proportion to one another, as their number of teeth are, so that the teeth in both wheels may be of equal size, and the spaces between them equal, and that either of them may turn the other easily and freely; it is required to find their diameter.*

"Here it is plain that the distance between the centres of the wheels is equal to the sum of both their radii in the working parts of the teeth. Therefore, as the number of teeth in both wheels, taken together, is to the distance between their centres, taken in any kind of measure, as feet, inches, or parts of an inch; so is the number of teeth in either of the wheels to the radius or semi-diameter of that wheel taken, in the like manner, from its centre to the working part of any of its teeth.

"Thus, supposing the two wheels must be of such sizes as to have the distance between their centres five inches; that one wheel is to have 75 teeth, and the other to have 33, and that the sizes of the teeth in both wheels is equal, so that either of them may turn the other; the sum of the teeth in both wheels is 108; therefore, say as $108 : 5 :: 75 : 3{\cdot}47$, and as $108 : 5 :: 33 : 1{\cdot}53$; so that, from the centre of the wheel of 75 teeth to the

working part of any tooth in it, is 3 inches and 47 hundredth parts of an inch; and from the centre of the wheel of 33 teeth to the working part of either of its teeth, is 1 inch and 53 hundredths of an inch."

Now, it frequently happens in practice, that we have the diameter of one wheel given as well as the number of teeth in it, and this wheel is required to turn another of a given number of teeth: hence arises the following Proposition:

Having the number of teeth in each of two wheels, and also the diameter of one wheel, it is required to find the diameter of the other, and, consequently, the distance between the centres of the two wheels.

We will here suppose, as in the last Proposition, that one wheel has 75 teeth, and the other 33 teeth, and that the diameter of the wheel of 75 teeth is 3·47 inches; here, then, by the rule of proportion, say as 75 : 3·47 : 33 : 1·53, the diameter of the wheel of 33 teeth; hence 3·47 + 1·53 = 5 inches, the distance between their centres. Now, with regard to the dividing plate of the engine, it appears that if the number of teeth we wish to cut in any wheel is not laid down in one of the concentric circles, or is not a multiple of any of the numbers there marked, we must have recourse to some other method than there shewn, for cutting the required number of teeth; and this, though not frequently wanted for the generality of clock-work, is sometimes necessary, and, indeed, indispensable, in the construction of orreries or astronomical clocks, and various other machines.

I will, therefore, here shew, how we may divide a circle, with ease, into any number of parts, which will enable us to make fresh points on our dividing plate, or divide the wheel itself, with very little trouble. I shall here remark, that the chief difficulty is in dividing a circle into any *odd* number of parts, as all the divisions on the plate are amply sufficient for the *even* parts. It is plain, that from any number of odd parts we may subtract a number which shall leave the remainder even. Thus, suppose we wish the circle to be divided into 59 parts; if we subtract 9, there will remain 50; therefore, to ascertain on the circle that proportion of its circumference which will contain 9 parts out of the 59, say as 59 parts is to 360 degrees, (the whole circle,) so is 9 parts to 54·9 degrees; therefore, if, by means of a sector, or scale of cords, we make an angle at the centre of the circle equal to 59·9 degrees, we shall enclose a space of the circumference equal to the 9 parts required; and as it is much easier, by trials with a pair of compasses, to divide this space on the circumference into 9 parts than the whole circle into 59 parts, we have an easy way to perform the operation; for having found one of the 9 parts, that distance will go 59 times in the circum-

ference, and, therefore, if holes are punched at that distance in another concentric circle on the dividing plate, it will answer the purposes intended; and thus we have a fresh division of 59 parts, or any multiple of it, as 118, 177, &c.

I would here suggest a method, which, as I have never seen either in actual practice or described in any author, I think would be a material improvement in the con truction of this engine, as it would do away with the necessity of the fixed divisions or holes in the dividing plate, as well as enable us with great accuracy to cut any required number of teeth, either even or odd, in our wheels. Let the edge of the dividing plate be cut into oblique teeth, so as it shall be turned round its centre by means of a screw working against it, in the same manner as many of our astronomical instruments are; and let the screw be such, that any number of turns round its axis shall move the plate once round its centre; for instance, let 300 turns of the screw turn it once round; then, it is evident that one turn of the screw round its axis will move the plate round a 300th part of its circumference: in like manner, any number of turns will move it a proportional part. Thus, we have for even numbers a ready method to divide the circumference into any number of parts, which are a multiple of 300. For instance, suppose we wish to divide the plate, or, which is the same thing, cut 50 teeth in any wheel, we divide 300 by 50; the quotient is 6, which shews that 6 turns of the screw will move the plate forward a space sufficient for one tooth; therefore, we have only for each tooth to turn the screw round six turns to divide the plate into the number of parts required. Now, the method for dividing the plate or, which is the same thing, to cut any number of odd teeth in the wheel, is analogous to this. But it will be necessary to have the head of the screw divided into a number of equal parts, say 100, and an index so fixed, that we shall be able to ascertain the number of turns, as well as fractional parts of a turn, we wish to give to the screw, in order to move the plate forward a space equal to the distance of the divisions we wish, in the dividing plate. Thus, suppose we wish to divide the plate into 56 equal parts, or cut 56 teeth in our wheel, we divide 300 by 56, the quotient is 5·35 nearly, which shews that for each tooth we must turn the screw 3 times round, and 35 parts of the hundred divisions into which the head is divided. By this method, any odd number of teeth may be cut; and to any Mechanic at all acquainted with the nature of the method of dividing a circle by the methods used on the arcs of mathematical instruments, this will be as simple as possible; and, if necessary, a stop might be so contrived to the screw, that, after it was once adjusted to the required number of turns, it would be only necessary to turn the screw for each

tooth, till it came to the required position; and thus proceed for every tooth round the circumference of the wheel.

I have not thought it necessary to illustrate this by an engraving, as, from what I have advanced, it will, I trust, be perfectly sufficient as a hint for the improvement of the instrument under consideration.

Example 105. *Page* 233.—The water-mill here described by our author, though ingenious in its construction, will be found, in actual practice, attended with many inconveniences. And first, as the weight of the spindle with its spiral leaf must, of necessity, be considerable, it follows, that the friction on the pivot at its lower extremity will be such as to occasion the want of very frequent repair, particularly if we add to the weight of the mill itself the weight of the water with which it is loaded. Another practical inconvenience is, that it will be found very difficult to make the spiral wheel work within the cylinder without considerable loss of water, and, consequently, a waste of power; and without taking into account the actual expense of the construction of the machine itself, when compared with its power and effects, it will be found, that the advantages its simplicity offers are more than counterbalanced by its defects; but that the principle of the spiral water-wheel may, under many circumstances, be of benefit, and worthy the consideration of the practical Mechanic, is not to be doubted, though it has hitherto not been applied with any material advantage.

I shall, in this place, describe another water-mill upon the principle of the pressure of water when confined in pipes, and flowing out at holes in their sides, thereby producing a centrifugal motion capable of turning a mill-stone, or moving any other machinery. This instrument, in its original form, is known under the title of *Barker's* or *Parent's-mill*, and improved by Mr. Rumsey. A very particular description of it, with the calculations necessary to estimate its powers, will be found in Gregory's *Mechanics*, page 106, vol. ii.

I shall here describe it with its improvements, and refer the reader for its original form to the above work, and also to Desagulier's *Experimental Philosophy*, vol. iii., page 459, or Ferguson's *Lectures* (Brewster's Edition, vol. ii., page 103.)

Fig. 1. *Pl.* I. represents the mill with Mr. Rumsey's improvement; A and B are the two mill-stones resting on the bed C; from the upper stone, A, proceeds an axis, or spindle, F, which is attached, and turns with it; the end of the spindle is fixed to a hollow cylinder G H, having a hole at G and H, on the opposite sides of it; this cylinder has a communication with the pipe IKK, the part of the pipe above I being fixed to the cylinder, and having a joint at I on which it turns, when the mill is in motion;

the dotted lines within the pipe, at I, represent a continuation of the spindle F, which works on a pivot within the pipe; the joint at I is made water-tight; at L is a reservoir of water, the contents of which descending along the pipe K ascend through I, into the cylinder GH, and are forced out at the holes G and H, which, acting on contrary sides of the cylinder, make it revolve, and, consequently, carry with it the spindle F and stone A; the whole is supported by the frame DE: there is, also, a contrivance to augment the orifice of the holes at G and H, by which means, the velocity of the mill is regulated.

It is evident, that the power of this engine depends, in a great measure, on the height of the reservoir L, and quantity of water flowing through the pipe K: hence, this machine is well calculated for those situations where the fall is great, and the quantity of water but small; it must be here confessed, that, however simple the action of this mill is, still the theory of its action is connected with some calculations somewhat abstruse in their nature, and which want of room will not enable us to do more than state, and refer for the calculation itself to the works above named. The several data required are, principally, the following:—*First*, the magnitude of the pipe which conveys the water from the reservoir: *Second*, the force with which the machine commences its motion: *Third*, the quantity of the centrifugal force in the hollow cylinder: *Fourth*, the inertia of the water which acts in opposition to the centrifugal force: *Fifth*, the acquired velocity of the water on issuing from the cylinder: *Sixth*, the ratio of the central force to the inertia; added to which, it is requisite to adjust the several pipes and cylinders, as well as the area of the apertures in the cylinders, so that a maximum of effect may be produced.

Example 106. *Page* 234.—As the subject of arches is one of considerable interest, and has, since the time of our author, Mr. Emerson, engaged the attention of men of science, I shall, (our pages not allowing us to enter minutely into the different opinions of various authors,) in the first place, give a list of some works that will amply gratify the curious on this point, and add another Table for the construction of any arch from any given extrados, or line of carriage-way across the bridge; and, first, the works I would recommend for perusal on this subject are,—Hutton's *Principles of Bridges*; Bossuet's *Recherches sur L'Equilibre des Voutes, Memoir de l'Acad. des Sciences*, 1774 and 1776; Bossuet's *Mechanique*, edition of 1802, page 383; Prony, *Arch-Hydraul.* tom. 1; Atwood's *Treatise on the Construction and Properties of Arches*; Gregory's *Mechanics*, vol. 1, page 130, and following; as well as *The Encyclopedia Britannica*, and *Supplement*, articles Bridge and Arch; and Hutton's and Barlow's *Mathematical*

Dictionary; also, Martin's *Circle of the Mechanical Arts* contain much on the subject selected from various authors, under the article Bridges.

With respect to the Table alluded to, it is calculated by Dr. Hutton, from a theorem on the Arch of Equilibration, that the span of the arch is 100, its height 40, and the thickness at the crown 6, and which will answer for any other arch whose span, height, and thickness, are to each other in like proportion. Now, we will suppose the extrados, or carriage-way, to be a straight line parallel to the horizon; and let a perpendicular line (to the horizon) be drawn through the crown of the arch, and lines drawn parallel to the horizon, and, of course, perpendicular to this line; then, if we consider the first line as the axis of the curve, the division of it, where the other line crosses, will be abscissas to the curve, and the lines themselves will represent the ordinates; then, if the extrados is an horizontal line, or nearly so, if we call the abscissa A, and the ordinate corresponding to it O, we shall, by the means of the following table, be able to construct the curve. And here we may remark, that we must measure the abscissa not from the vertex of the curve, but from the surface of the extrados, or carriage-way, as the number 6 is always added to the value of the abscissa.

O	A	O	A	O	A
0	6·000	21	10·381	36	21·714
2	6·035	22	10·858	37	22·948
4	6·144	23	11·368	38	24·190
6	6·324	24	11·911	39	25·505
8	6·580	25	12·489	40	26·894
10	6·914	26	13·106	41	28·364
12	7·330	27	13·761	42	29·919
13	7·571	28	14·457	43	31·563
14	7·834	29	15.196	44	33·299
15	8·120	30	15·980	45	35·135
16	8·430	31	16·811	46	37·075
17	8·766	32	17·639	47	39·126
18	9·168	33	18·627	48	41·293
19	9·517	34	19·617	49	43·581
20	9·934	35	20·665	50	46·000

When the extrados differs materially from a straight line, this Table will not answer; but as, in general, it is best to make the carriage-way nearly, if not quite, level, the Table will be found, in actual practice, very convenient.

Example 107. *Page* 238.—As the description here given of the *weighing engine* is attended with many inconveniences, both with respect to the delay occasioned by fixing the chains, and the damage likely to accrue from lifting the waggon by means of chains, as well as the general inconvenience from the clumsy form of the engine itself; the engine, as here described, has been superseded by one of much more convenient structure, and which I shall here shew. Let ABCD (*Fig.* 2. *Pl.* I.) represent a strong frame, or box, about twelve or eighteen inches deep, with a solid and thick bottom of oak; the whole sunk into the ground and firmly bedded, so that the edge of the box shall be level with the surface of the road over which the carriage is to pass. There is an opening in the side of the box in AB, to admit of a strong iron lever, or bar, EF; nearly at the end F is fixed a fulcrum pin, formed like the nail of a balance, having a sharp edge downwards, and resting on two circular arches of hardened steel. A little further from the end, and in the centre of the box, is another fulcrum pin, whose sharp edge is upwards: the two strong iron bars, G*d*G and H*d*H, resting on it at *d* and *d*; in the underside of these bars are four strong obtuse angled cones near the ends GG and HH, which rest on four strong pieces of iron, having a very hard steel cup to receive the cones on the top of these bars; and rather nearer the centre of the box are also four conical pieces of steel, as *a*, *a*, *b*, *b*; on the cover of the box are four similar strong pieces of iron, with steel cups, to correspond and rest on the cones *a*, *a*, *b*, *b*; the end of the bar or lever EF projects from the box, and is usually enclosed in the weighing house: it has a pin at *e*, *e*, which is attached to a steel-yard, or a pair of scales, by which we ascertain the weight on the platform or lid of the box on which the carriage or waggon rests. *Fig.* 3. *Pl.* I. is a section of the box, with its bars, or levers, and cones. LM represents the platform, or top of the box; *n*, *m*, two studs or blocks fixed to it with hollow steel faces, resting on the obtuse cones *a*, *b*, which are fixed to the bars GH; the cones *o*, *p*, fixed near the extremities of the bars, rest on the pieces *x*, *y*, with hollow steel faces, and fixed firmly to the bottom of the box IK. The other ends of the bars GH rest on the pin *d*, *d*, going through the lever F. The ends of the bars GH are generally bent, so that the points of support *a*, *o*, *d*, and *d*, *p*, *b*, are in the same horizontal plane, as well as the sharp edge of the pin CC, *fig*. 2. The machine thus constructed is evidently nothing but a combination of levers in the nature of a compound steel-yard, and, therefore, this machine is well adapted to weigh heavy weights, as waggons, &c. loaded with goods, &c. The studs at *x* and *y* have generally a little rim raised round their edge, in which some oil is poured, that the damp and dust, which unavoidably enters between the lid or platform,

may not injure by rust, or otherwise, the accuracy of the instrument. It is usual, in some modern instruments, to attach the end of the lever E to a bar, or chain, connected with wheel work, or a spring steel-yard similar to that shewn in *Fig.* 2. *Pl.* G, which enables the person at the weigh-bridge to ascertain, without trouble, the number of stones, pounds, &c. the carriage is above the weight allowed by Act of Parliament, and thus readily to fix the required toll.

Upon much the same principle as this, is constructed the common weighing machine, used for the purposes of commerce.

At page 264 of these Notes, under the observations on *mechanical powers,* I mentioned the funicular machine, as sometimes included amongst the simple mechanical powers; I shall, therefore, give some description, accompanied with propositions, regarding its effects in overcoming resistances.

This machine is thus noticed in Barlow's *Mathematical Dictionary.* "FUNICULAR MACHINE (from *funiculus,* a *rope*), is a term used to denote an assemblage of cords, by means of which two or many powers, sustain one or many weights. This is classed, by some authors, amongst the simple mechanical powers, and is the simplest of them all." He then goes on to shew, that, in order to find the equilibrium of forces, we must make use of the method of composition of forces to reduce the several powers acting on the cord, to a single power which acts in the direction of the cord, and refers to the article in Gregory's *Mechanics;* but still, to the mechanical reader, the funicular machine is not itself described, nor the uses to which it is applied. The machine, then, in its most simple form, is merely a rope, or cord, having one end fixed immoveable, and the other fastened to the weight to be moved; the power is any force applied to the cord between the weight and the fixed end of it, which endeavours to draw the cord out of the straight line joining these points. In the investigation of the power of this machine, we must suppose the cord to be without weight, perfectly flexible, and of no thickness, but merely an imaginary line. Thus, every rope in a ship is a funicular machine, and the method used to belay a tack, or that of fastening the cords round a loaded waggon, is but the action of the funicular power, if the term may be used. I shall now lay down a few propositions to illustrate the theoretical computation of the funicular machine; and for those who wish more on the subject, I would refer them to a work entitled *Traite Elementaire de Statique, par Gaspard Monge,* pp. 101 to 114, the second edidition, and is a work which may be consulted by the Mechanic (who is acquainted with the French language) with great advantage, as the demonstrations are particularly plain, and easy to be comprehended.

PROPOSITION I.

Let ABC (*Fig.* 1. *Pl.* K.) be a cord, held at A and B by any force; and let another cord, as BD, attach to it by a sliding knot or ring at B; then, if a force is applied at D in any direction, the ring at B will slide along the cord ABC till it rests in a point where the forces at A, D, and C are a just balance for each other; or, which is the same thing, the tension of the parts AB, BC, and BD, of the cords will be equal, or the different forces acting at ADC will be in equilibrio; and this will always happen when the angle ABC is bisected by a line drawn in the direction of DB, as BE; that is, when the angle ABE is equal the angle CBE, the forces at A, D, and C are in equilibrio.

Demonstration.—Suppose, in the figure, that the cords AB, DB, and CB are in a state of equilibrium. Let us resolve the two forces AB and DB into one equivalent force; thus, as the tension on the cord AB is equal to the tension on the cord DB, set any distances *a*B, *b*B, equal to each other, to represent these equal tensions, and complete the parallelogram *a*C, *b*B; then, from the principle of the resolution of forces, the diagonal *a*, *b*, will represent the effect of the joint forces *a*B and *b*B; in the same manner, the diagonal *a*, *b*, will represent the joint forces of *b*B and *d*B, by drawing the parallelogram B*deb*; again, in the same manner, draw the parallelogram *a*E*d*B, and its diagonal EB will represent the joint forces of *a*B and *d*B. Now, as the tension of all the cords, to make the equilibrium, are equal, the sides of the several parallelograms are made equal; it is, therefore, evident, that the angles ABD and CBD are equal, but the angle ABC is also divided into two equal parts by the diagonal EB; therefore, the angle EB*a* is equal the angle EBD; hence, angle EB*a* + angle ABD, equal angle EB*d* + angle CBD, or EBD are in one right line, and, of course, DB continued to E divides the angle ABC into two equal parts; consequently, when the forces ADC are in equilibrio, the direction of DB will be such, that being produced to E, the angle EBA must be equal to the angle EBC, which was to be demonstrated.

Cor. 1. Hence, it is evident, that as the cord DB is at liberty to move along ABC by means of the ring at B, if any force is applied at D, the ring will slide on the other cord, till the direction of DB is such, that the angle ABC will be bisected by DB produced.

Cor. 2. If DB is fixed to any point B in the cord ABC, and if the line of its direction does not bisect the angle ABC, the forces exerted, the forces will not be in equilibrium; hence, by varying the direction of BD, we may exert more or less force on the points A and C.

Cor. 3. The locus of the point B will always be in the circumference of an elipse, whose foci are at A and C, and whose largest or transverse diameter is the whole length of the cord ABC.

Cor. 4. The three forces at A, D, and C, when they are in equilibrio, will be in the same proportion to each other as the sides of the triangle E*a*B, that is A : C : D :: *a* B : *a*E : EB.

Cor. 5. As the sides if triangles are in the same proportion to each other as the sines of their opposite angles, we shall have the following proportions with respect to the powers A, D, C, to each other, viz. AB : BC : BD : : sine∠ *a*EB : sine∠ EB*a* : sine∠ E*a*B, and in the proportion of their supplements ABD, CBD, and ABC.

Proposition II.

If a cord is suspended from two fixed points, and another cord is fixed to it in any point, the force applied to that point to produce an equilibrium, or make the tension of all the cords equal, will be in proportion to the angles formed by the meeting of the cords in the point where they are attached to each other.

Let ABC be a cord attached to the point A and C, and let another cord BD be firmly fixed to the point B; I say the force applied at B must be (to produce an equilibrium) in the same proportion that the angle ABC has to the angle CBD, added to the angle ABD, or, more properly, the angle ABC is reciprocally as the sines of the angles ABD and CBD, that is, the greater the angle ABC is, the less the sine of the angle ABD and CBD. This is evident from Cor. 5. of the last Proposition; for these angles are in the same proportion as the forces, or as the angles EAB, ABE, and AEB, which was to be shewn.

Cor. 1. Hence, also, as EB represents the tension of BD, AE the tension of BC, and EC the tension of AB; we see that a small force at B (when the angle ABC is great) will be sufficient to keep in equilibrio a great force at A and C; and hence the power of the funicular machine.

Cor. 2. Hence, also, as every rope must have some weight, it is impossible, unless the power at A and C are infinite, to keep it stretched in a right line; and, hence, a very small power exerted on the rope ABC will be able to overcome a great resistance at its extremities, when it is nearly laying even between the points A and C, whether those points are perpendicular, parallel, or oblique to the horizon: for as AE, EC, and EB represent the forces when in equilibrio, EB may be very small in comparison to AE or EC.

Cor. 3. The same reasoning holds good, if two or more powers

are applied to the rope ABC; for we may revolve them all into one power, by the method of the *revolution of forces*, and then consider them as one single force.

Cor. 4. Hence, it is evident, that if the rope ABC is slackened, so that the angle ABC becomes very small, or, that the diagonal EB of the parallelogram AECB is greater than AE, no power will be gained by applying a force to the point B; and, therefore, the less the diagonal EB, the greater the power of the machine.

Cor. 5. Hence, it is evident, that this machine is calculated to raise very heavy weights but a small distance; for, by exerting any force at B, we increase the angle EAB, and consequently the power is lessened, till another equilibrium is obtained in the position of the ropes.

Note.—These two Propositions are sufficient to explain the theory of the *Funicular Machine*. I shall now endeavour to shew its practical utility, by an idea of a machine, which, for simplicity and comparative power, will, in the absence of more powerful and complicated engines, be found of great use to raise heavy bodies for small distances; and the reader will observe, that it is by this mechanical power, that on shipboard ropes are hauled taut, or packages are corded tight.

Let AB (*Fig.* 2. *Pl.* K) be two posts, firmly fixed in the ground, &c. and let a cord proceed from C (where it is held by the hand) through two holes at C and D, and proceed to E, where it is fixed to the body W we wish to move, either by drawing on the ground, or up an inclined plane G. The action of the machine is this: the man at C gives the rope a turn or two round the post A, to prevent it being drawn out of the hole; then another man (or more) lay hold of the rope at O near the centre, and, pulling it in the direction OP, causes the weight W to move, when it is pulled so far, that the power at O is in an equilibrium with the weight or resistance; if the weight is sustained from rolling back, the man at C again looses the hitch round the post, and pulls the rope tight; he then twists it round the post again; thus the rope being again stretched, the power is once more exerted at O, and so on, till the weight is moved as required.

Fig. 4. *Pl.* K. represents another modification of the same engine, when the weight is required to be lifted from off the ground: but in this case, as the direction of the rope is changed from an horizontal to a vertical position, we must have recourse to the pullies at *a*, *b*, *c*, *d*, as the figure shews; the rope ABPD is fixed to the weight at W, and is then twisted round the post at E: there must, in this method of applying the furnicular power, be some contrivance to keep the rope in the position to which it is drawn by the power at P, or, which is the same thing, to prevent the weight descending after it is raised by the power at P: or, otherwise, by

slackening it at D, in order to take a fresh purchase round the p
at E, the weight would descend to its former position. This con
vance may be somewhat similar to that described at page 308 in
Appendix, and may be placed between the pullies *b* and *c*. T
this machine acts by alternately tightening the rope, and apply
the power; and we may here observe, that the farther the p
AB, (*Fig*. 3.) or the pullies *cd* (*Fig*. 4.) are apart, the greater
power.

Perhaps, the greatest advantage this machine possesses is, t
the combined efforts of many individuals may be applied by me
of a rope having a loop or ring sliding on the point P; or, if
consider it a simple mechanical power, the combined efforts
many may be used much better than in that of any other sim
mechanical power; though, at the same time, it is wanting in m
of the conveniences of these. I have here, therefore, noticed
more with a view of drawing the attention of the Mechanic, t
that of shewing any advantages it possesses over other powers;
the principle may, without doubt, in many cases, be applied w
considerable convenience; and as it is so very simple, it may, w
combined with others, be of great utility, particularly when m
complicated engines are not at hand, and we have the exertio
several individuals whose muscular strength we wish to apply w
the greatest advantage.

As these Notes and Observations have already extended t
length that I had not contemplated, and as the subject of mech
cal engines is of itself an inexhaustible mine, rich with preci
metal, and worthy the labour of extracting, I think I cannot
better than recommend to my readers to consider, with at
tion, what has already been attained; and to remember, that h
ever severe the toil is, or difficult the surface to penetrate, we s
be amply rewarded by the possession of the treasure of knowle

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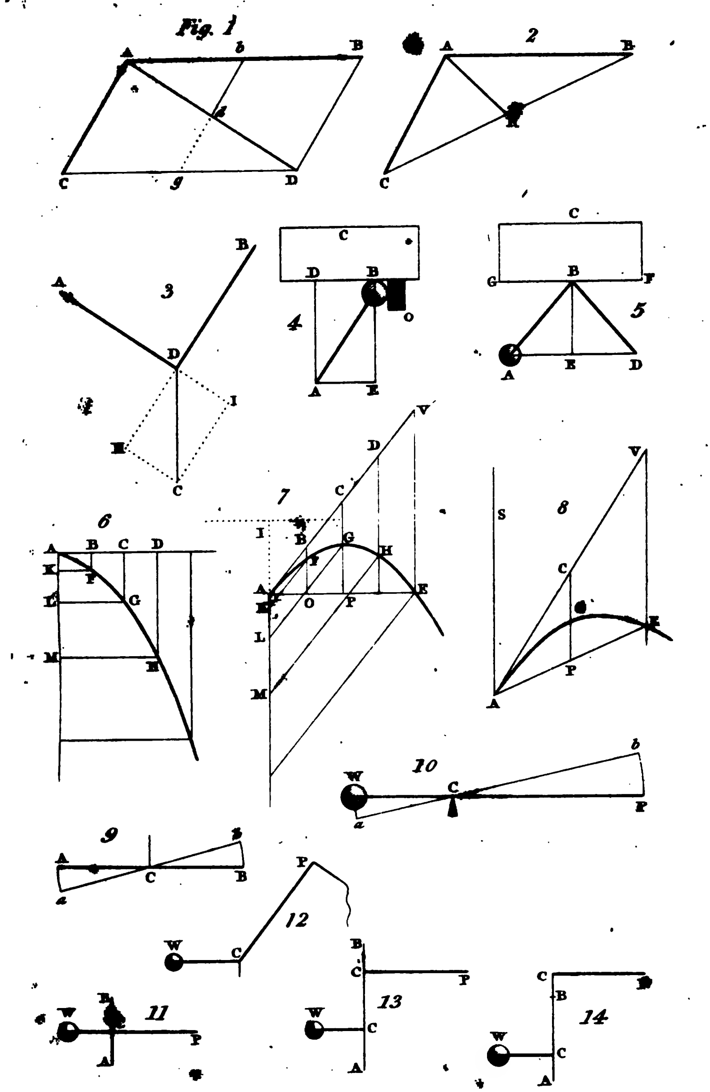
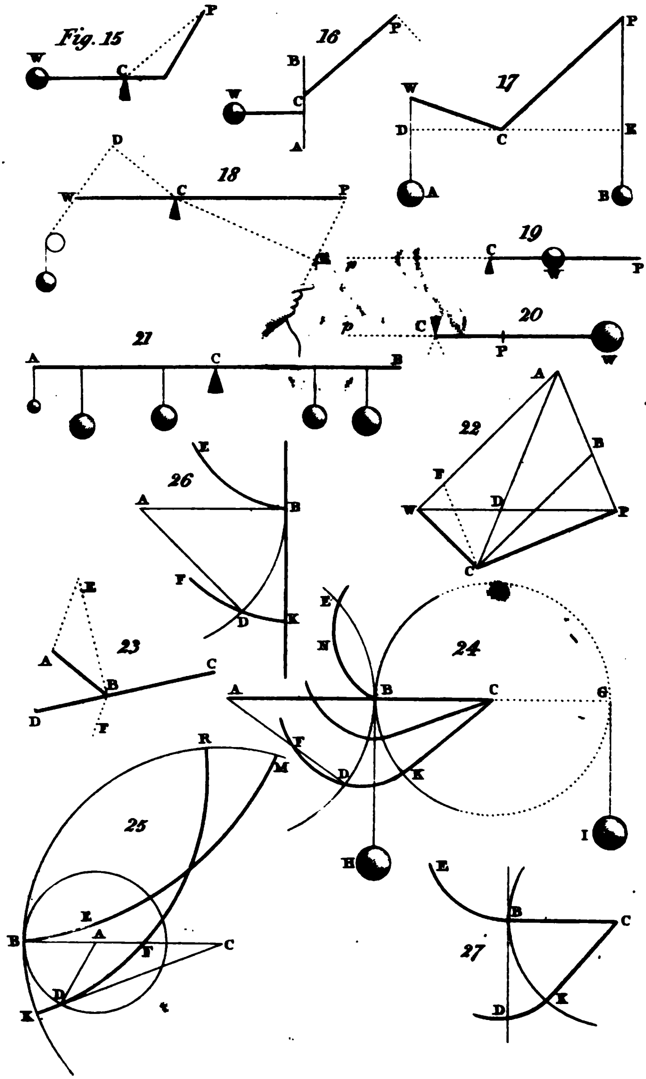


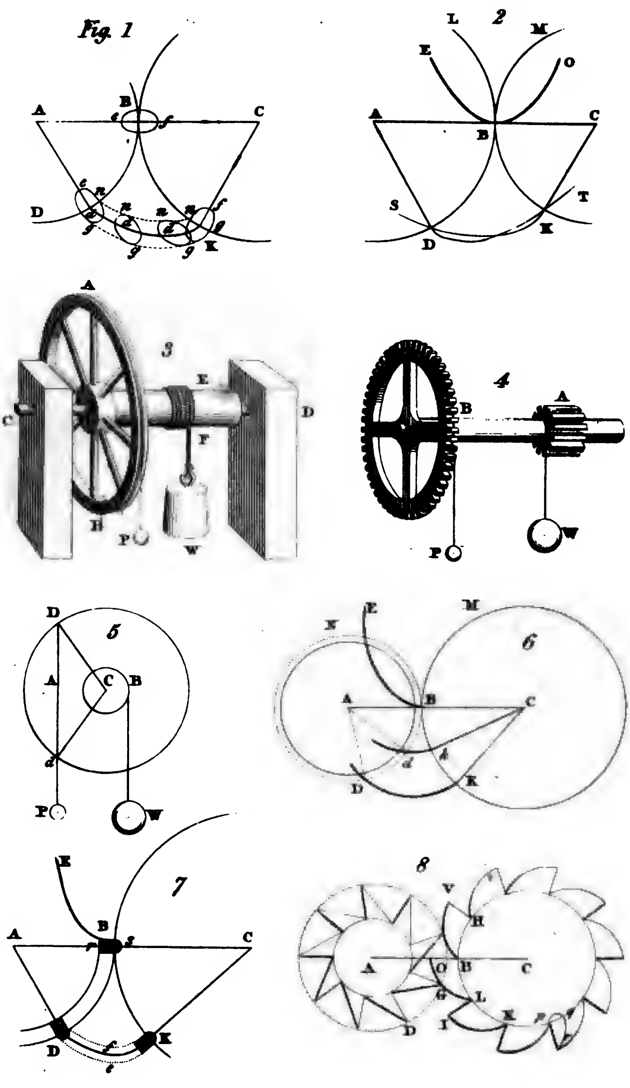
Fig. 1
A b B C g D
2
A B C
3
A B D C I
4
C D B O A E
5
C G B F A E D
6
A B C D K F L G M
7
V D C I B G H A O P F L M
8
V S C P A
9
A C B a b
10
W C a b P
11
W B P A
12
P W C
13
B C P W A
14
C B W C A

Pl. 1.

P. R. H.



Pl. 2.



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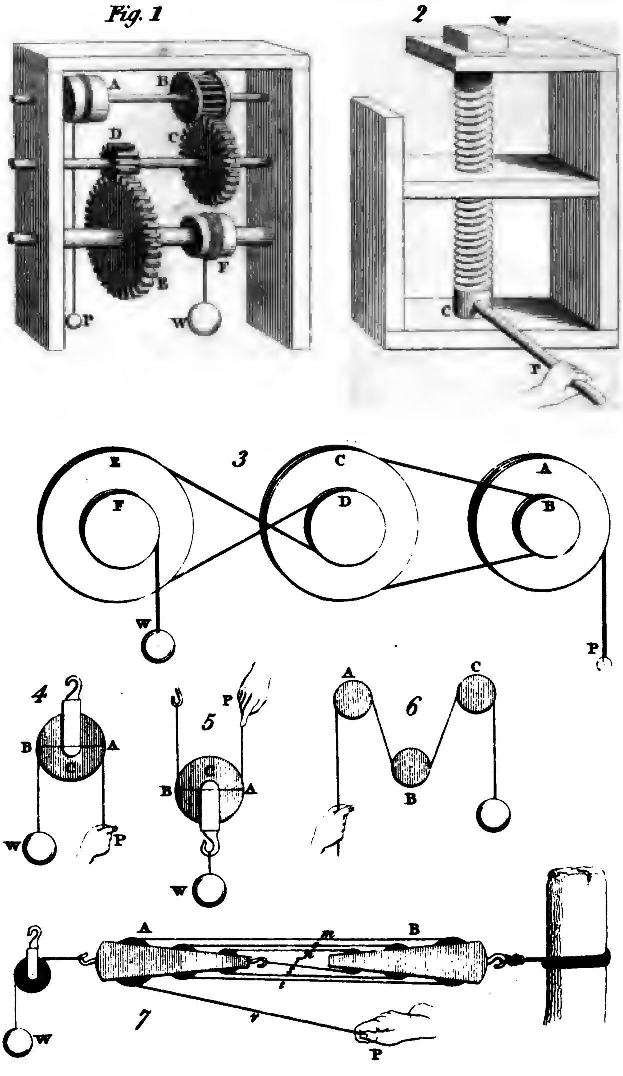
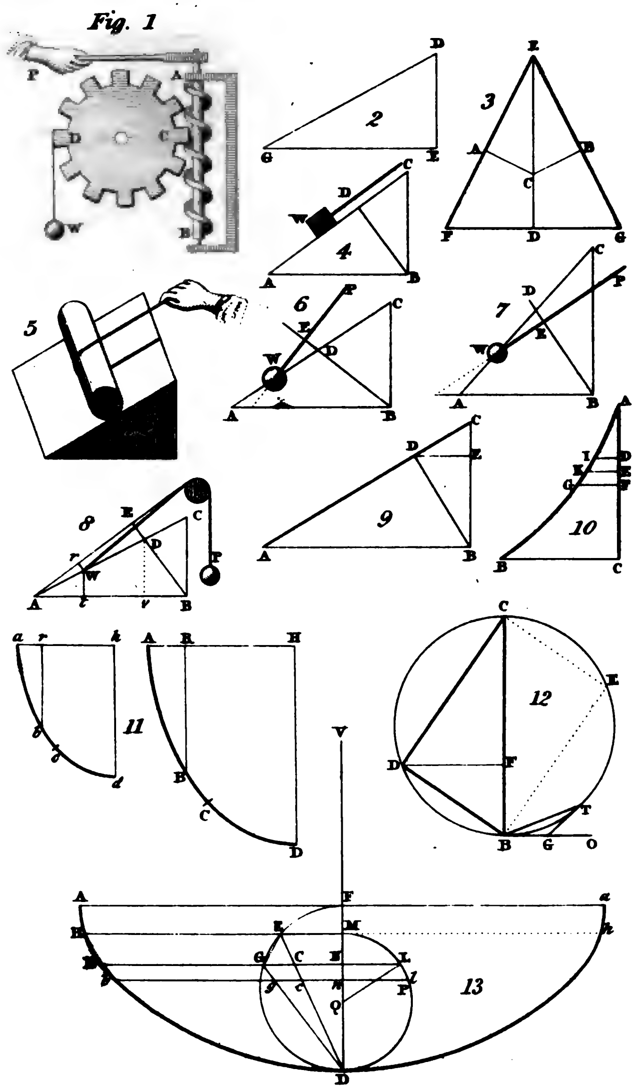


Pl. 4.



Pl. 5

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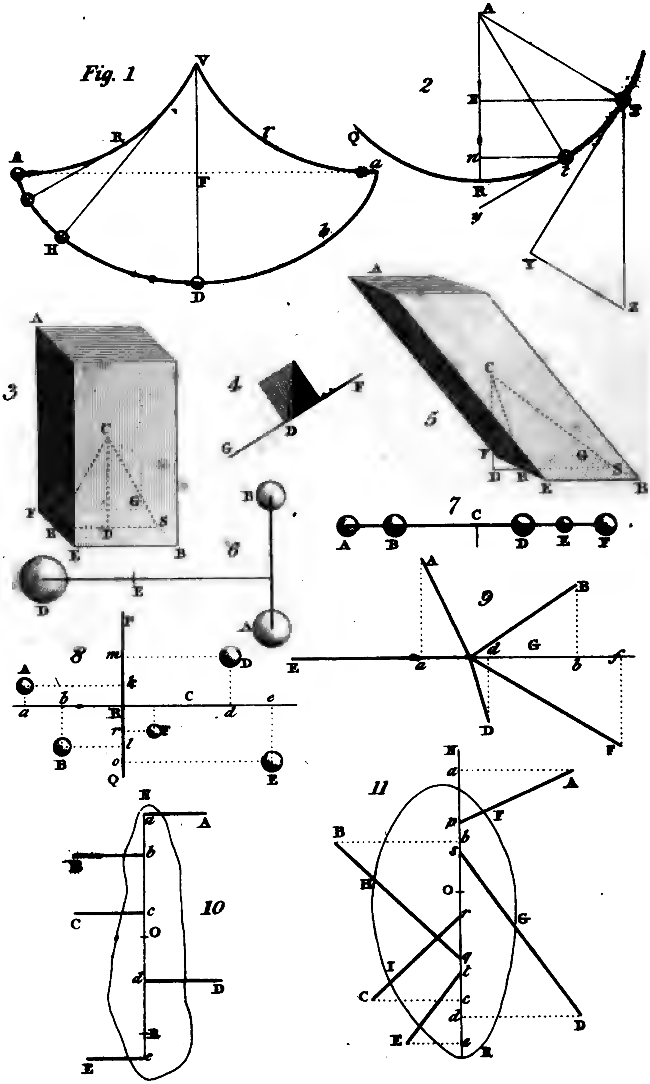

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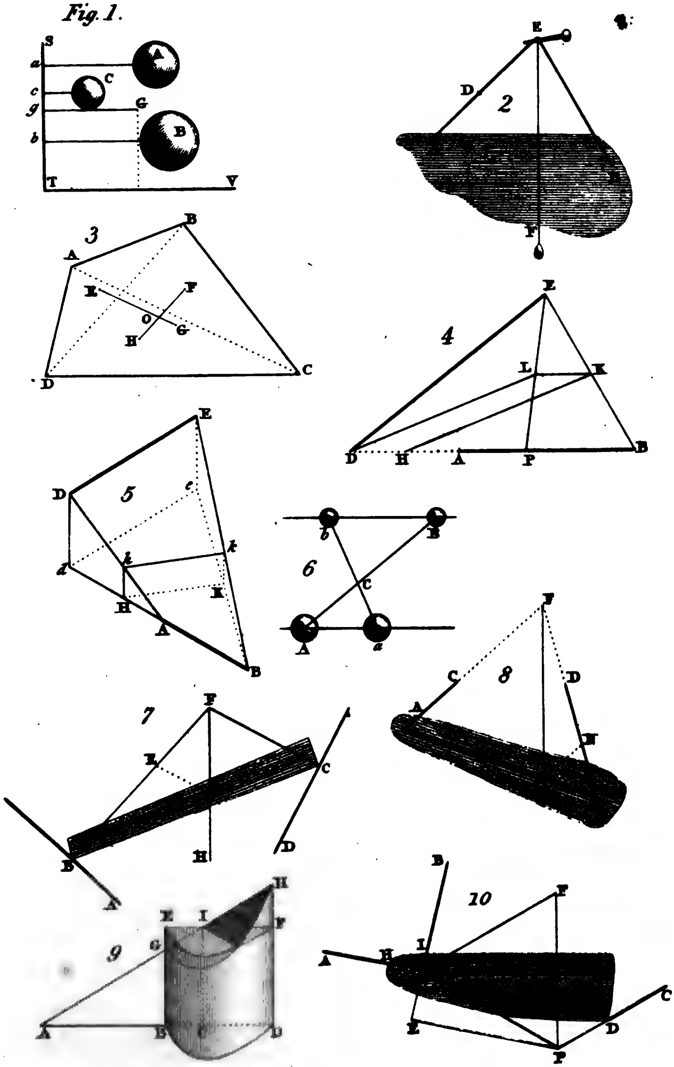

Fig. 1.
S
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F
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G
H
D
C
4
E
L
K
D
H
A
P
B
5
E
D
e
k
d
k
K
H
A
B
6
b
B
C
A
a
7
F
E
C
B
H
D
A
8
F
C
D
A
E
9
E
I
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10
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F
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A
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E
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C
P

Pl. 7.

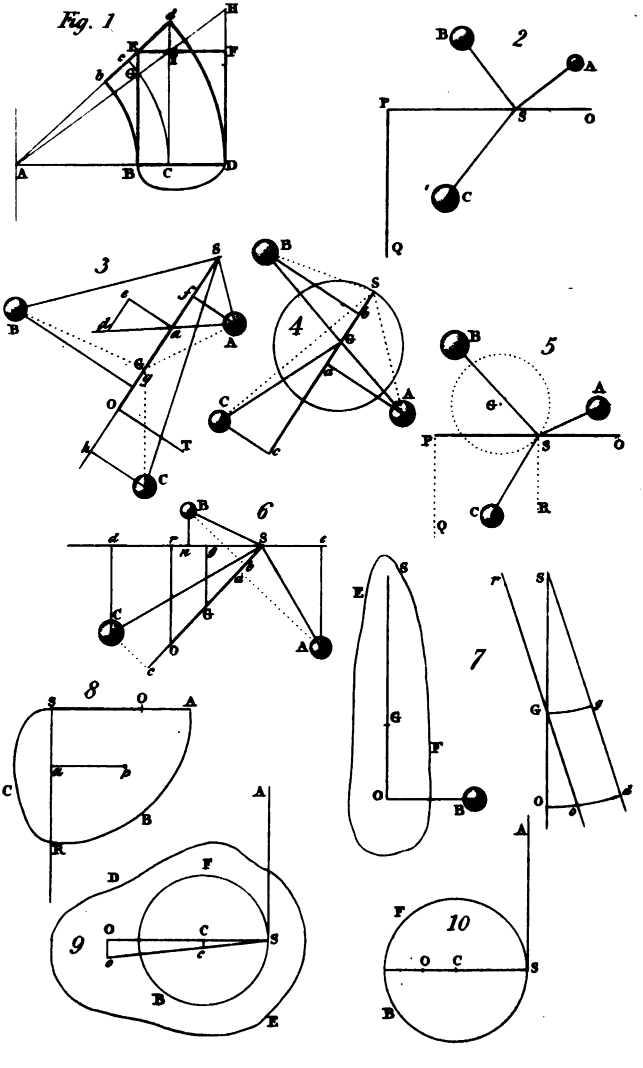
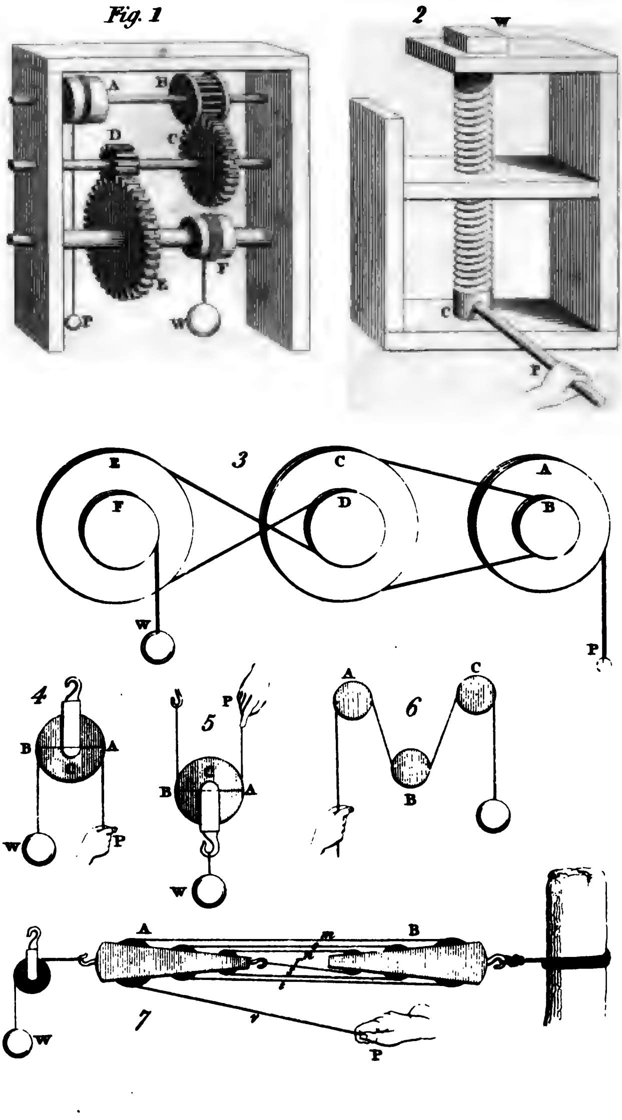


Fig. 1
A B C D E F G H I
b c d
2
A B C P Q S O
3
S B A C G O T
a d e f g k
4
B S A C G
a b c
5
B A C P Q S O R G
6
B S A C G O
d r n e a b c
7
S E F G O B
r g o
8
S O A C B R
a p
9
A D F O C S B E
c o
10
F A O C S B

Pl. 8.



Pl. 4.

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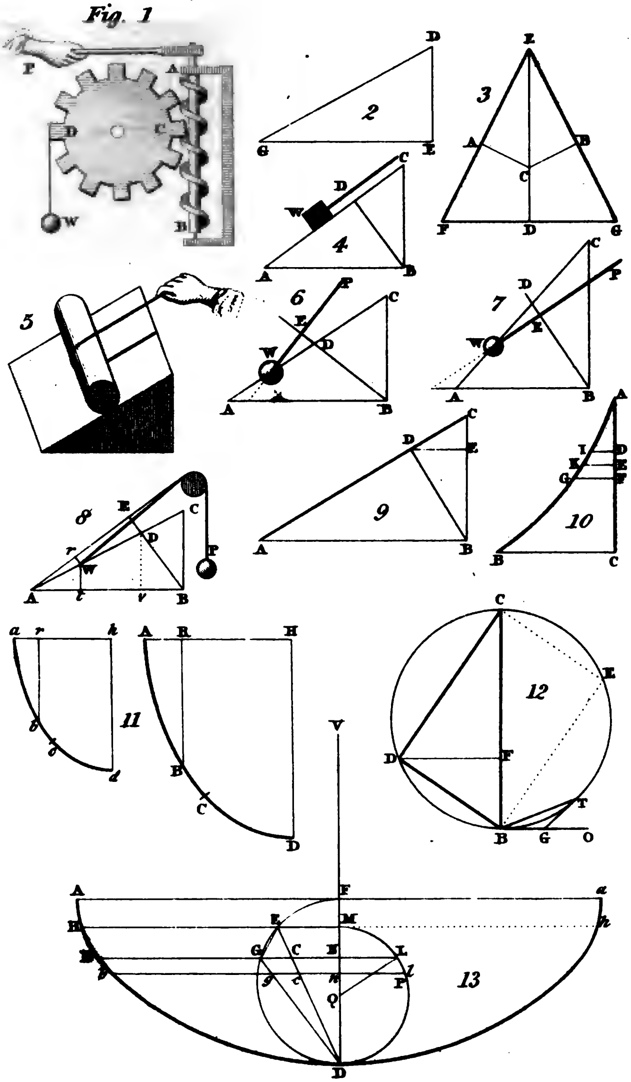
Fig. 1
F
A
D
C
W
B
2
D
G
E
3
A
B
C
F
D
G
4
C
D
W
A
B
5
6
P
C
E
D
W
A
B
7
C
D
P
E
W
A
B
8
E
C
D
P
r
W
A
t
v
B
9
C
D
E
A
B
10
A
I
D
K
E
G
F
B
C
11
a
r
h
b
c
d
A
R
H
B
C
D
12
C
E
D
F
T
B
G
O
V
13
A
F
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M
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Q
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Pl. 5

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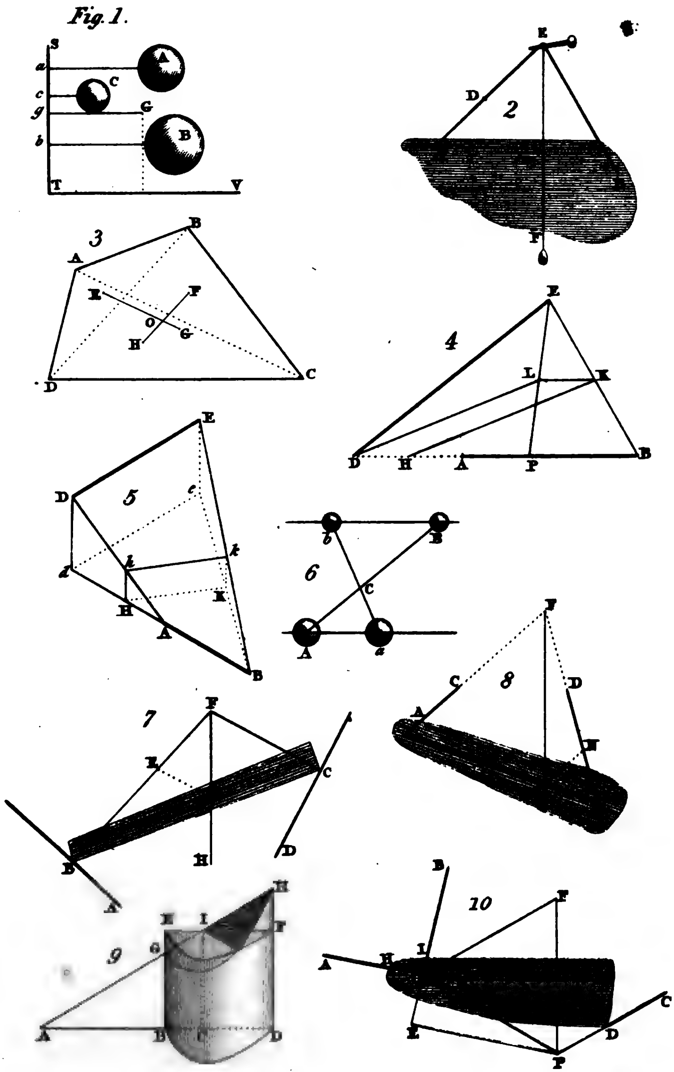

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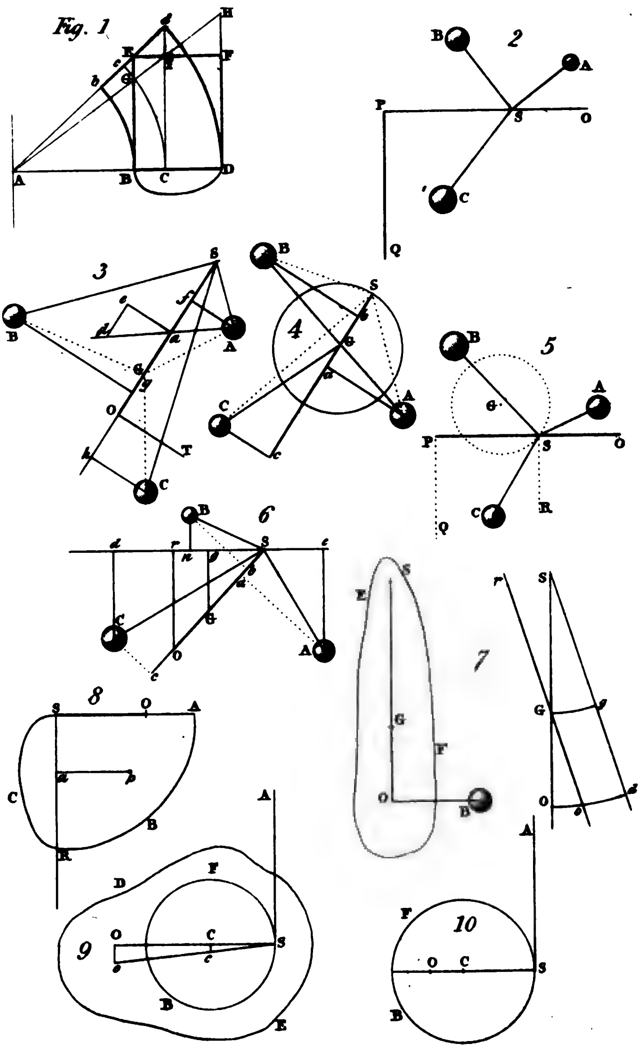

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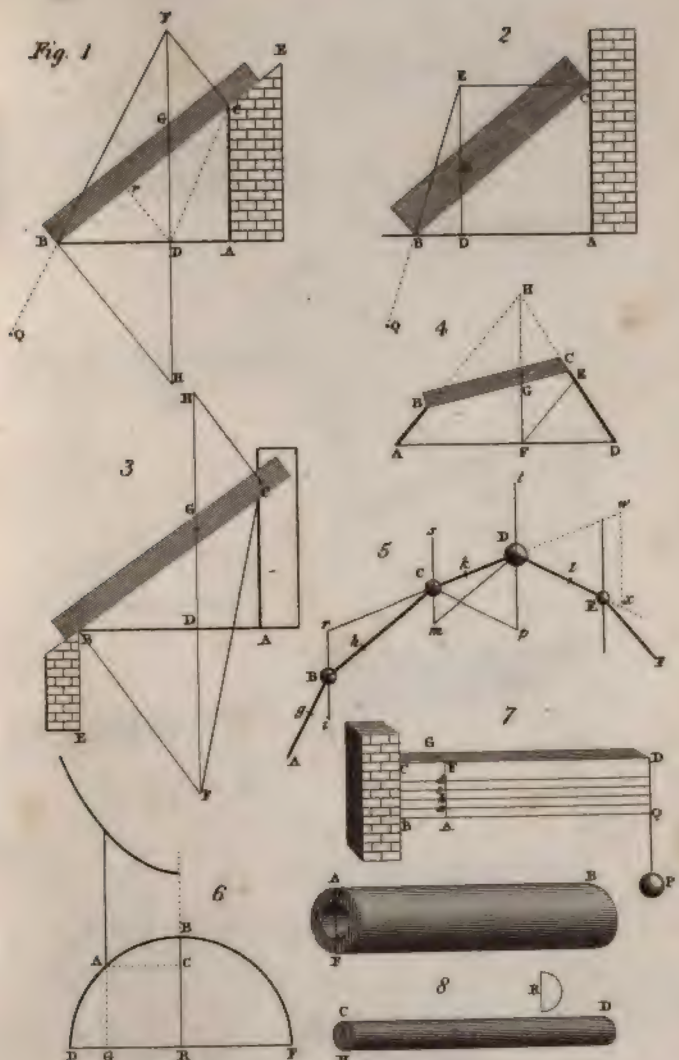


Pl. 9

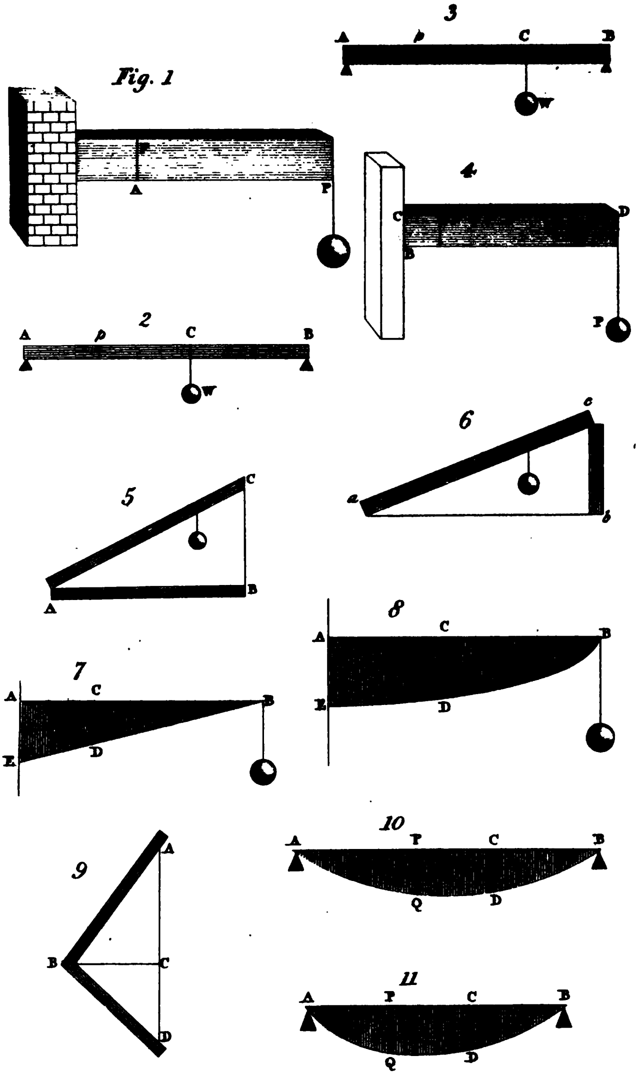
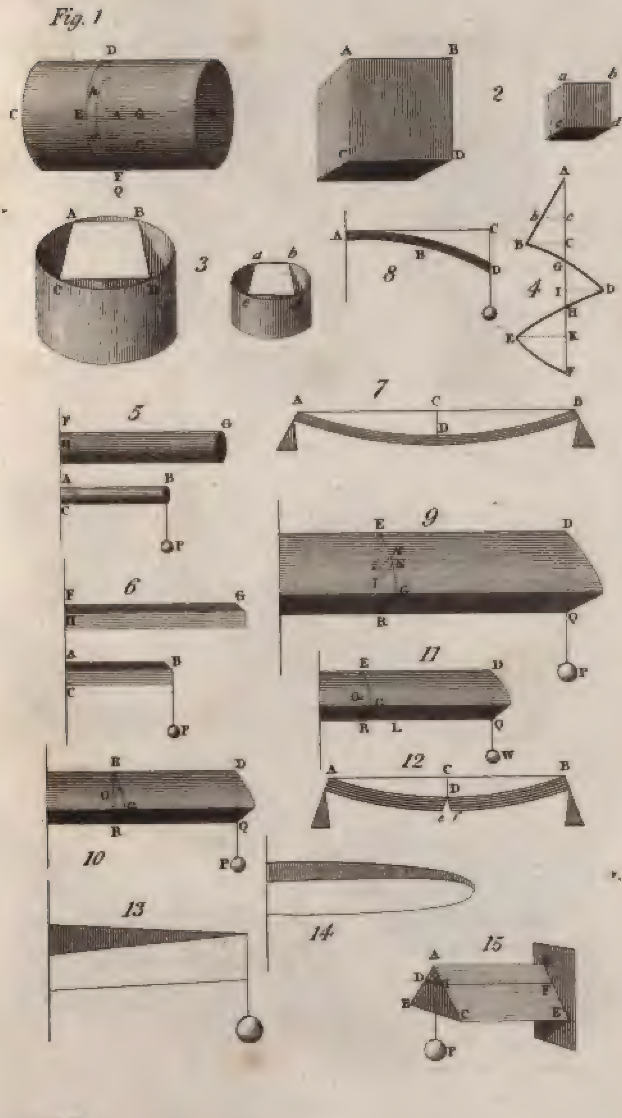
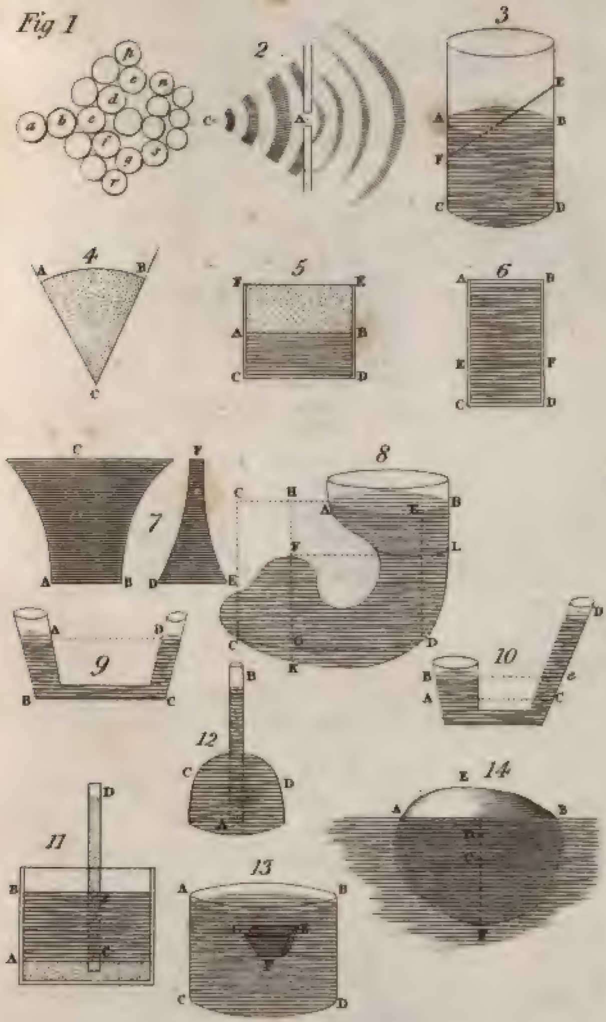


Fig. 1
A
P
2
A p C B
W
3
A p C B
W
4
C
B
D
P
5
A B C
6
a b c
7
A C B
E D
8
A C B
E D
9
A B C D
10
A P C B
Q D
11
A P C B
Q D

Pl. 10.



Pl. II.



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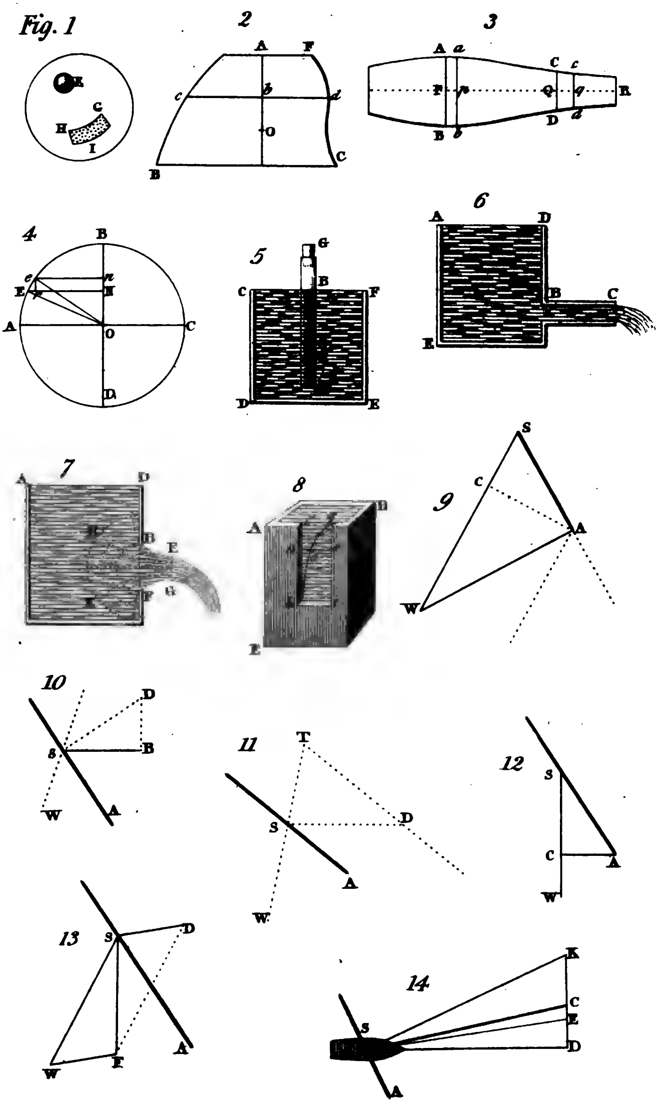

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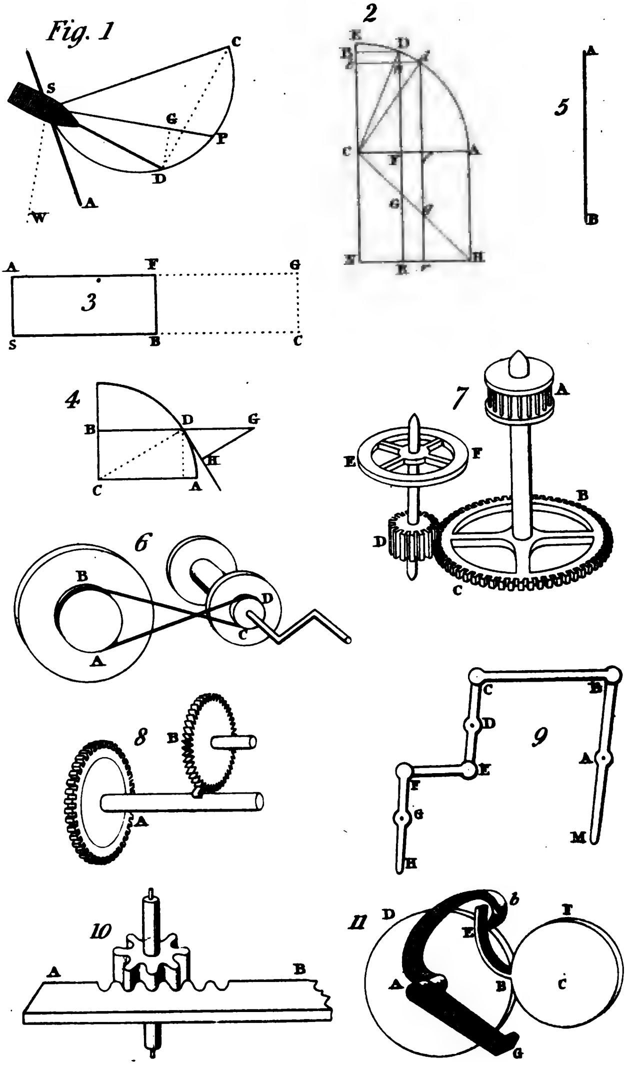
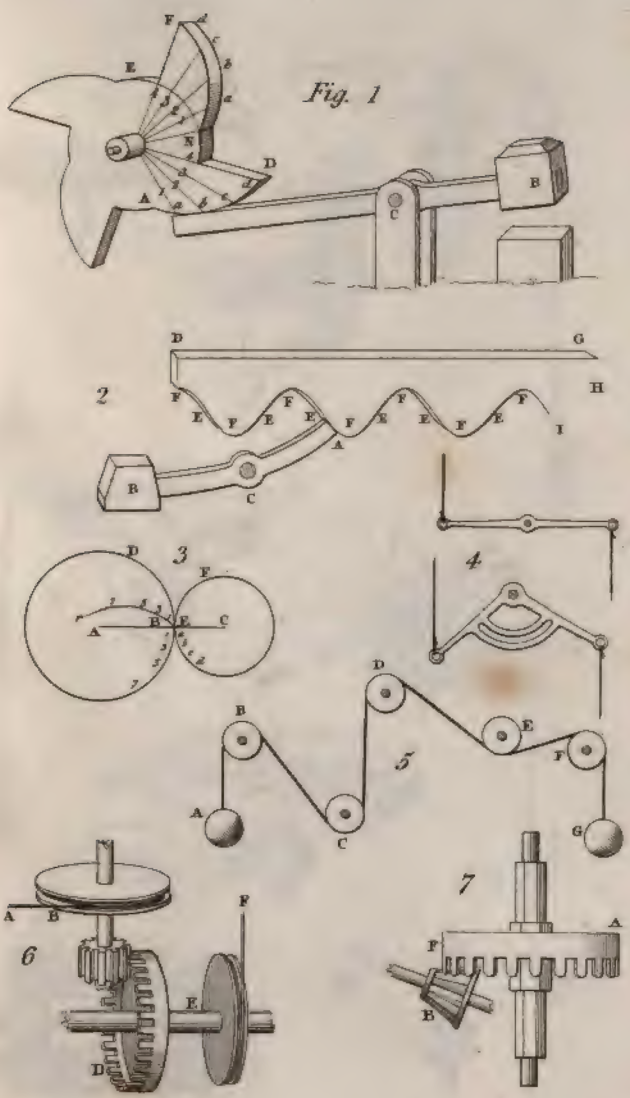


Fig. 1
S C G P D A W
2
E D C F A G H
5
A B
A F G
3
S B C
4
B D G H C A
7
A E F B D C
6
B A D C
9
C B D E A F G H M
8
B A
10
A B
11
D b T E A B C G

Pl. 14.



Pl. 15.

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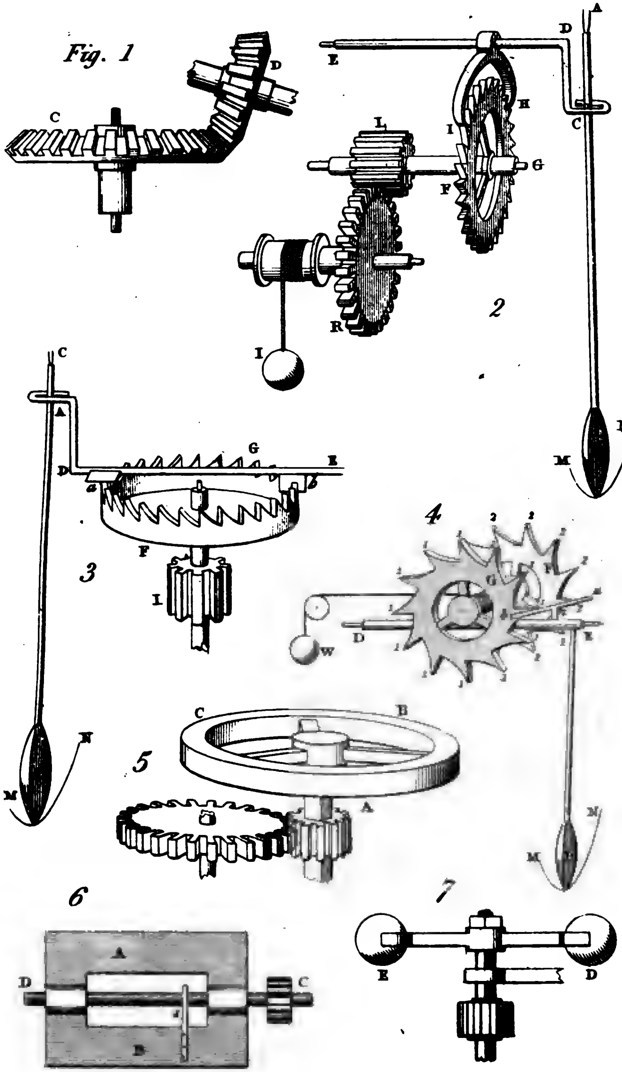

Fig. 1
C
D
2
A
D
E
C
H
I
L
G
F
R
I
M
N
C
A
D
G
E
a
b
F
3
I
M
N
4
G
F
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M
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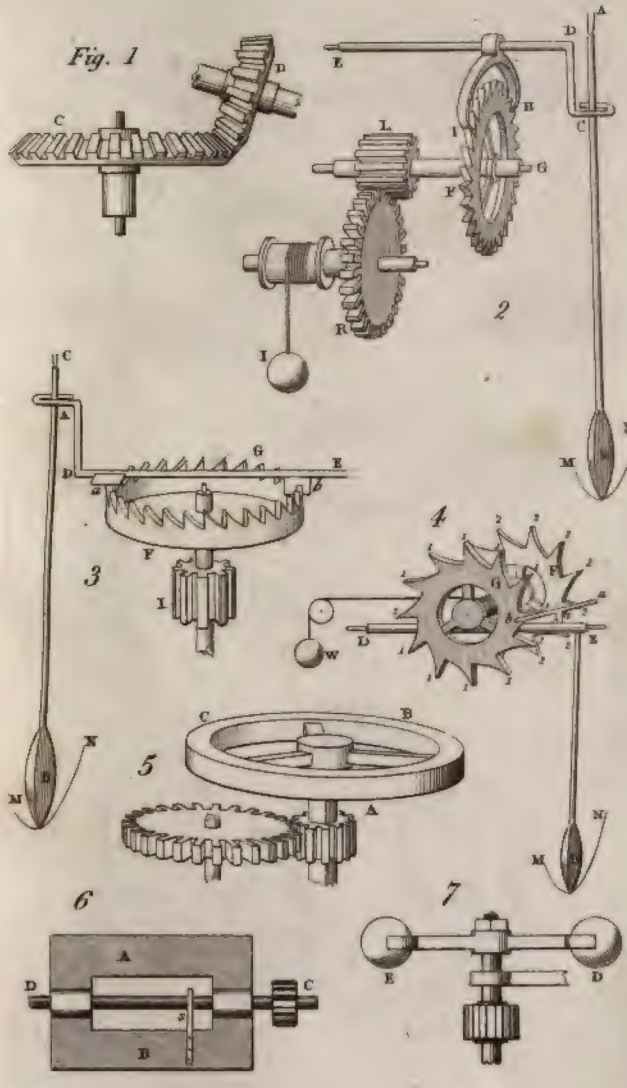
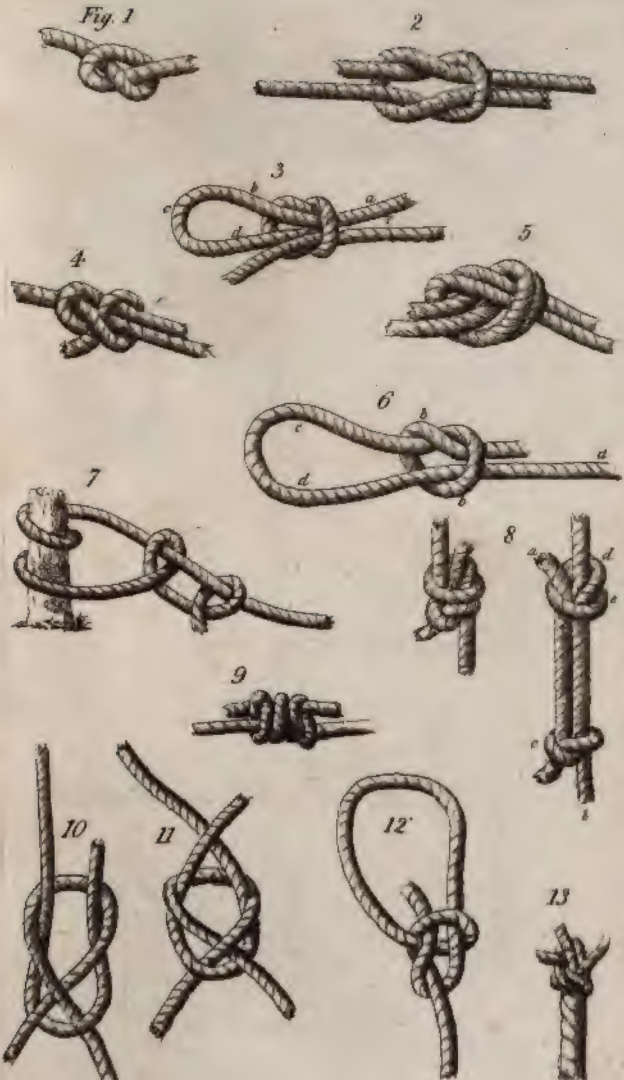
D
C
B
7
E
D

Pl.16.



Pl.16.



Pl. 17.

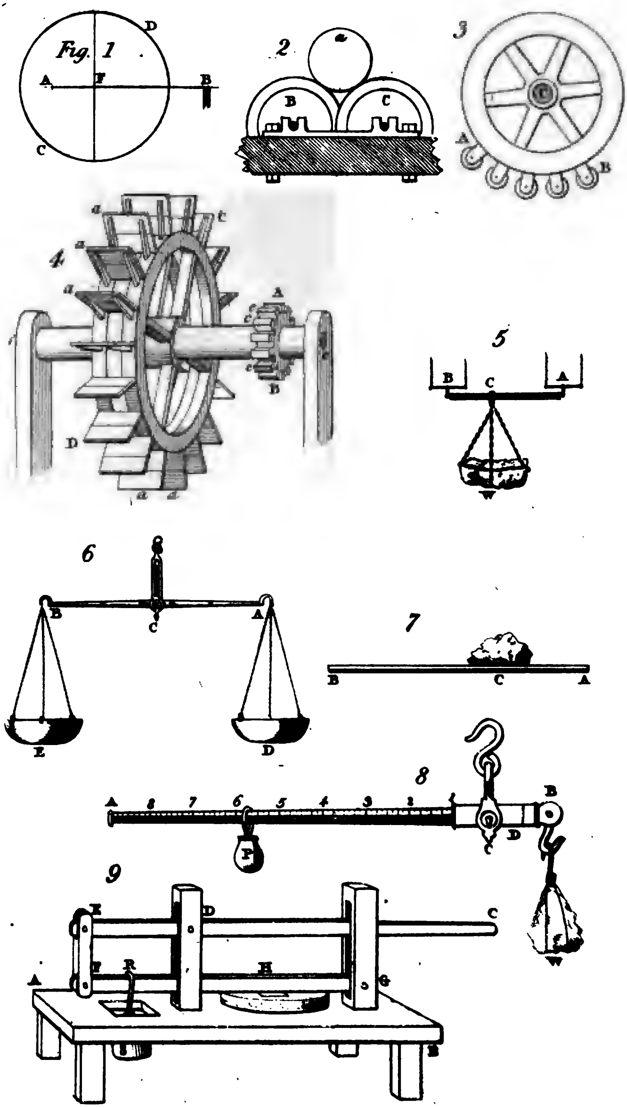


Fig. 1
A
F
B
C
D
2
a
B
C
3
A
B
4
a
c
A
D
B
5
B
C
A
W
6
B
C
A
E
D
7
B
C
A
8
A
8
7
6
5
4
3
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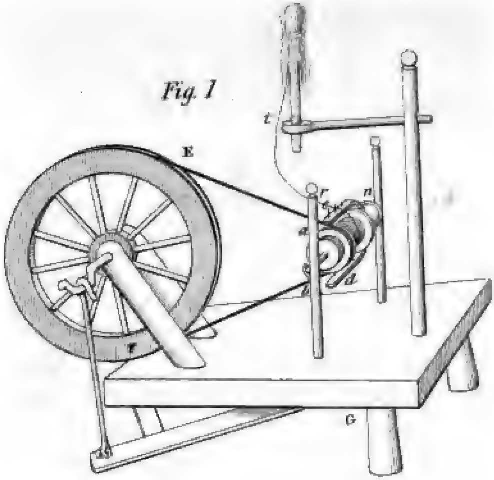
B
D
P
W
9
E
D
C
A
F
R
H
G

Pl. 18.

Fig. 1



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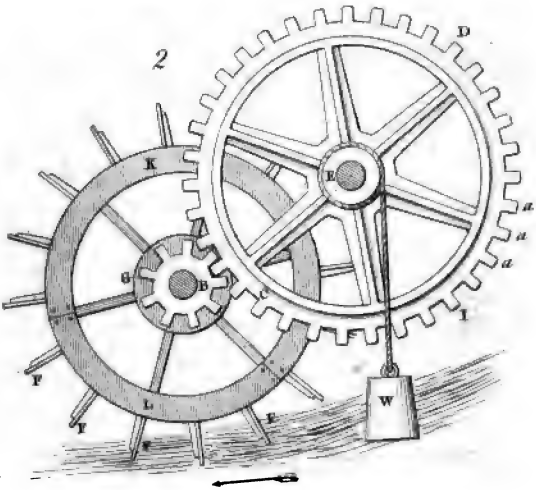
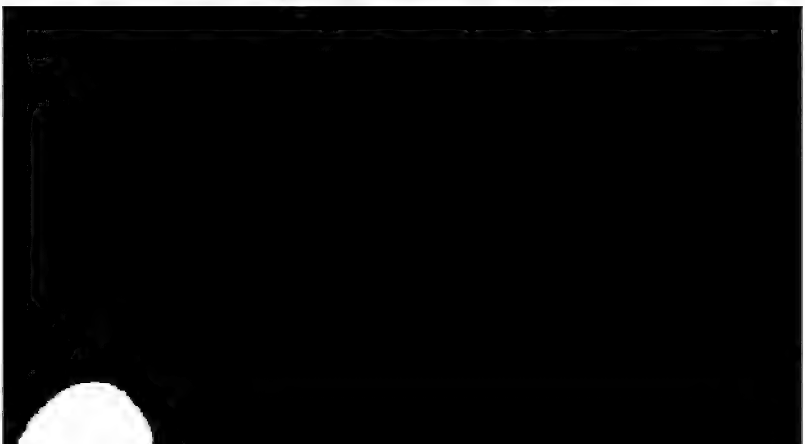


Pl. 19.



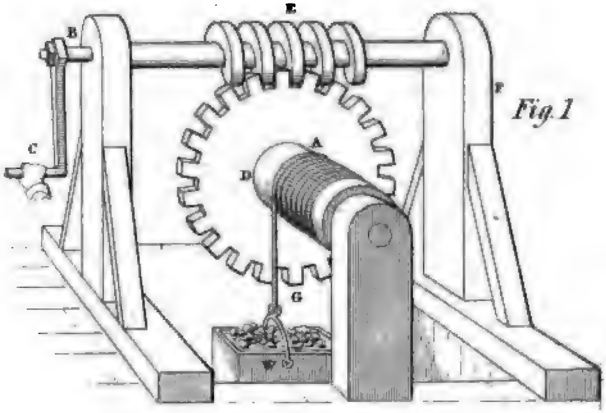
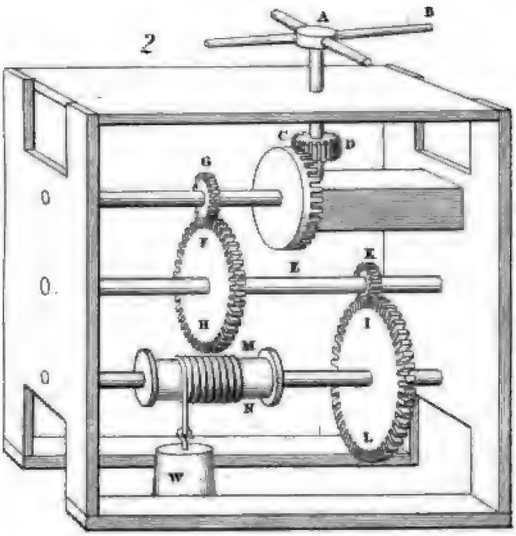
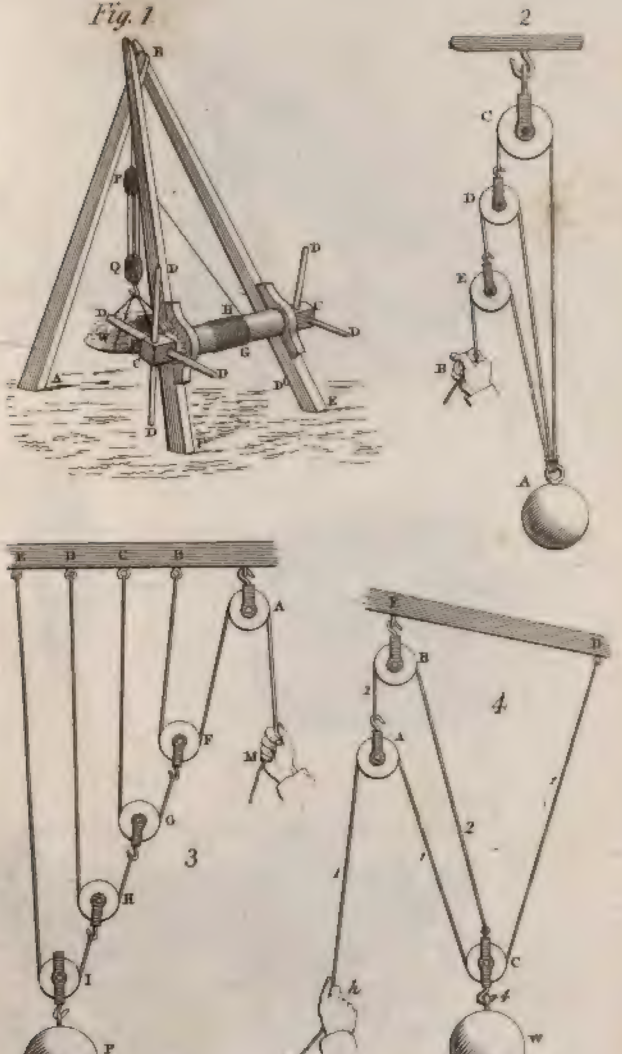


Fig. 1



2

Pl. 20.



Pl. 21.

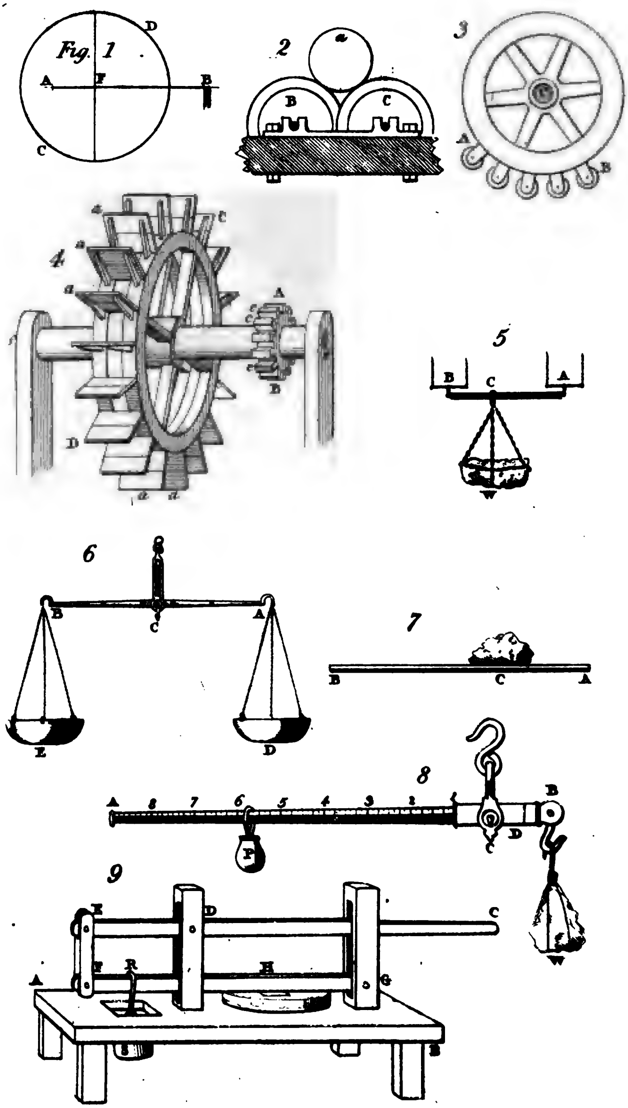
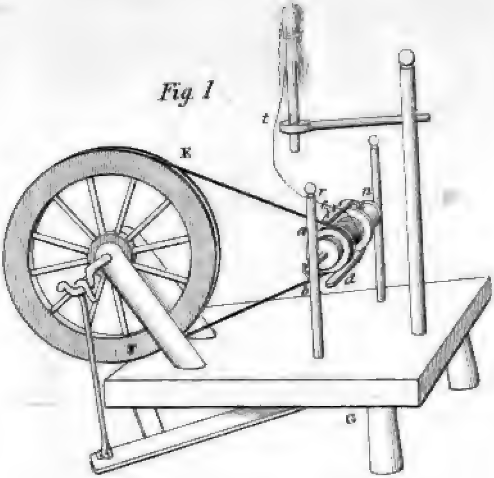


Fig. 1
A F B
D
C
2
a
B C
3
A B
4
A
B
C
D
a
5
B C A
W
6
B C A
E D
7
B C A
8
A 8 7 6 5 4 3 2 1 B
P
D
W
9
E D C
F R H G
A
S
B

Pl. 18.

Fig. 1



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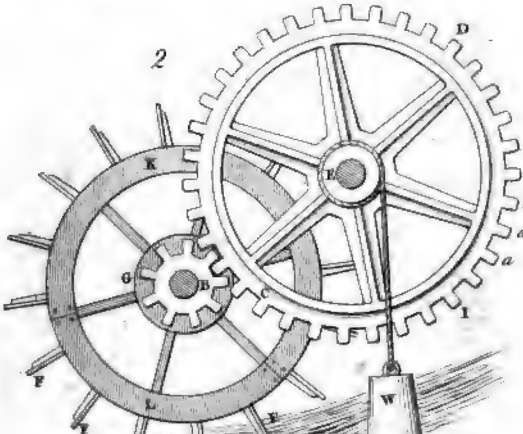

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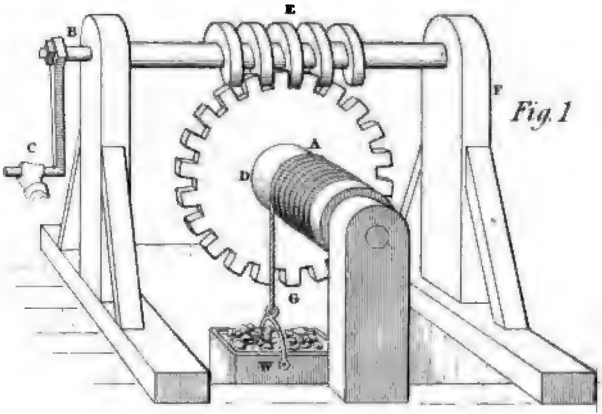
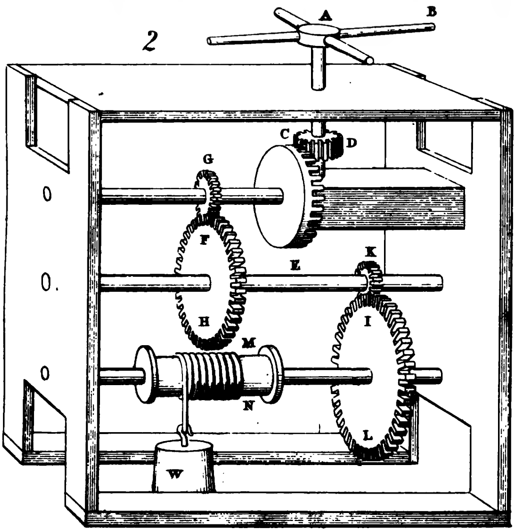


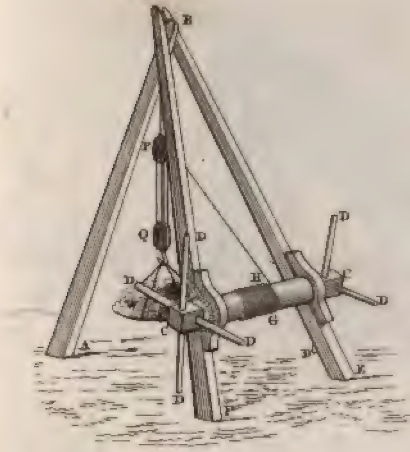
Fig. 1

2



Pl. 20.

Fig. 1



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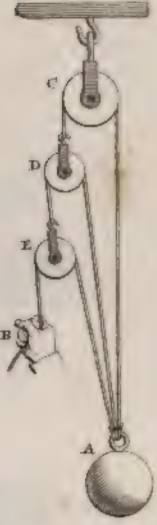
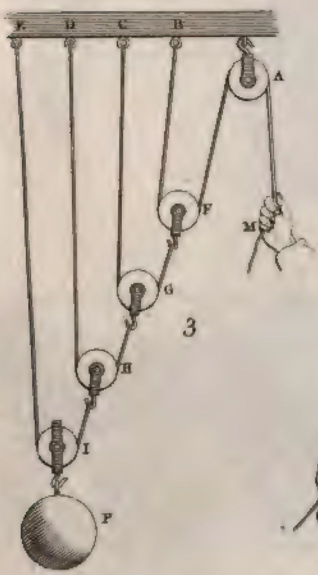
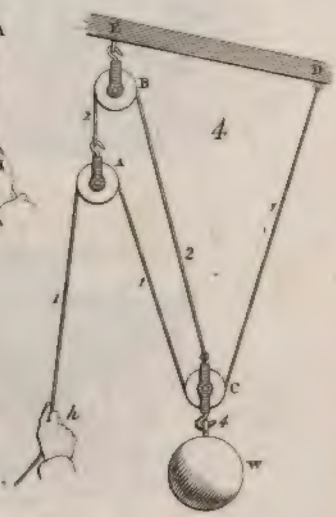
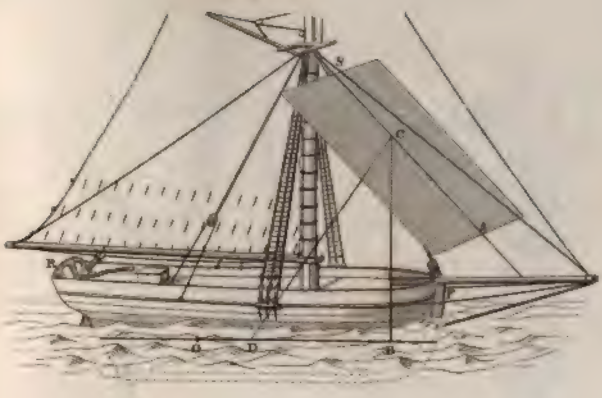






Pl. 21.

Fig. 1



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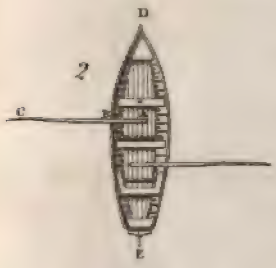

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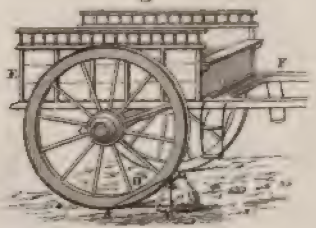

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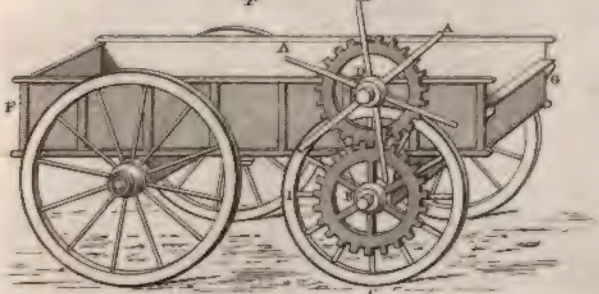

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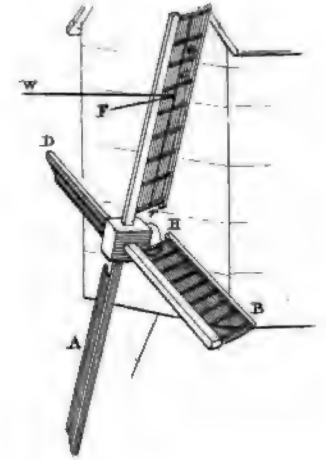
Fig. 1



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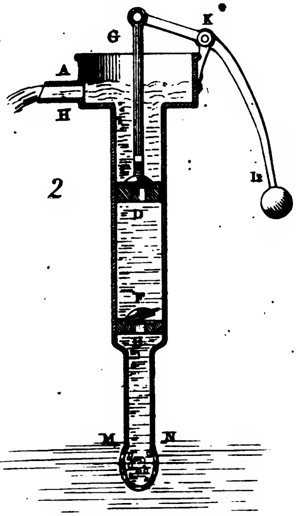

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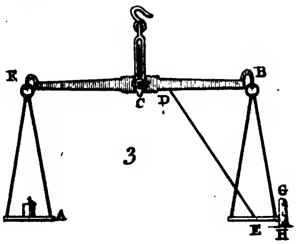

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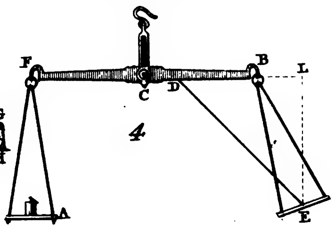

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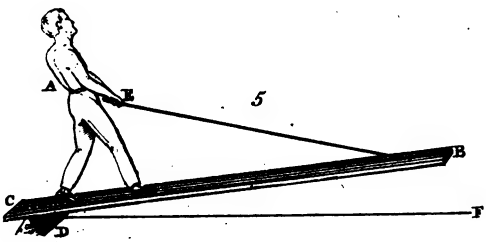

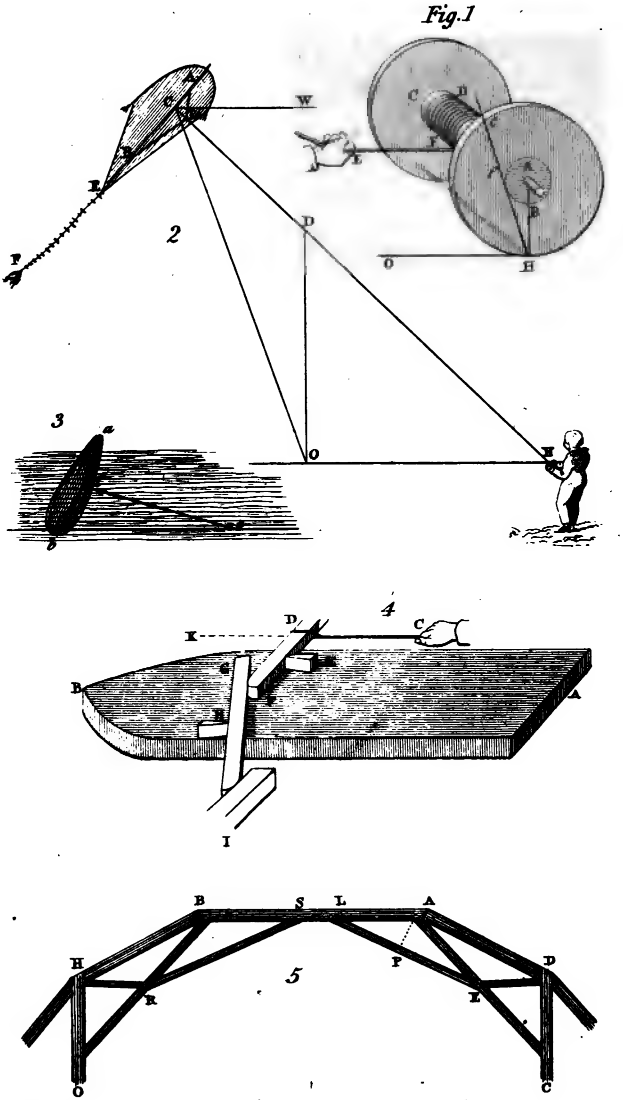

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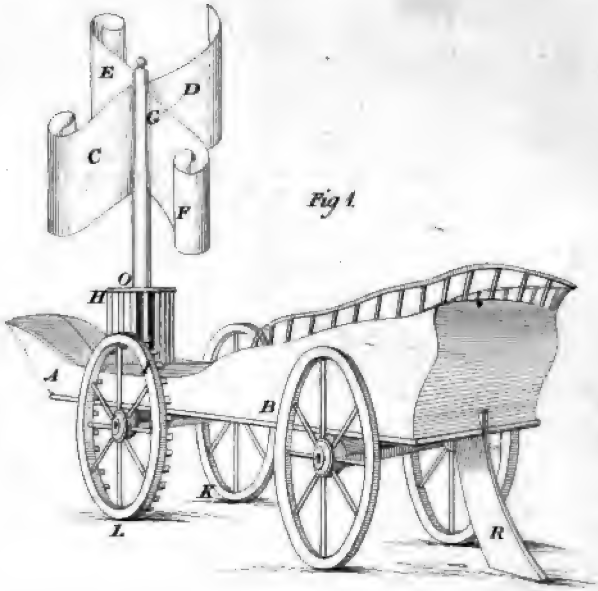

Fig 1.

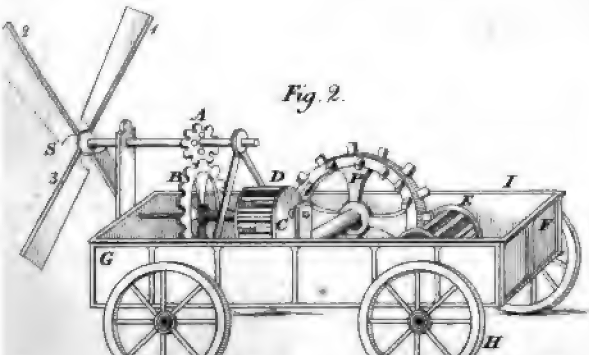

Fig. 2.

PL. 25.

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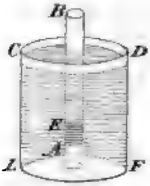

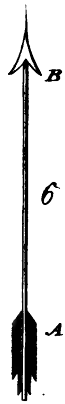

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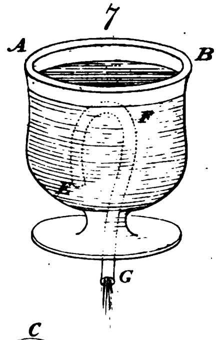
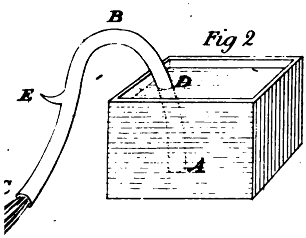
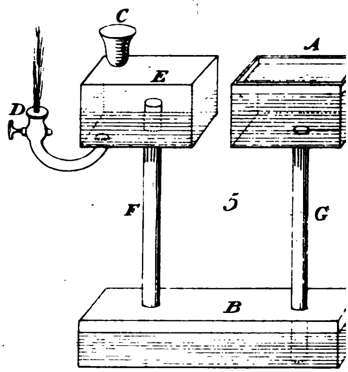
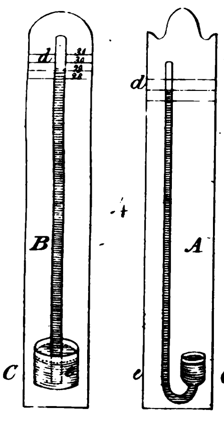


Fig 2







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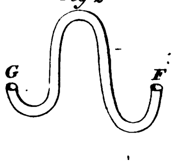
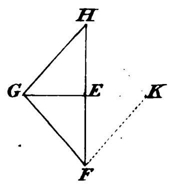
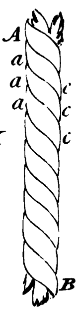
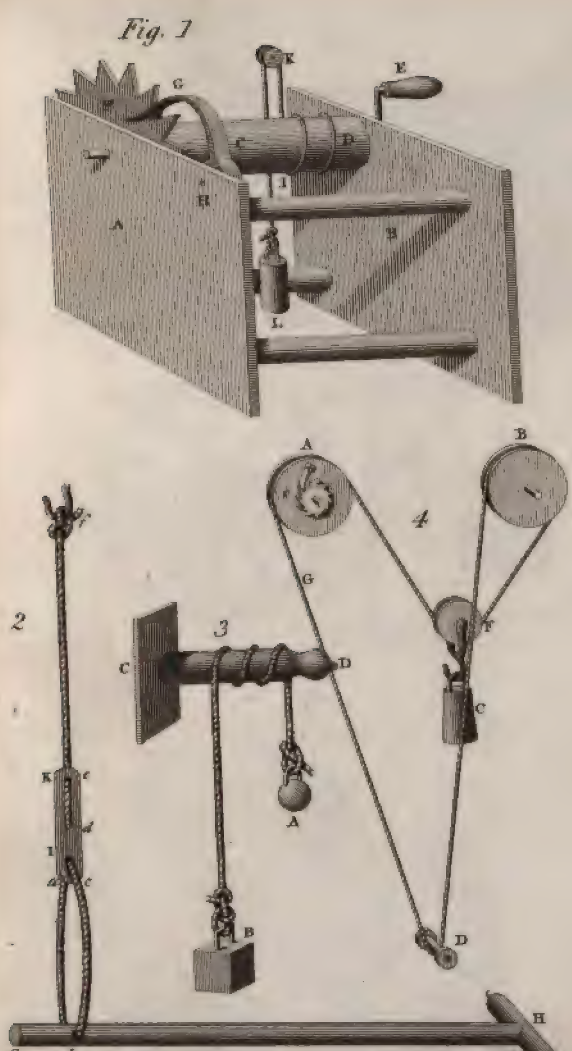






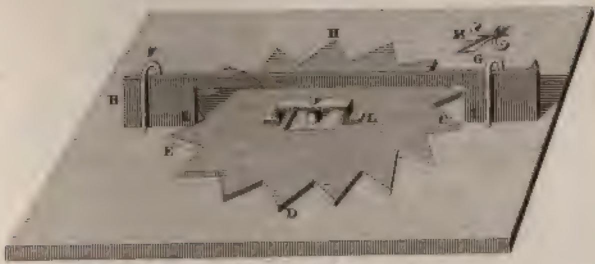
PL. 26.

Pl. 27.



Pl. 28.

Fig. 1



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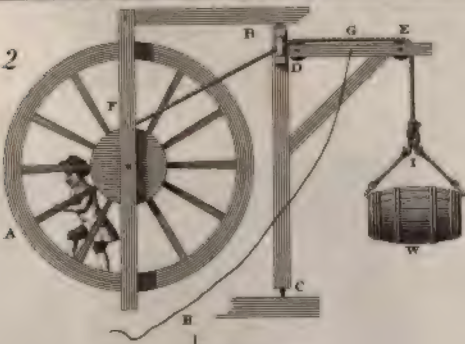

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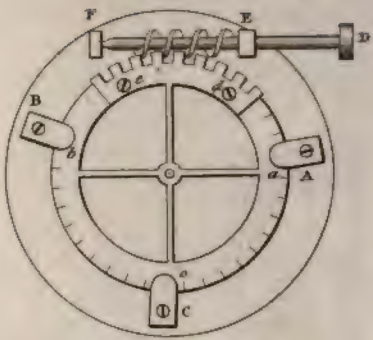
Pl. 29.

Fig. 1



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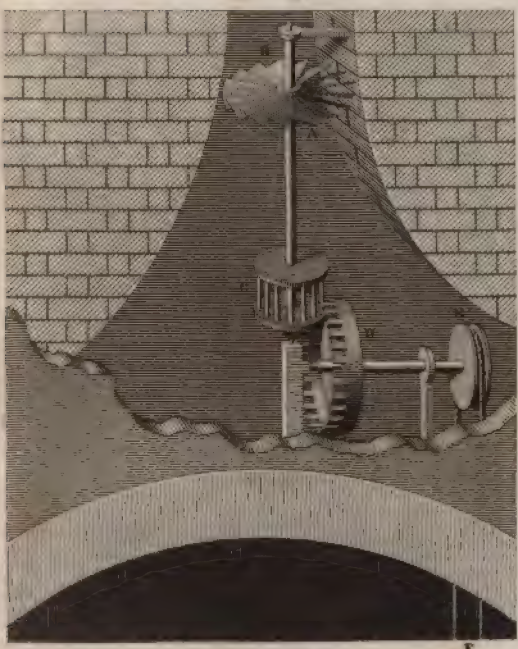
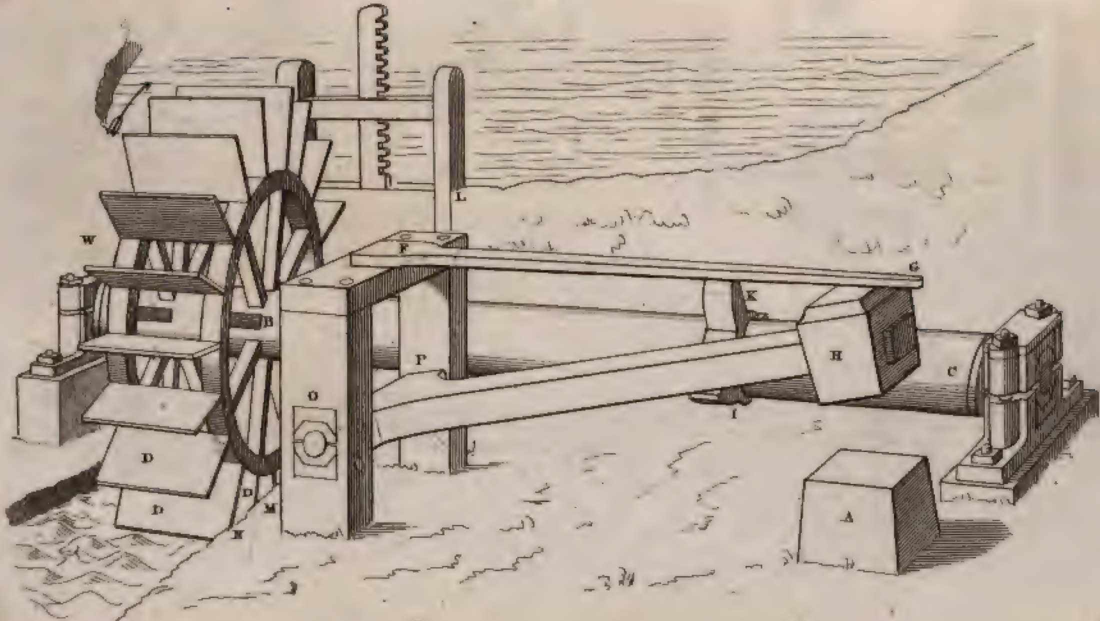


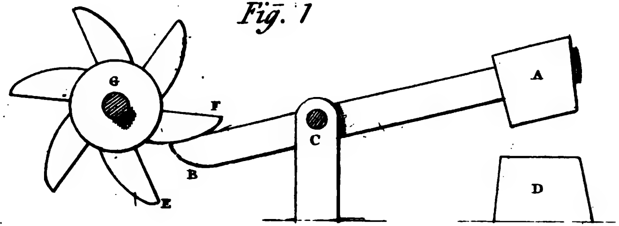
Pl. 30.

Fig. 1



Pl. 31.

Fig. 1



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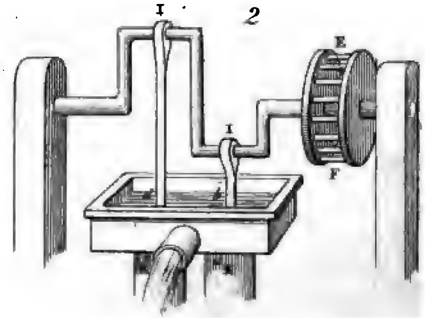

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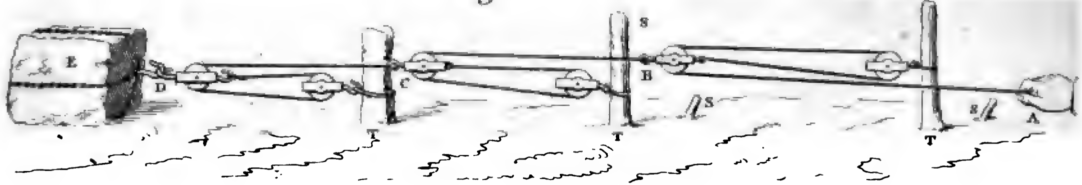
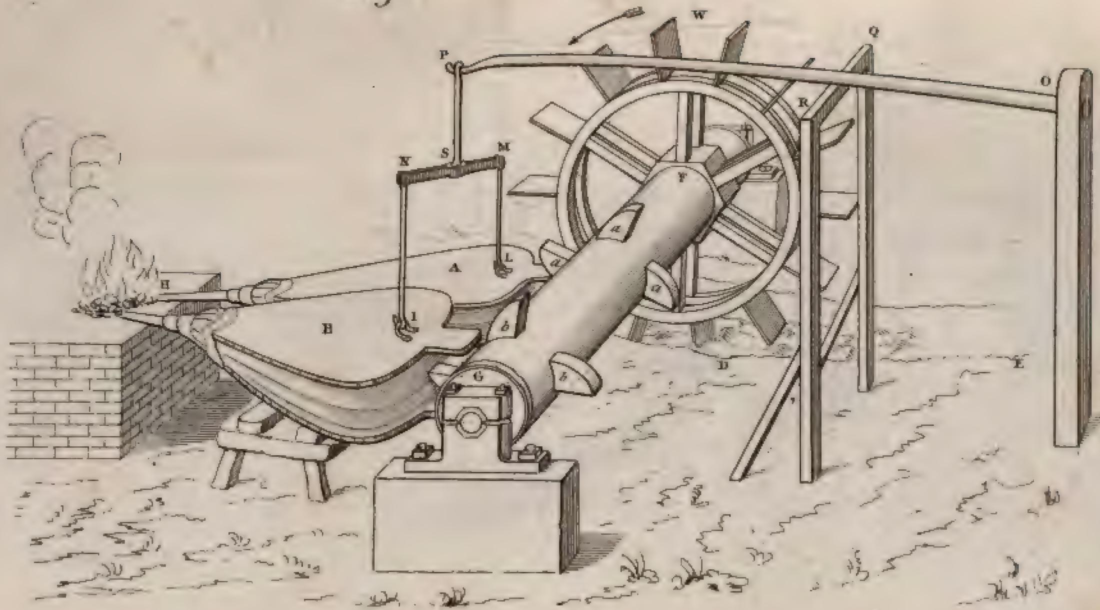


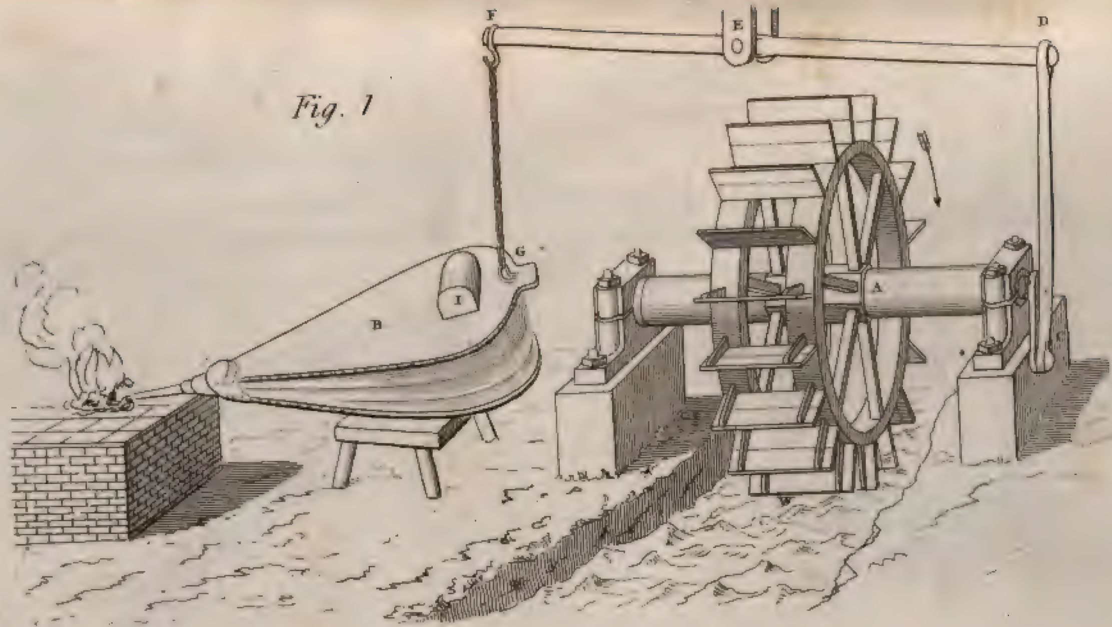
Pl. 32.

Fig. 1



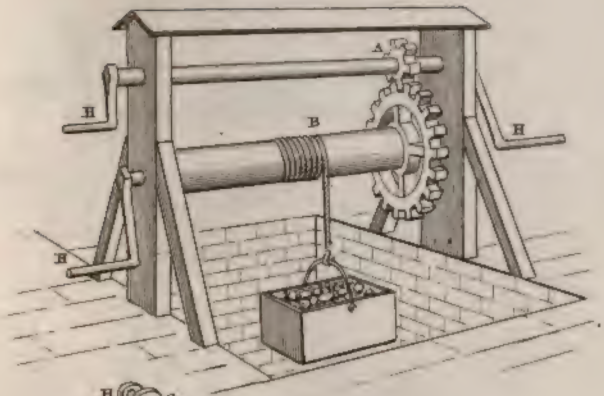
Pl. 33.

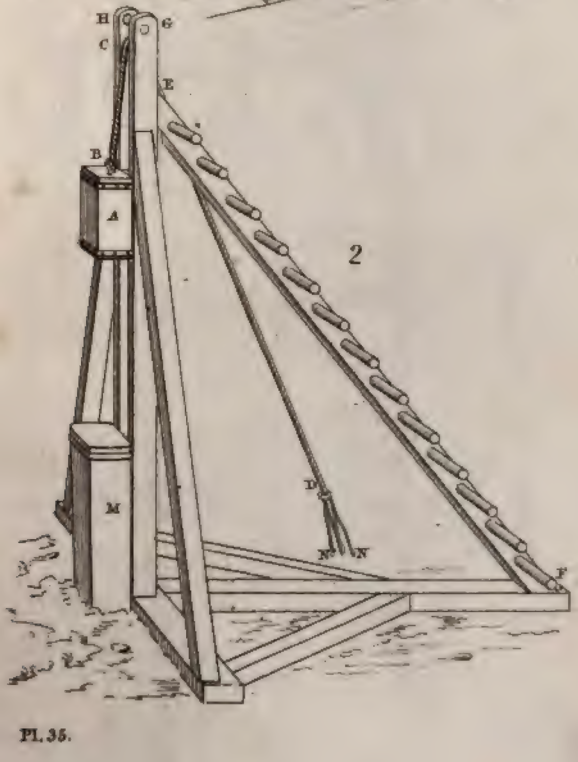
Fig. 1



Pl. 34.

Fig. 1





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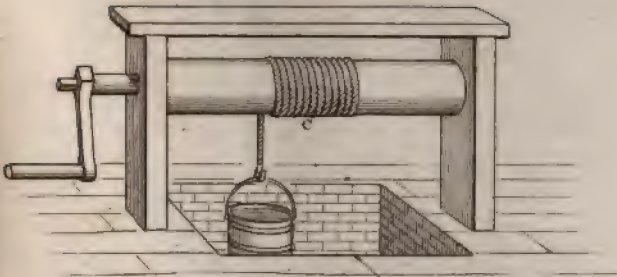
Pl. 35.

Fig. 1



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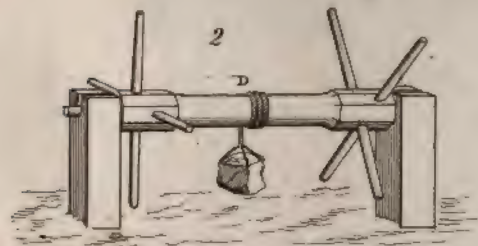

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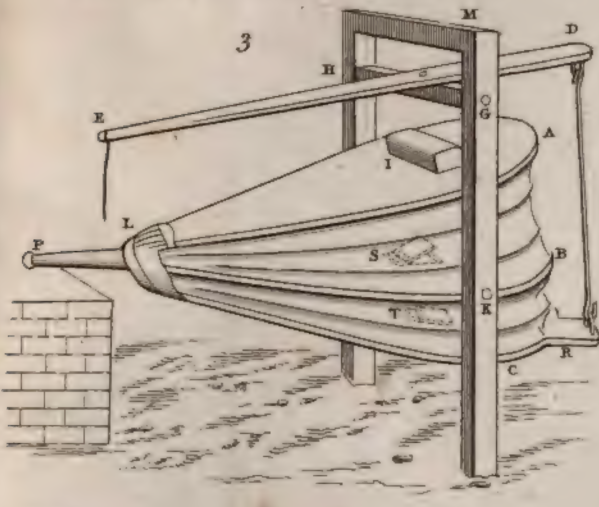

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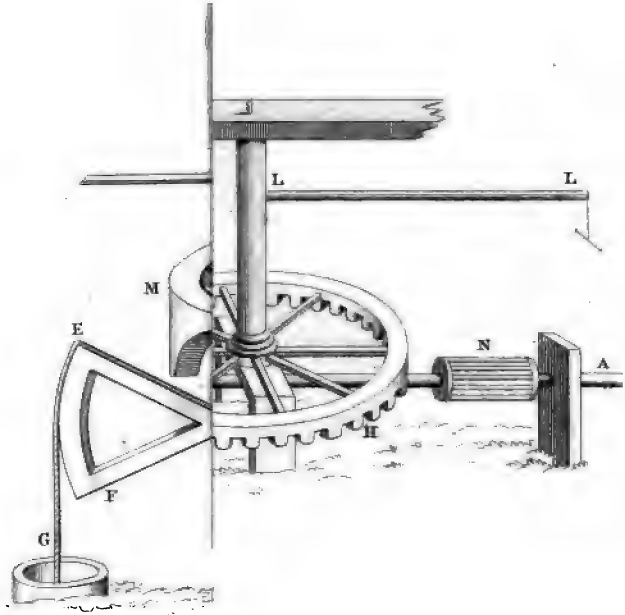
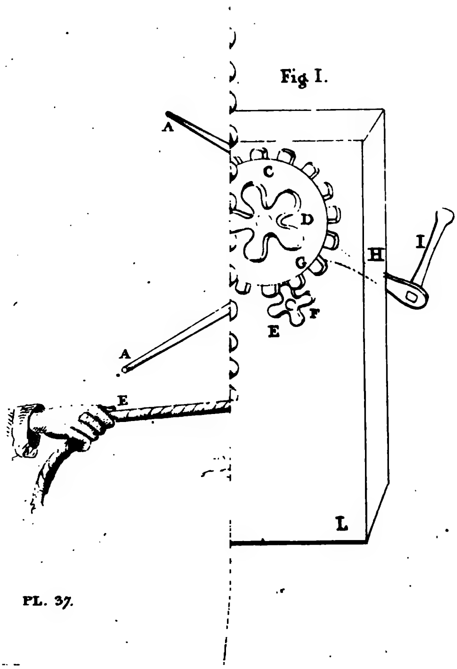
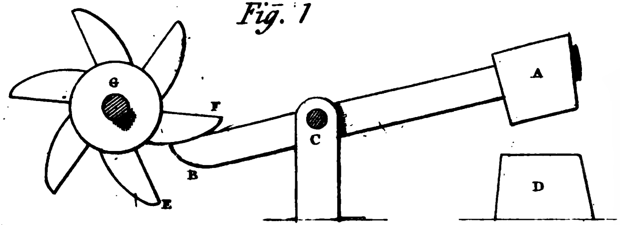


Fig I.



PL. 37.

Fig. 1



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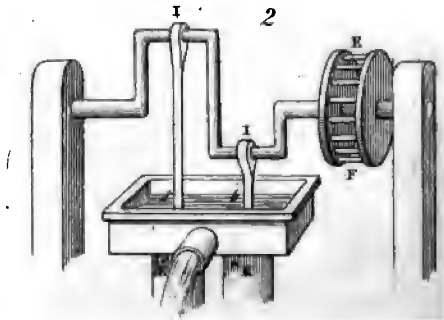

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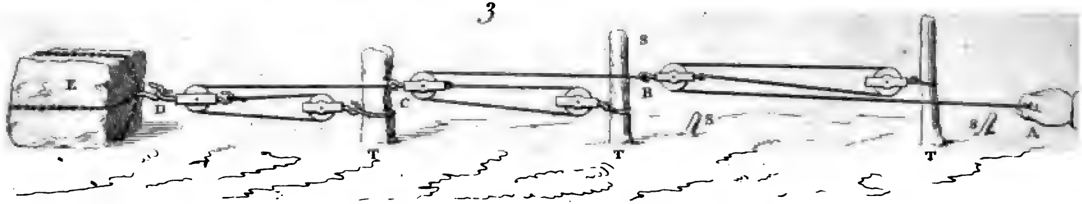

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Fig. 1

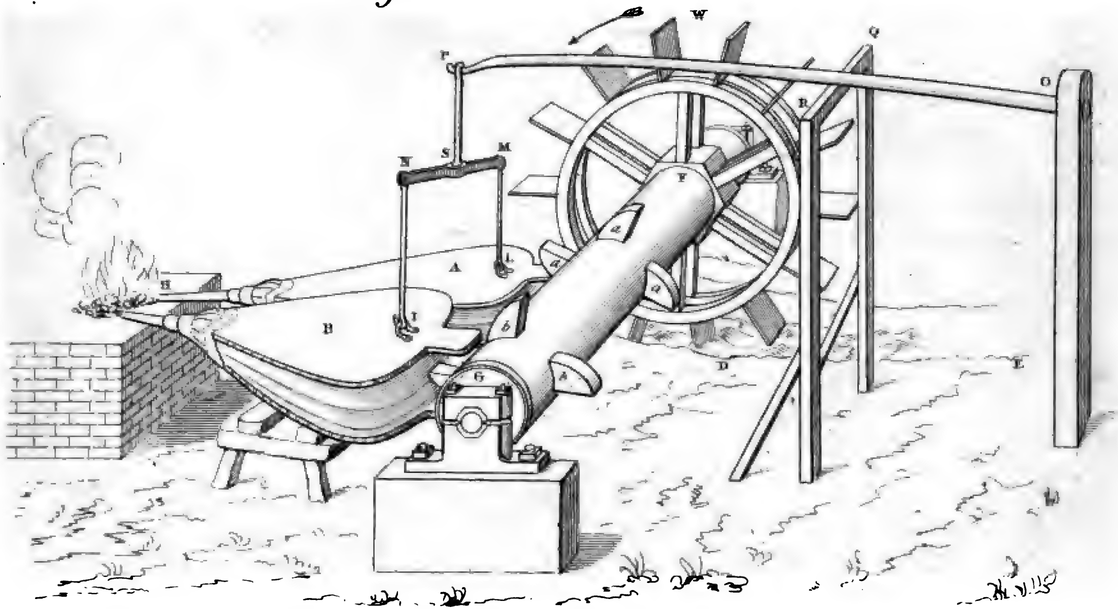

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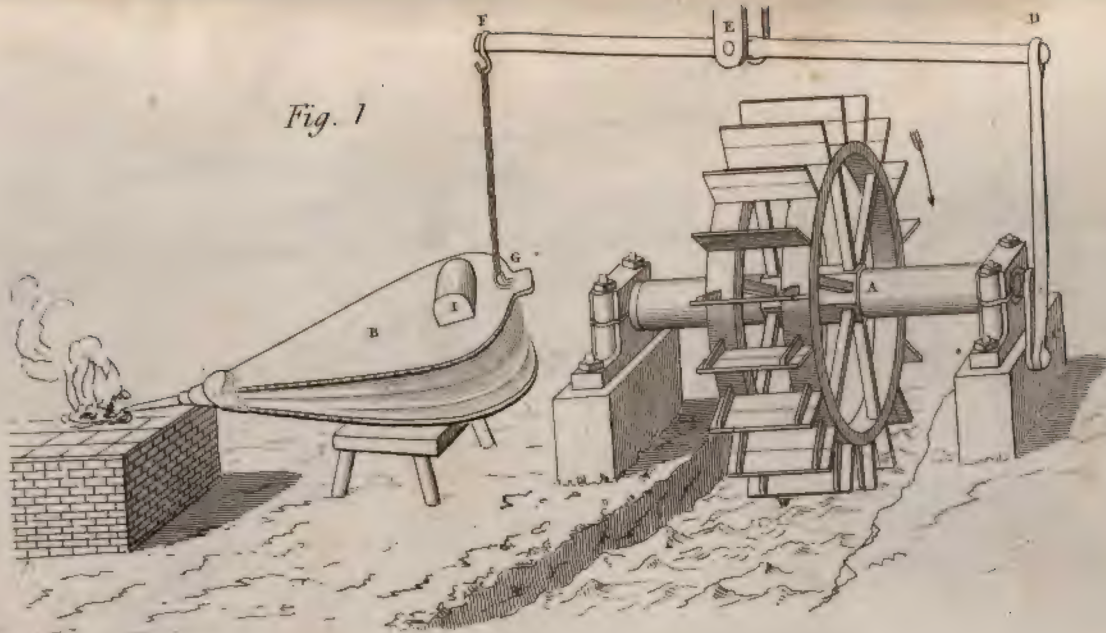
Fig. 1



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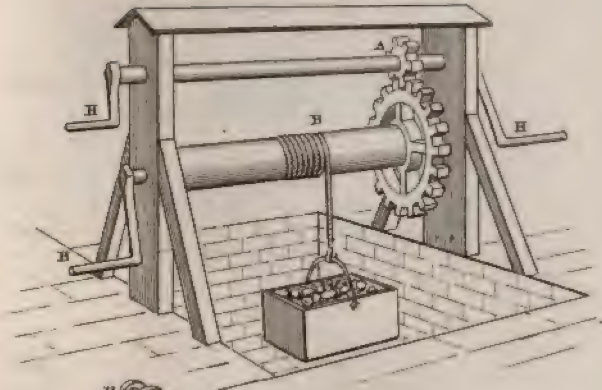

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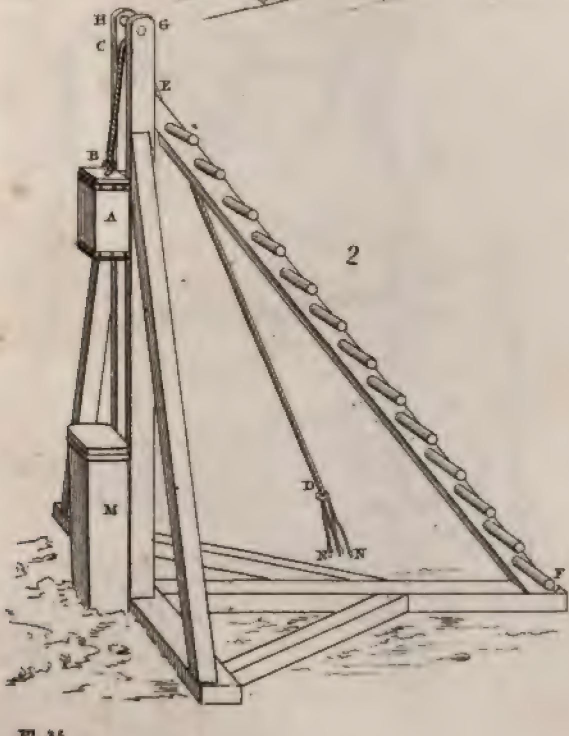

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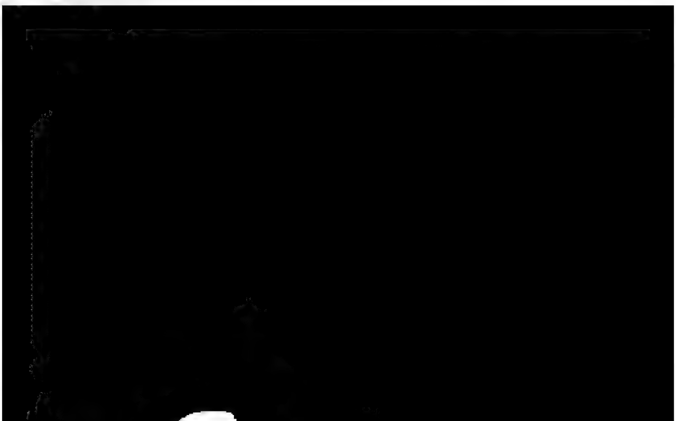
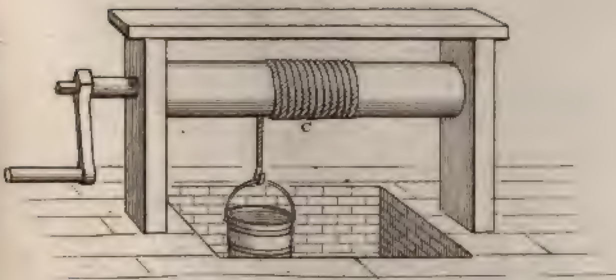


Fig. 1



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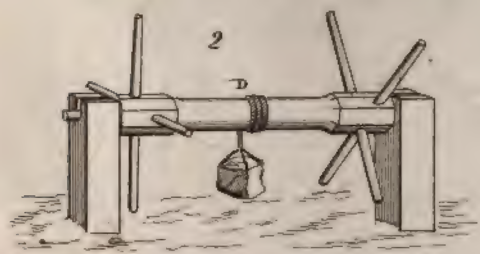

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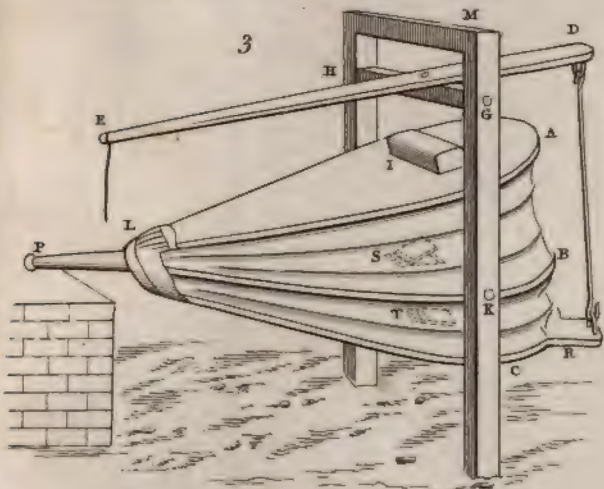

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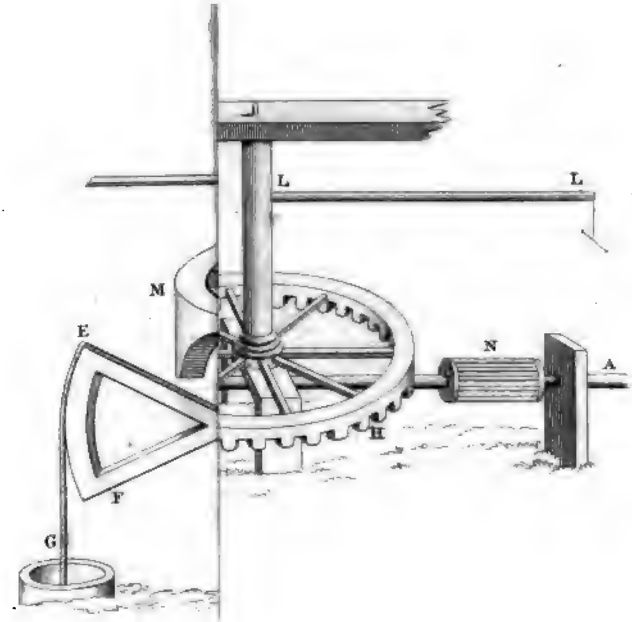

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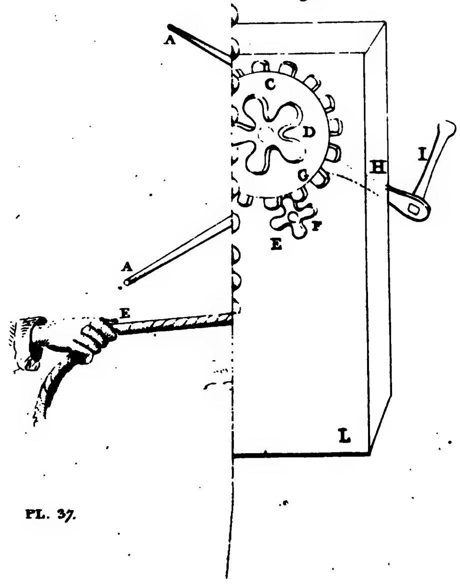
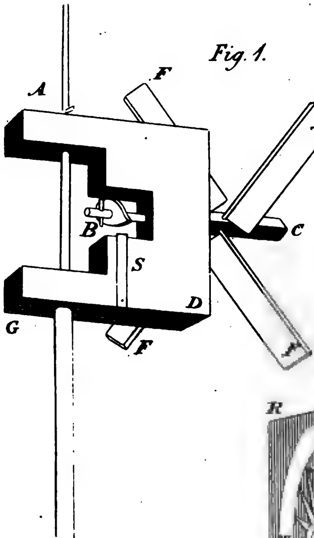


PL. 37.

Fig. 1.



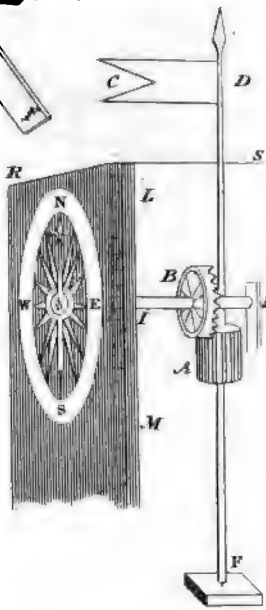
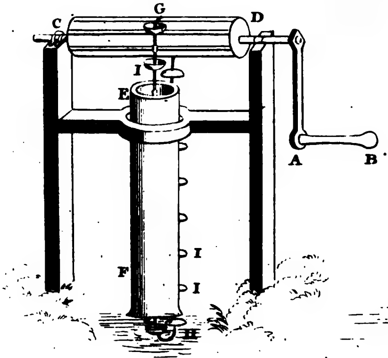


Fig. 2.

Fig 3.



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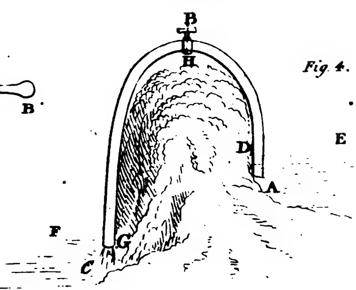

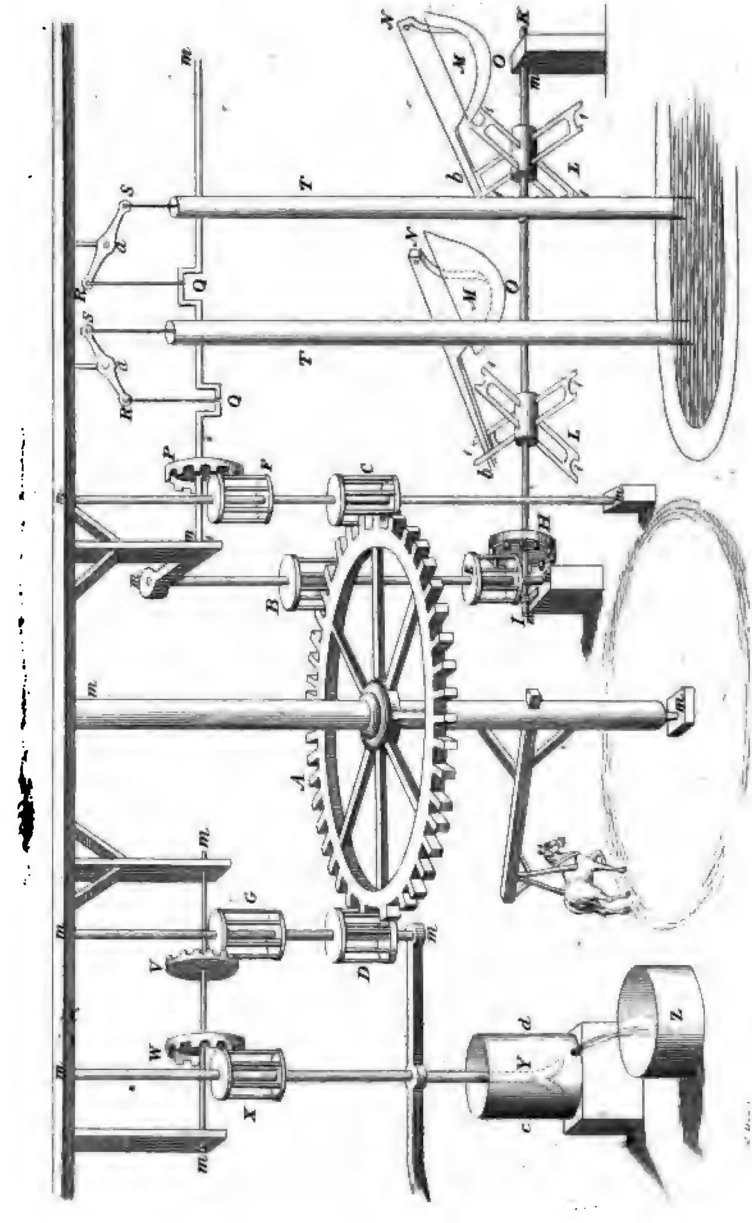

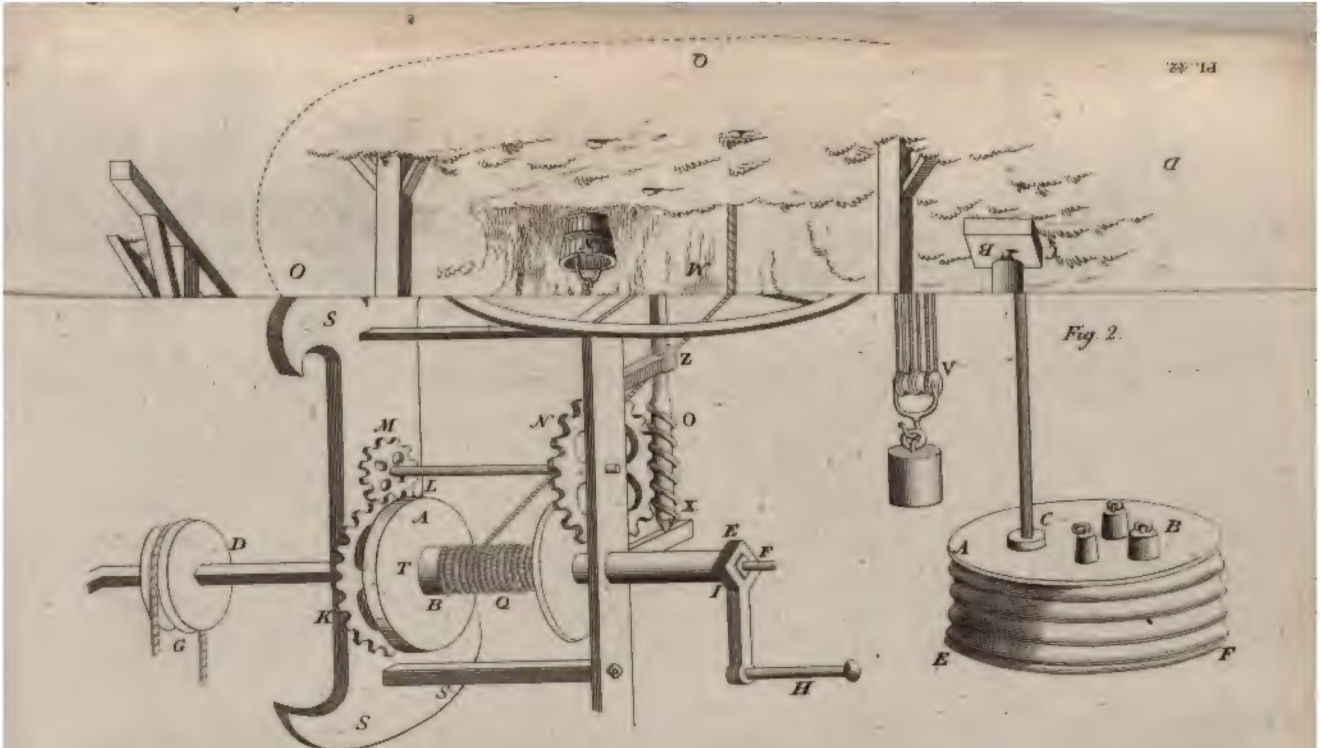

PL. 43.

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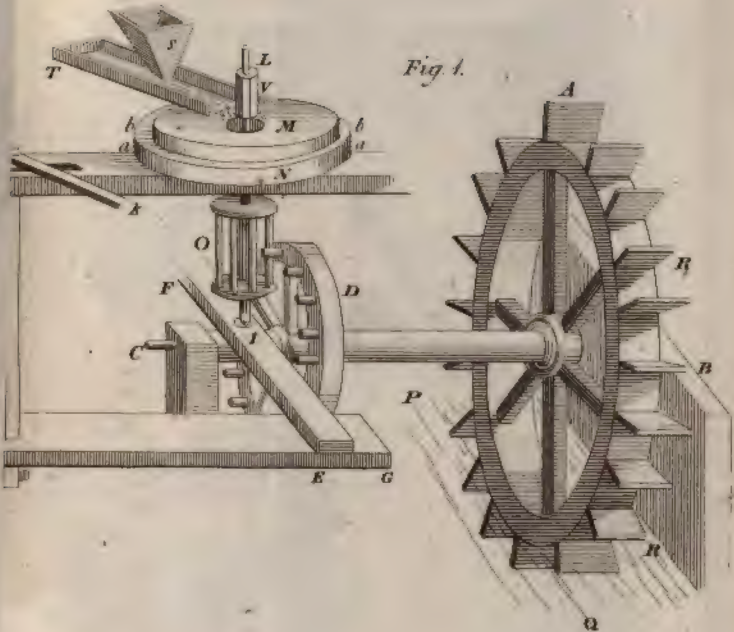
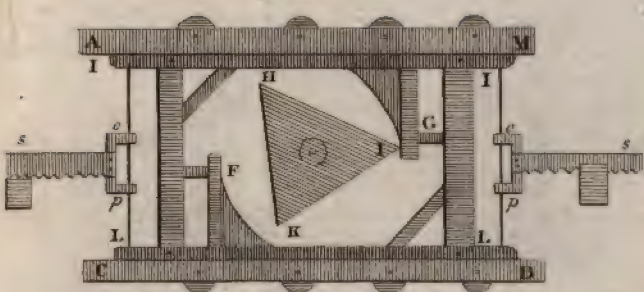


Fig. 2



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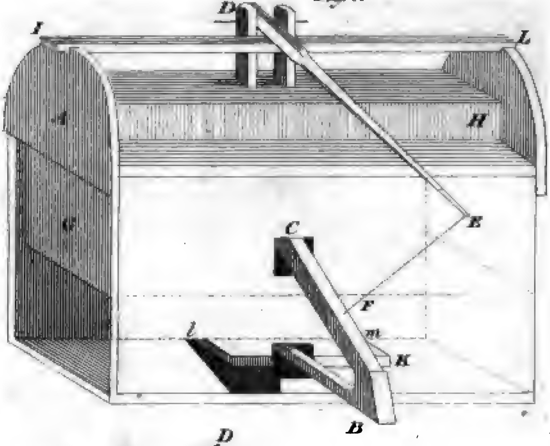

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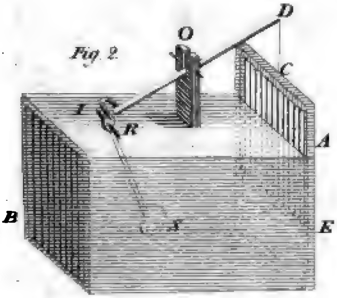

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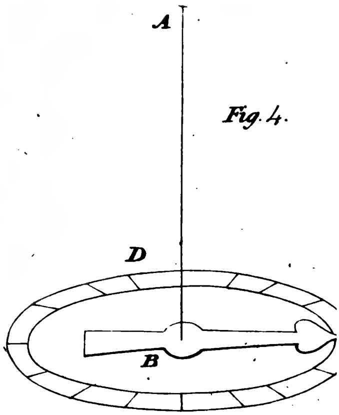

Fig. 2.
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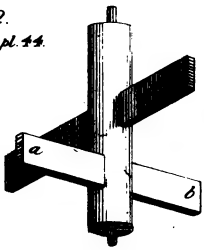

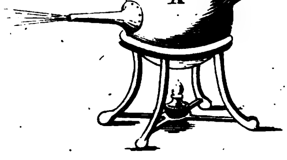

Fig. 3.

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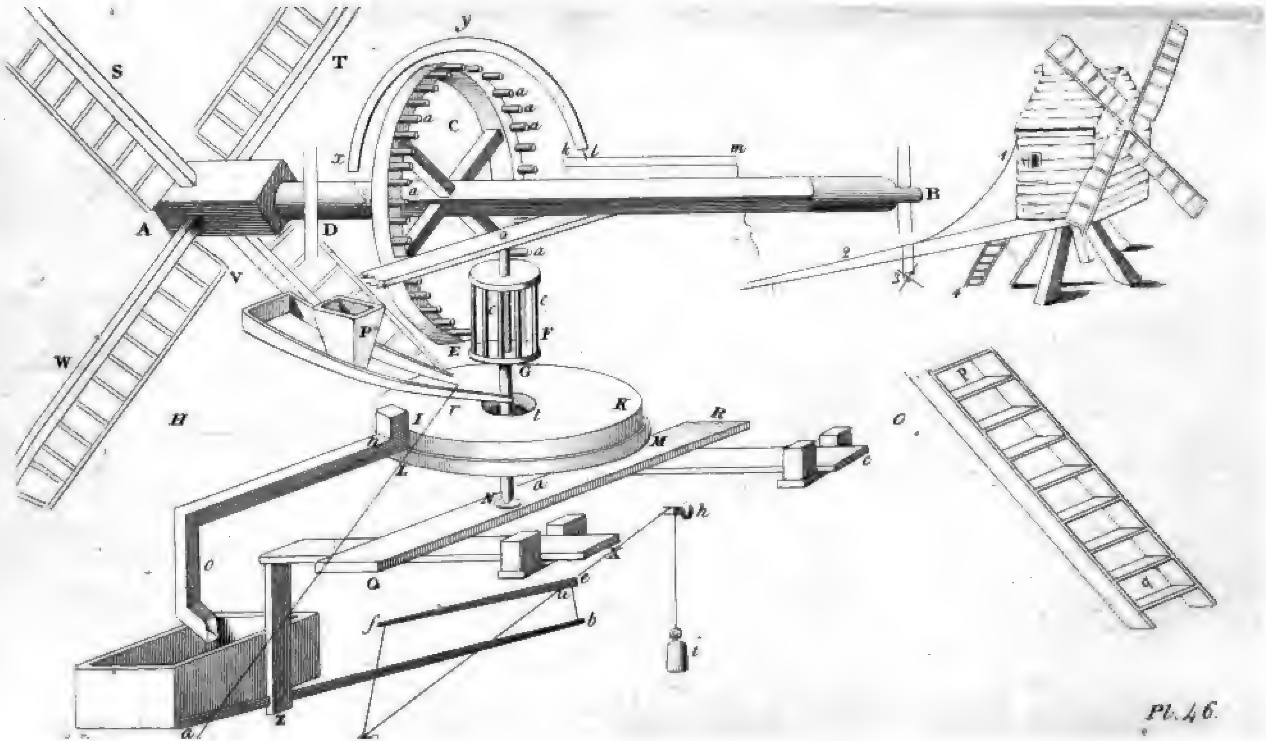

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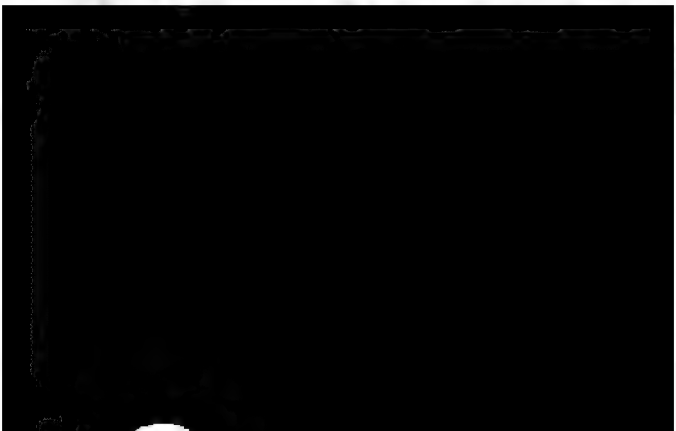
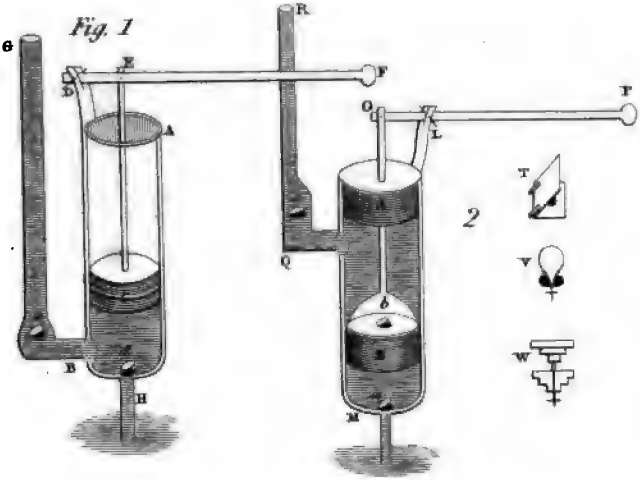


Fig. 1



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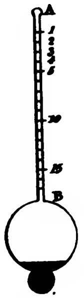

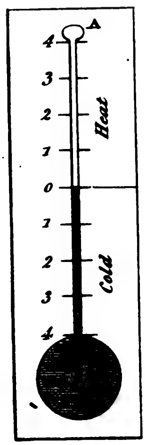

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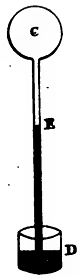
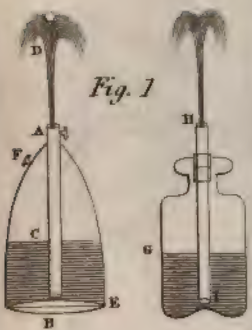


Pl. 47.

Fig. 1



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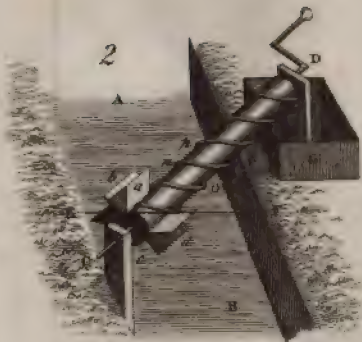

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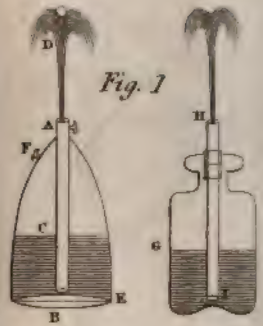
Pl. 48.

Fig. 1



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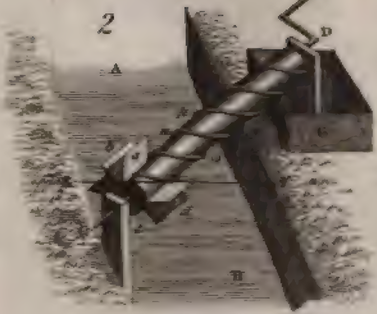

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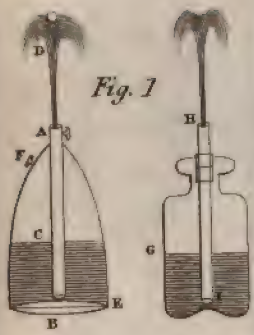
Fig. 1



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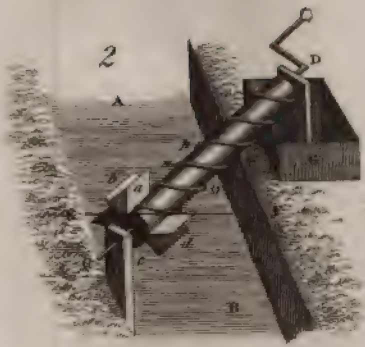

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Pl. 48.

PL. 42.

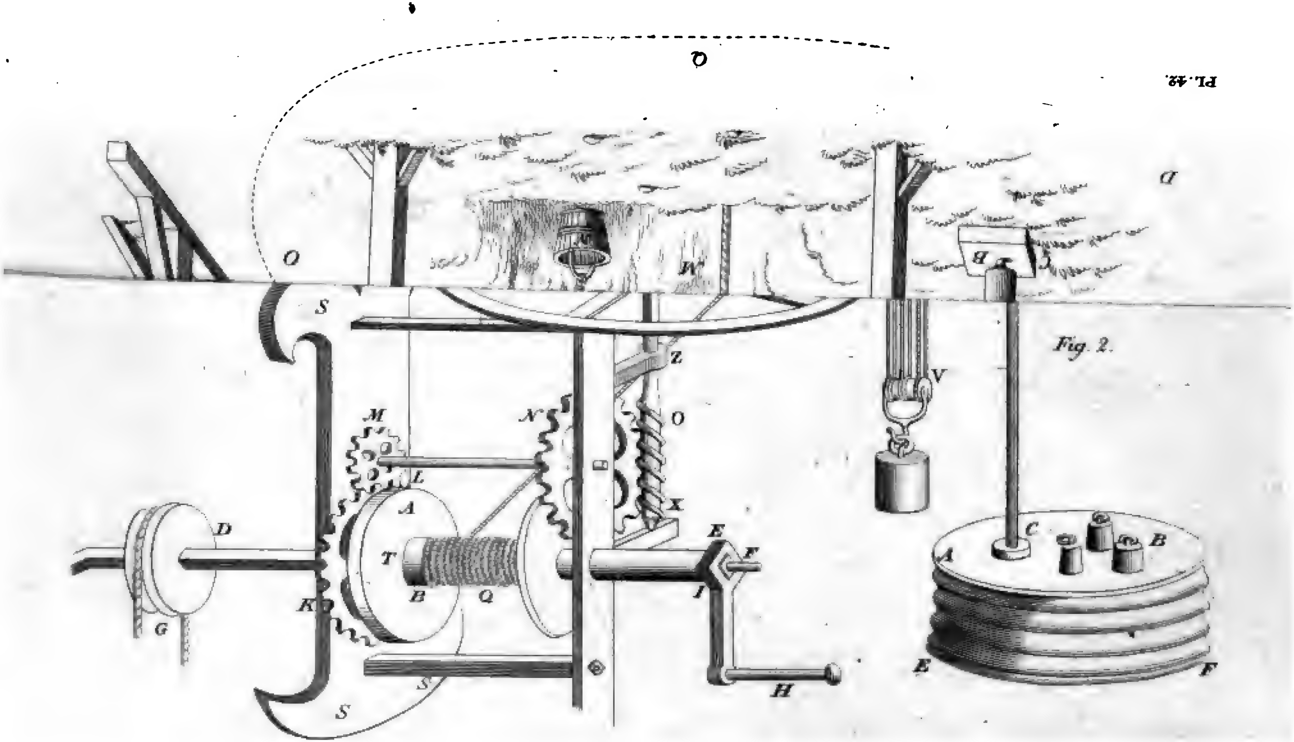

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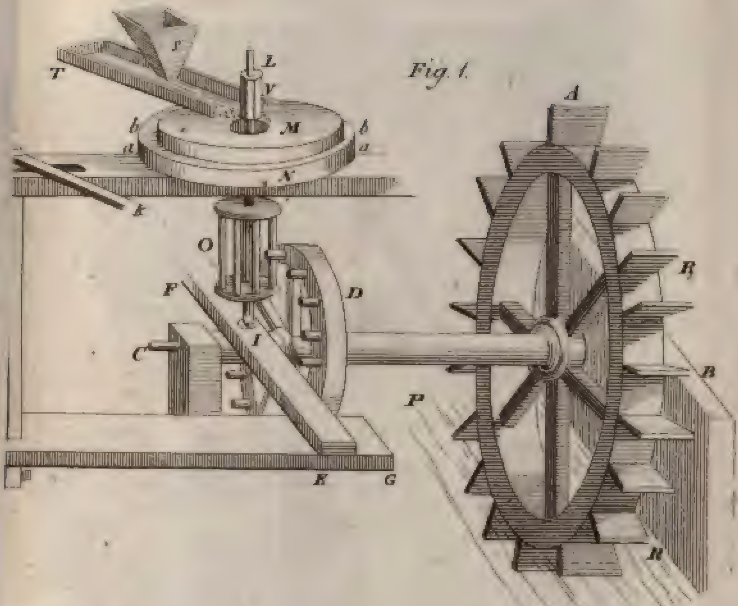
Fig. 1.



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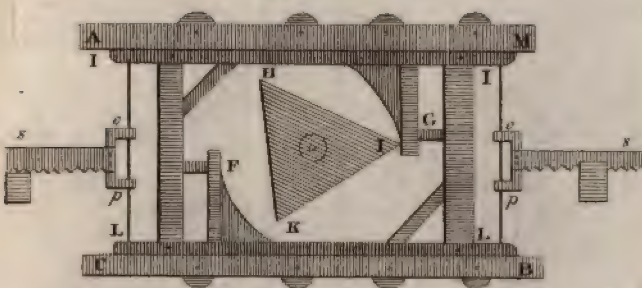

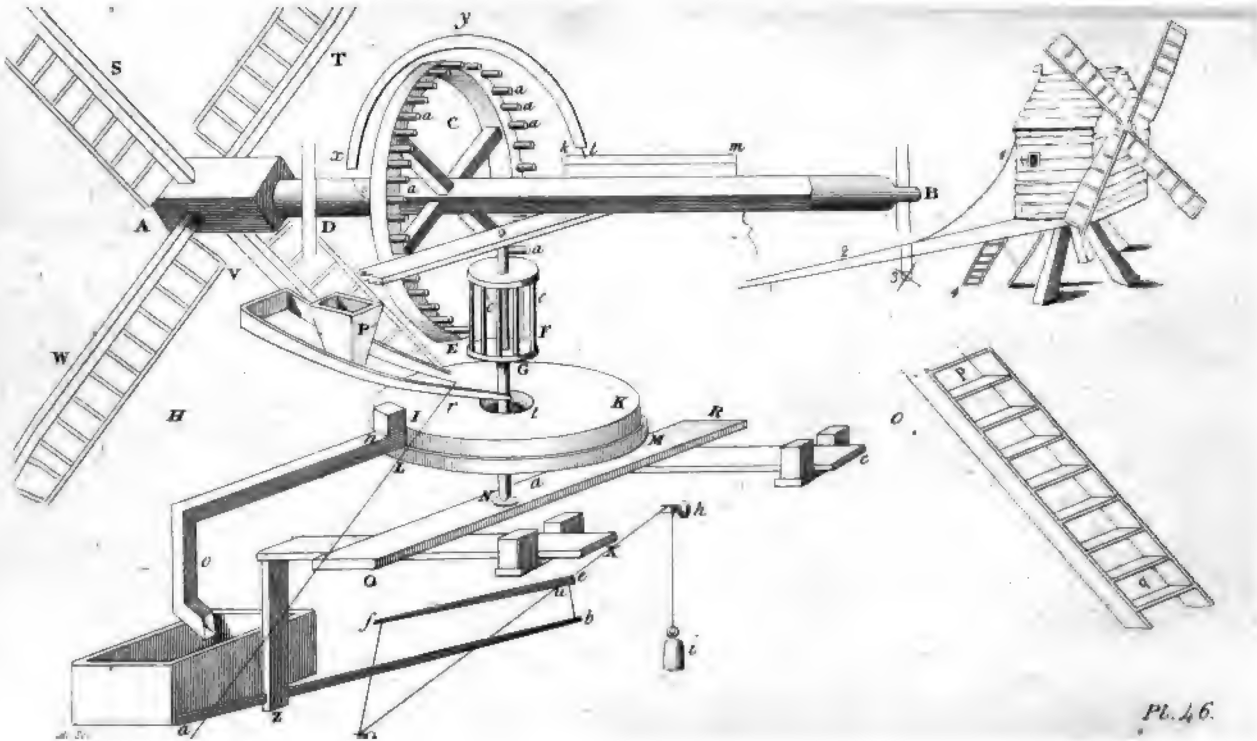

Pl. 46.

Fig. 1

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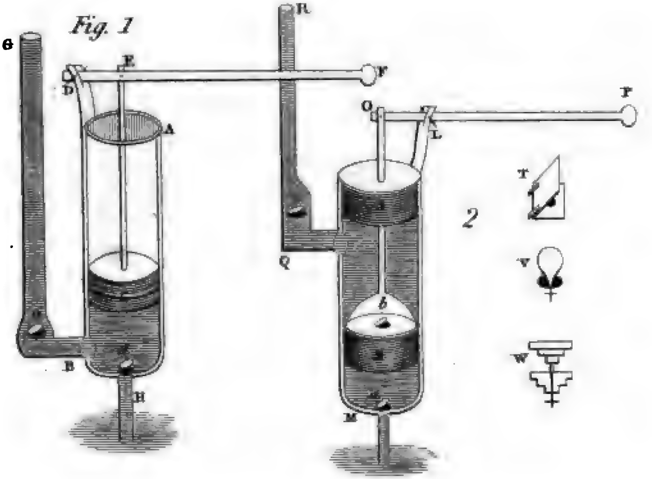

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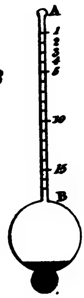

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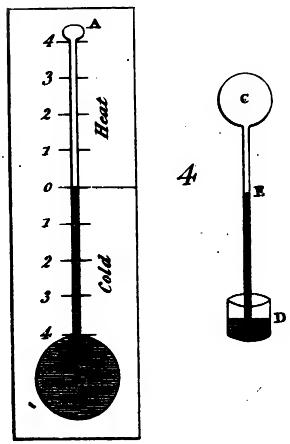
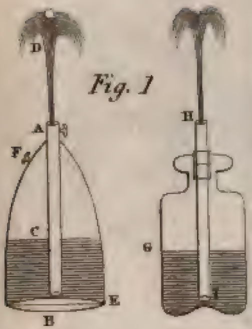


Pl. 47.

Fig. 1



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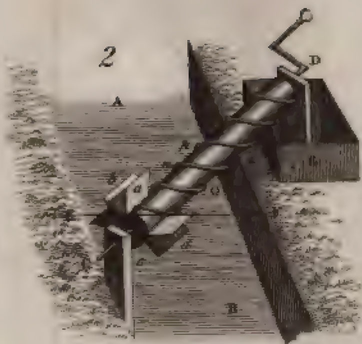

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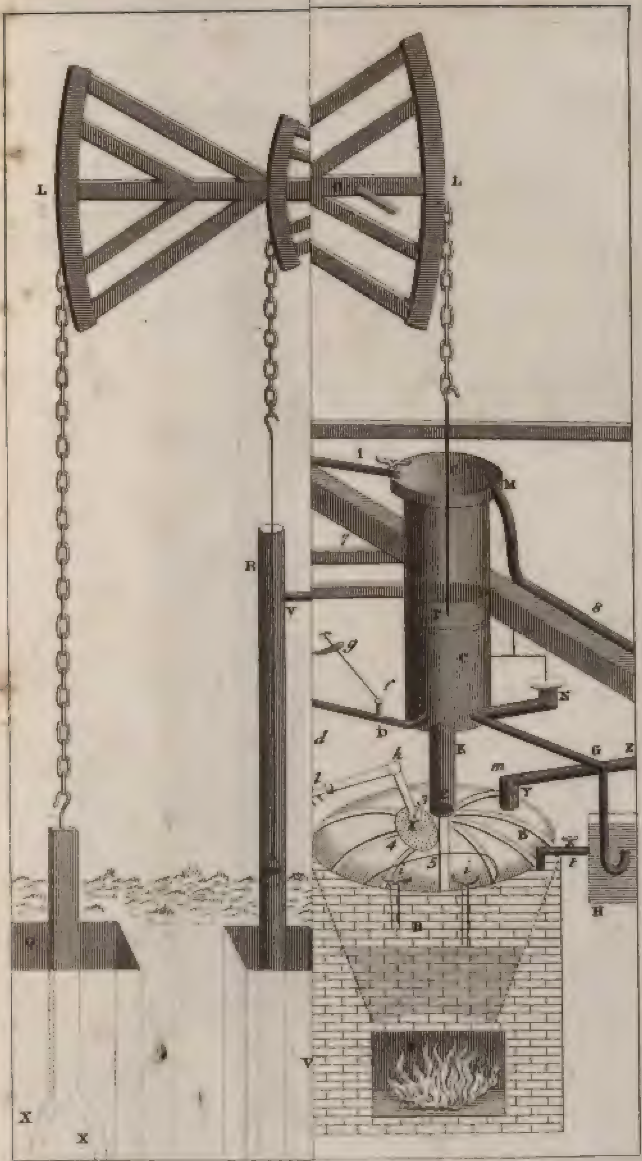

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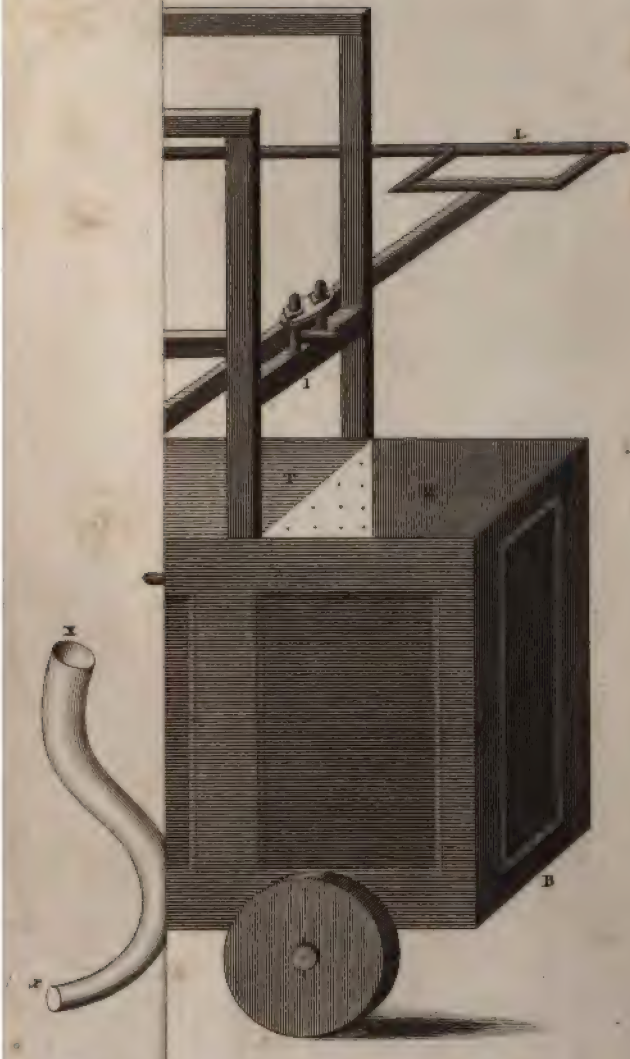

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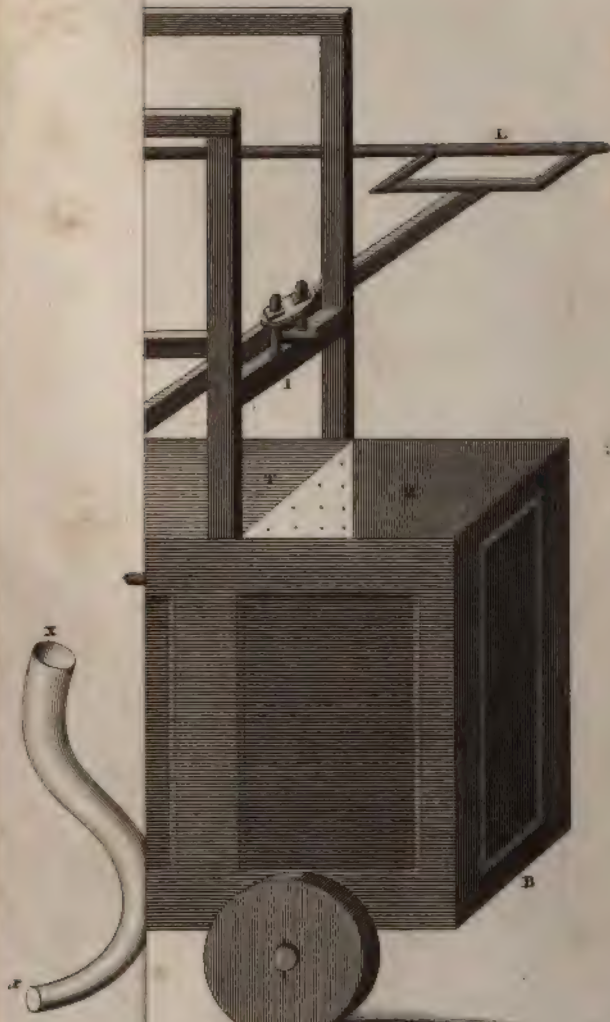

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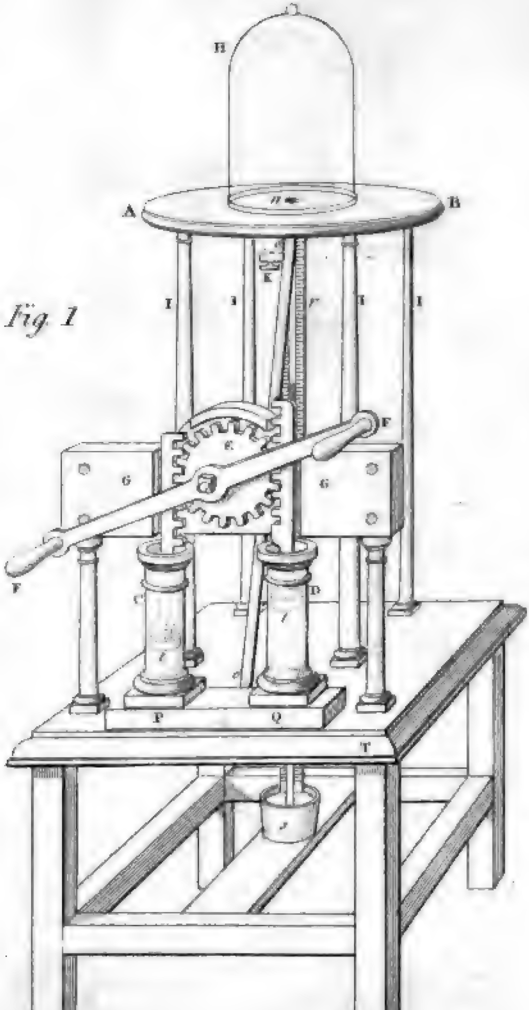
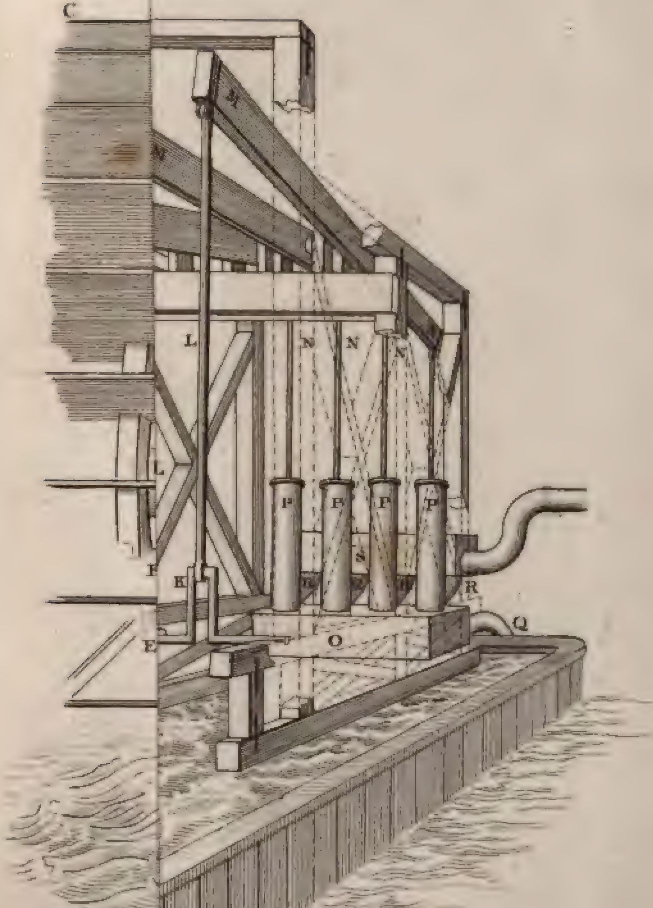
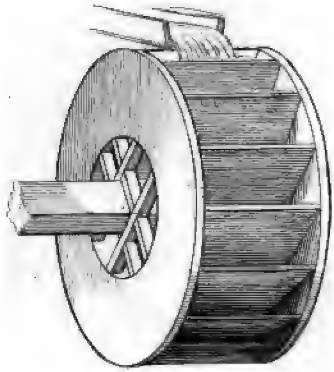
Fig. 1



Pl. 52



e. 54.



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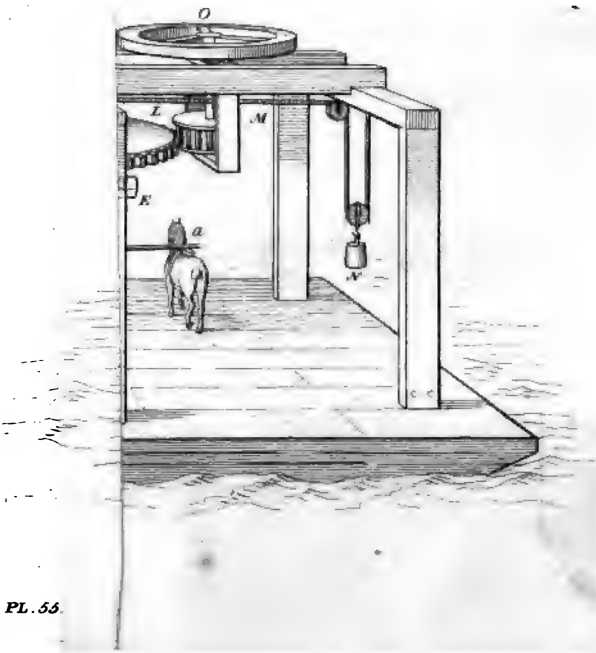
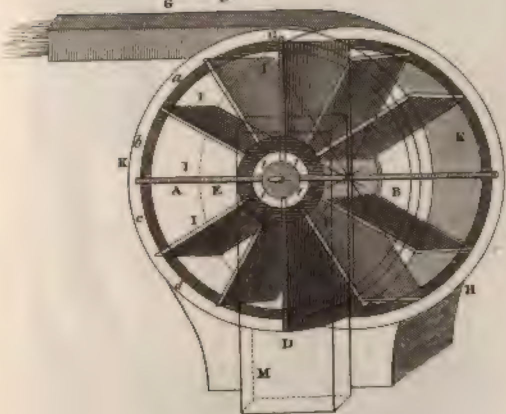


PL. 55.

Fig. 1



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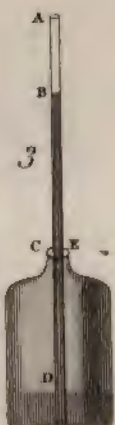
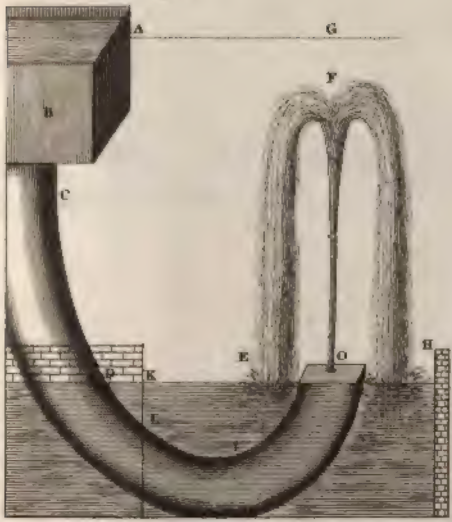


Pl. 56.

Fig. 1



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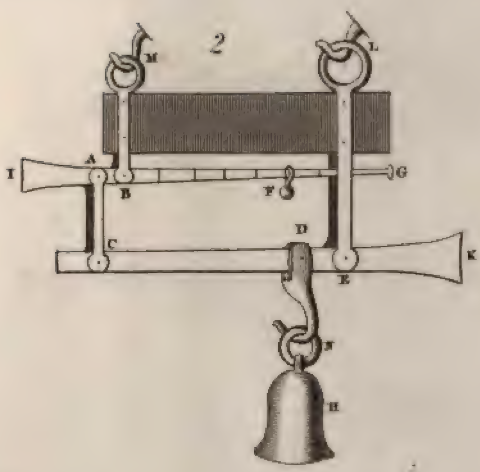
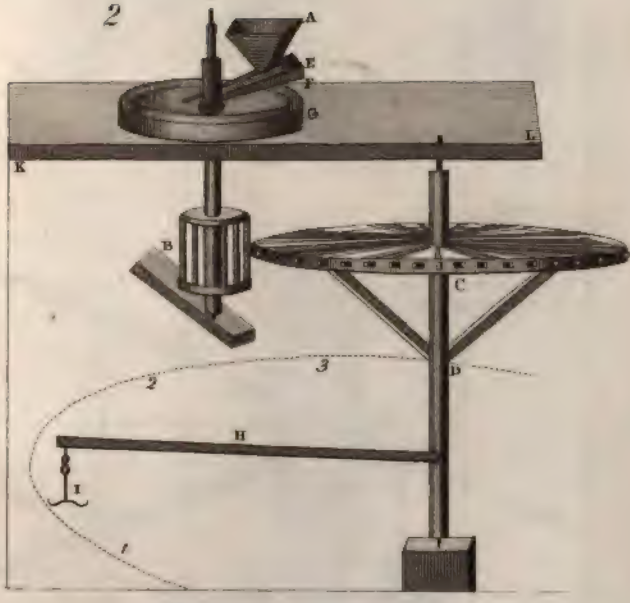


Pl. 57.

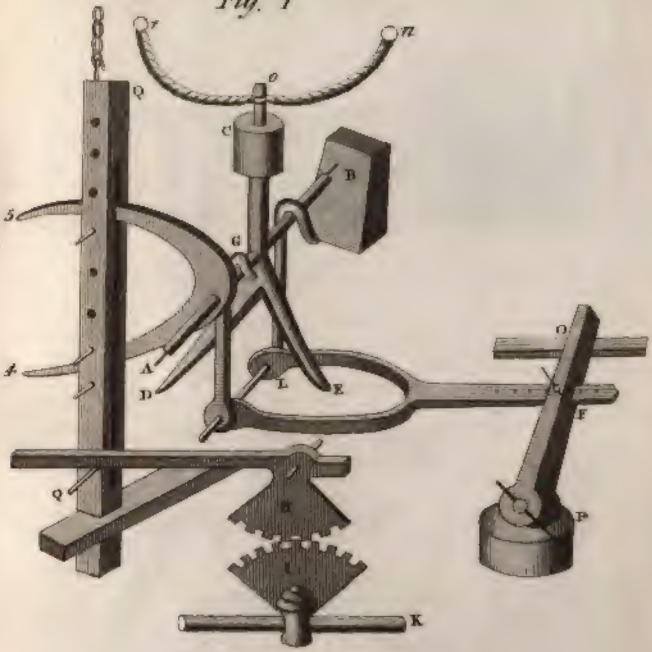
Fig. 1

2



Pl. 58.

Fig. 1



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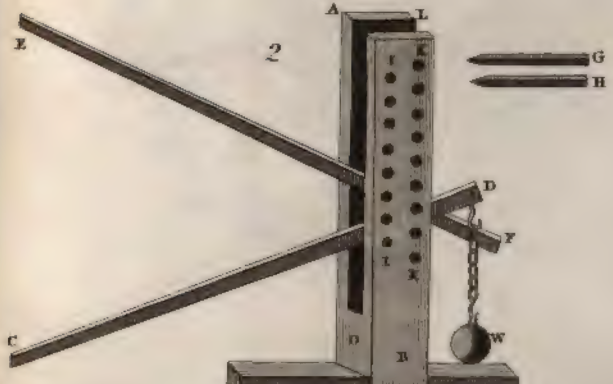

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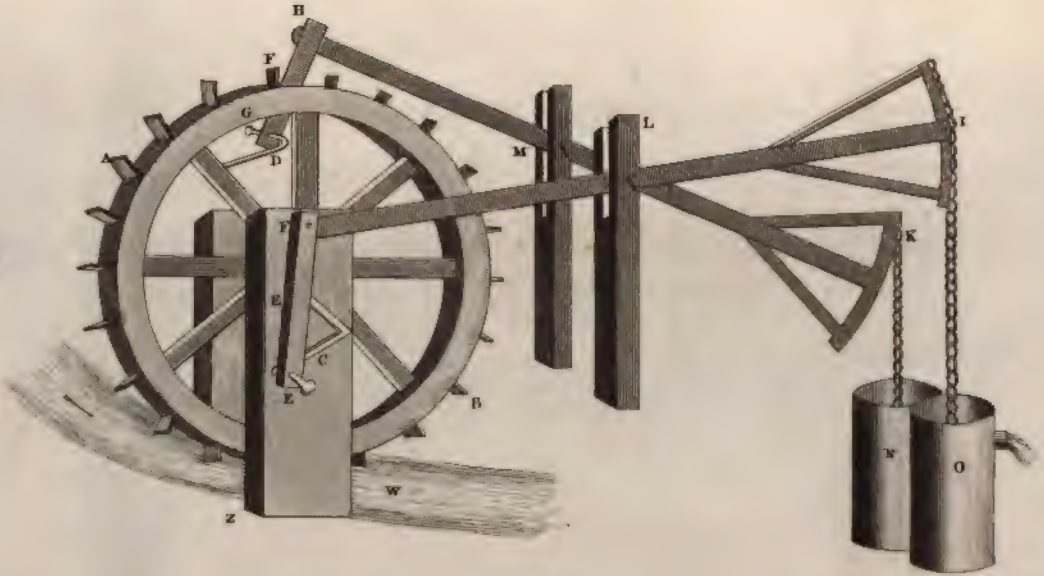

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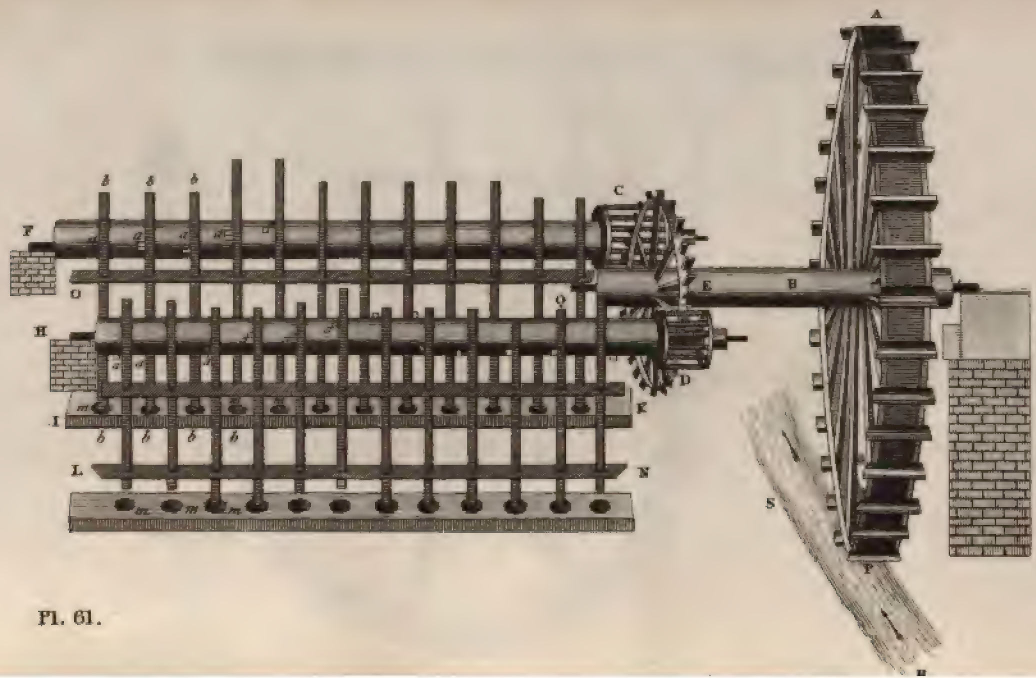

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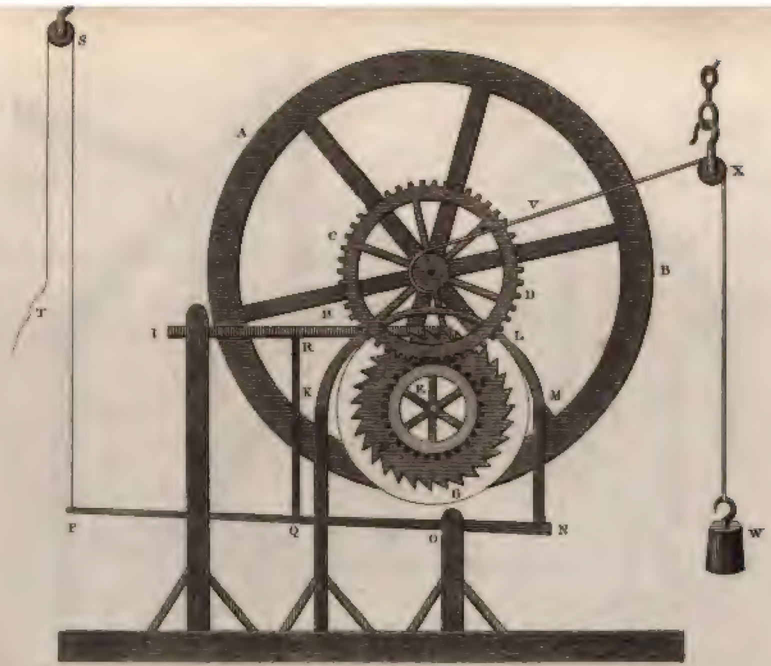
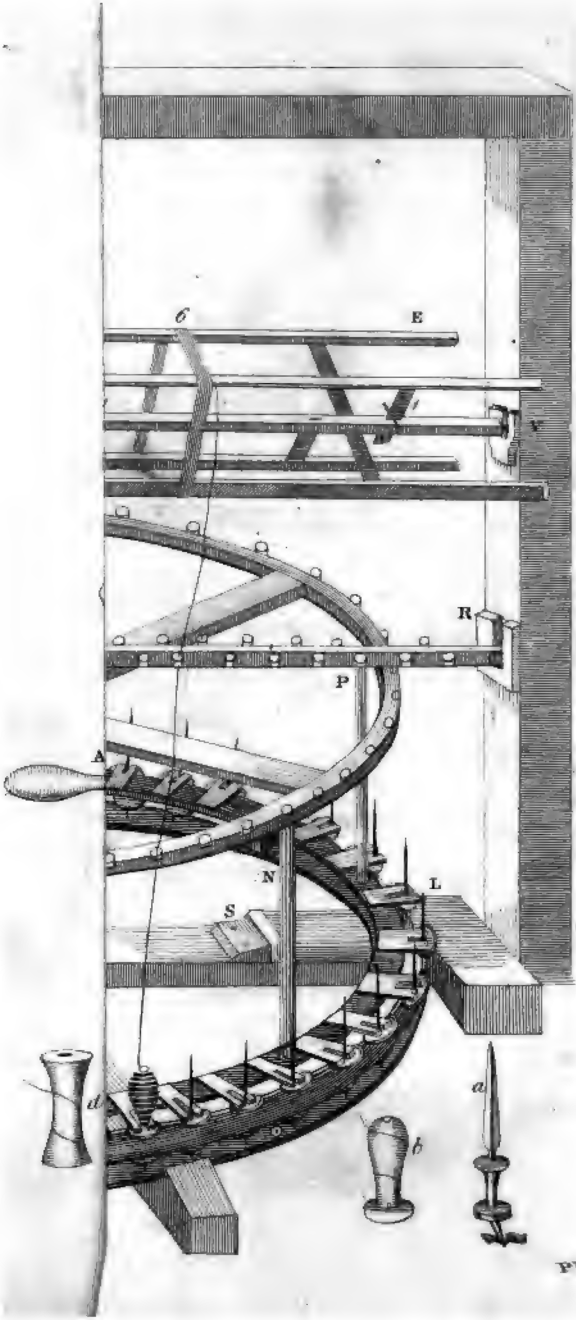
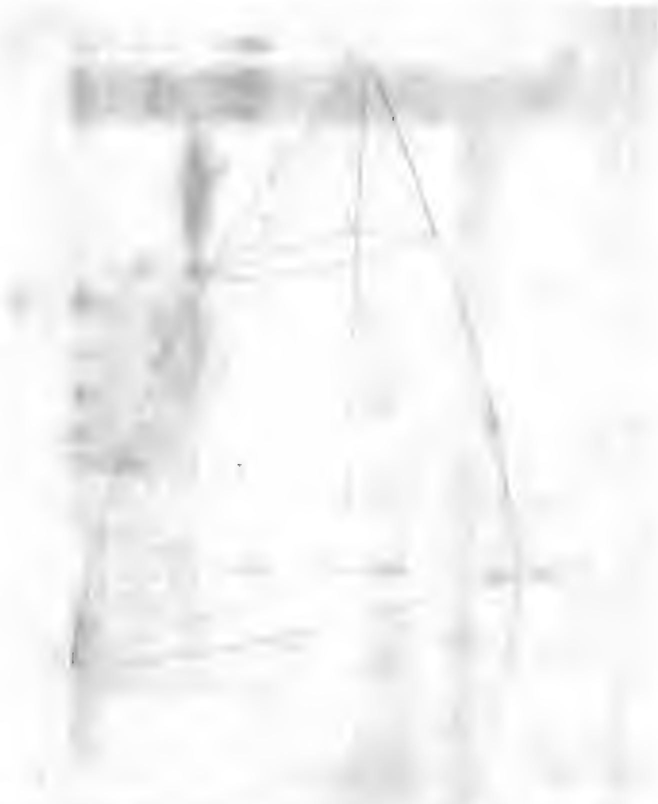


Pl. 62.



PL.64.



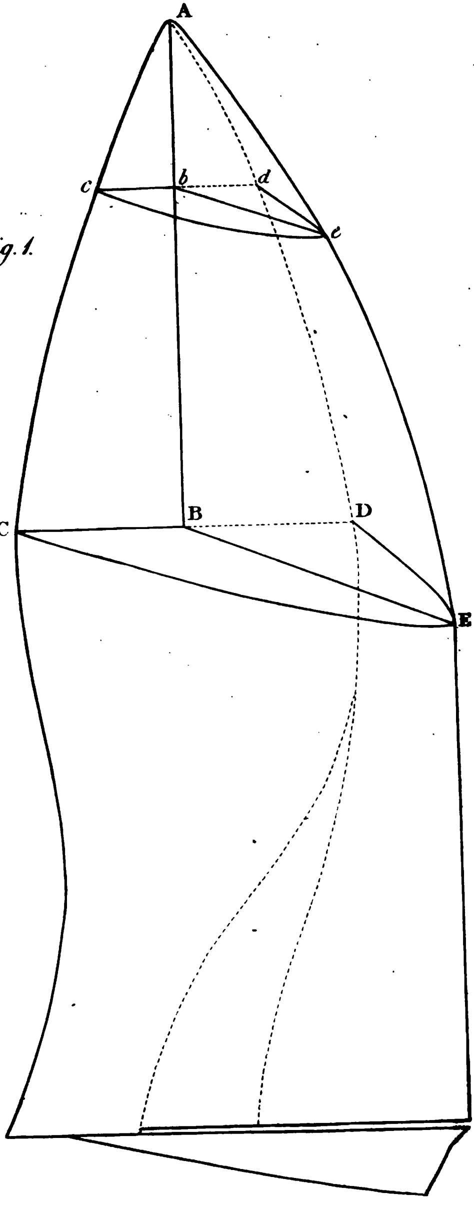
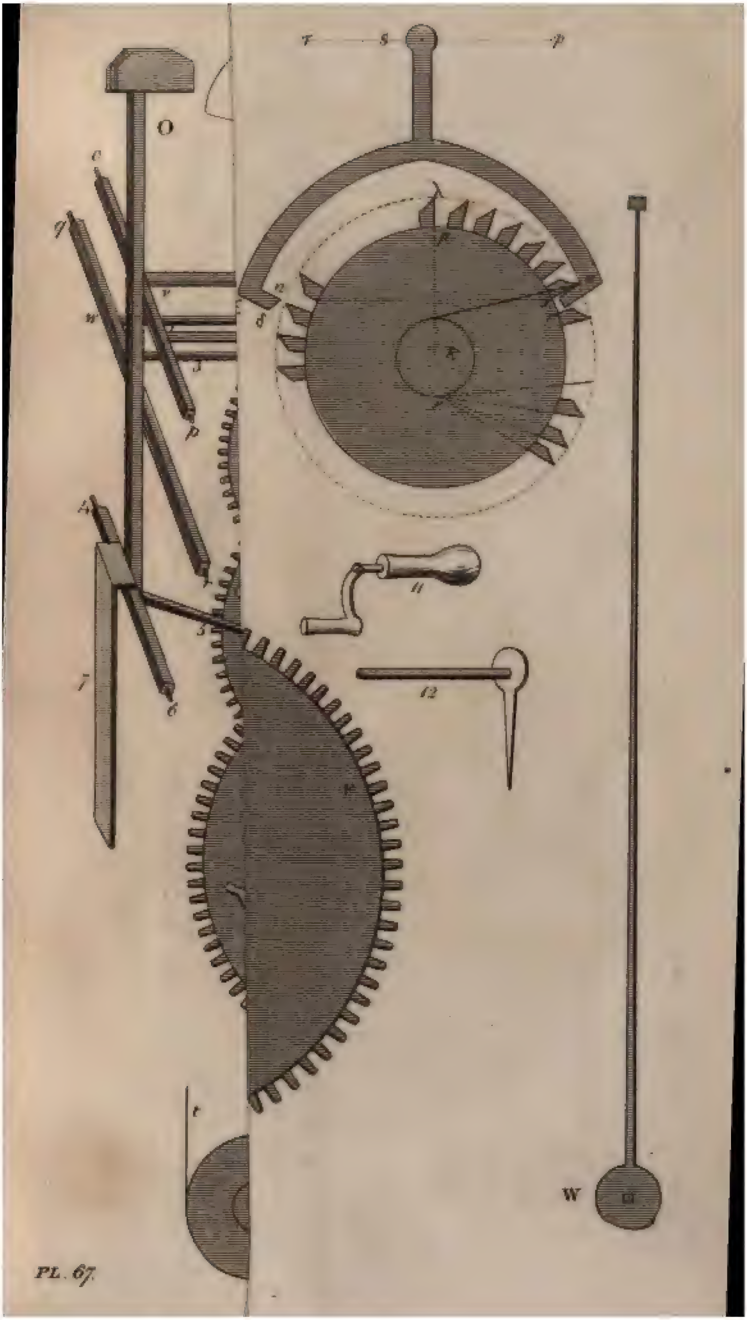



Fig. 1.

PL. 65.



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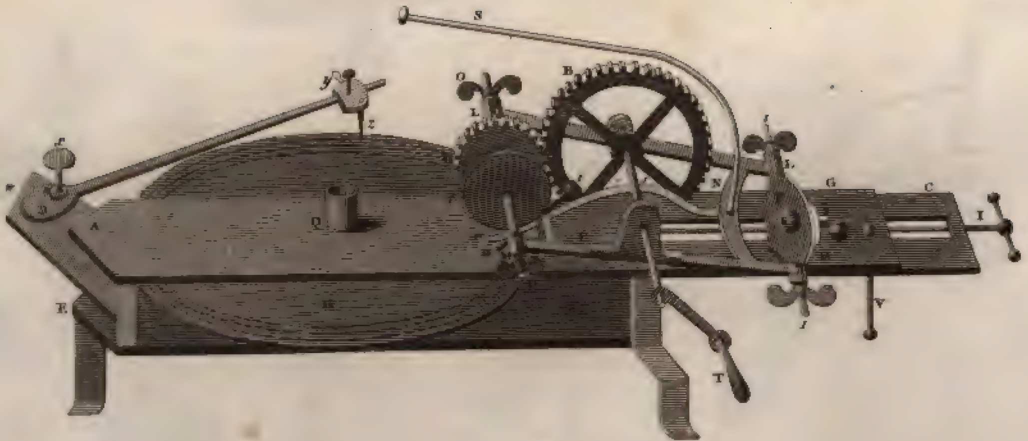

Pl. 68.

Fig. 1

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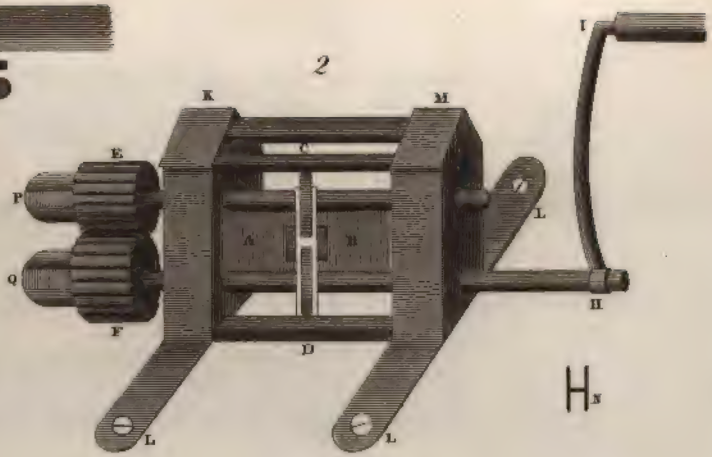
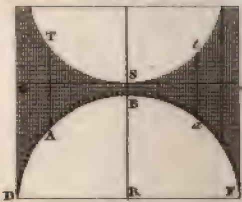


Pl. 69.

Fig. 1.



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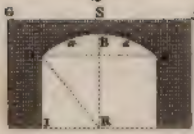

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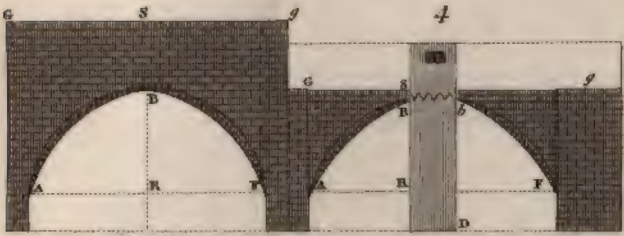

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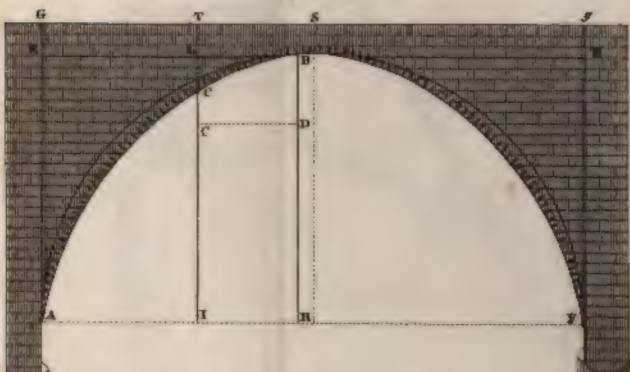

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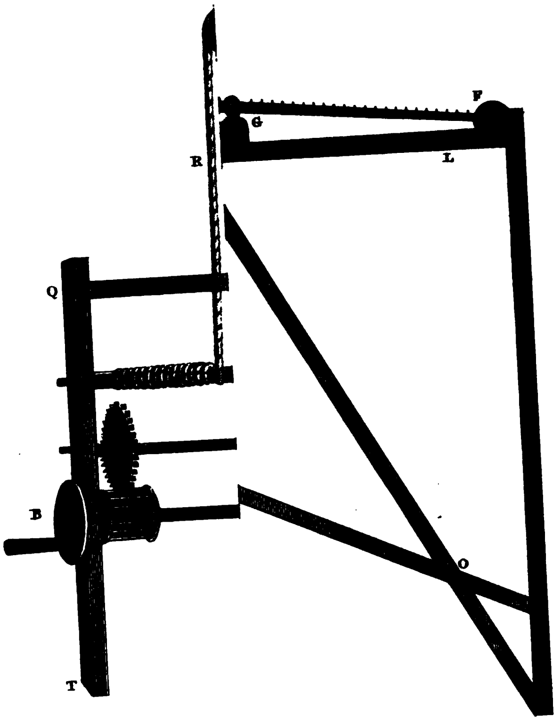
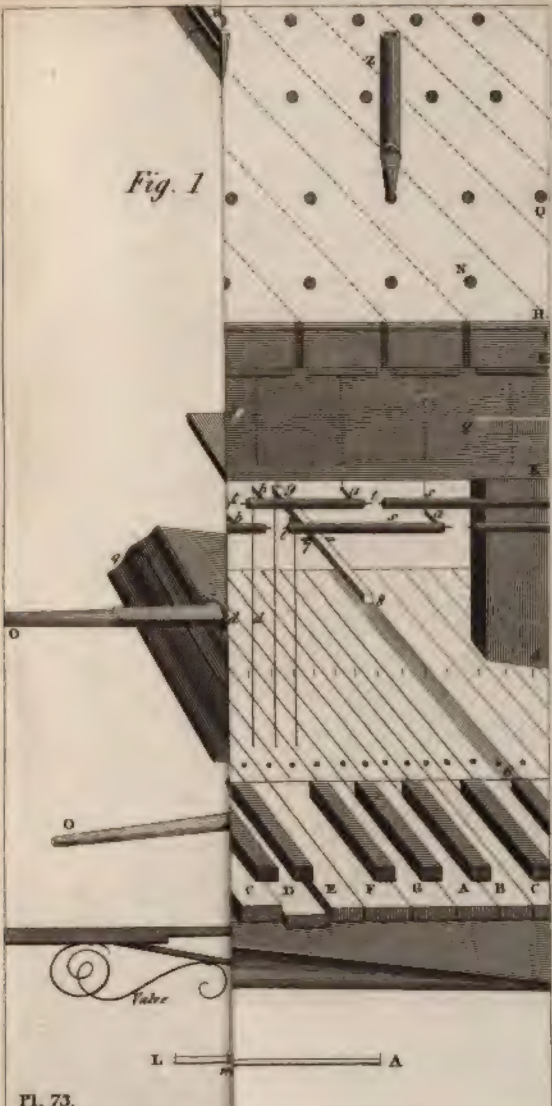
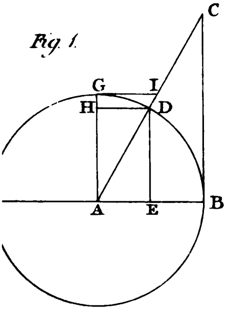


Pl. 72.



Pl. 73.

Fig. 1.



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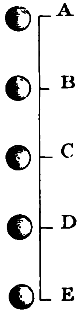

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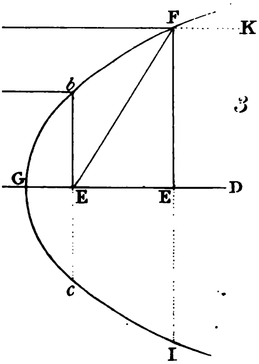

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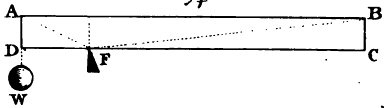

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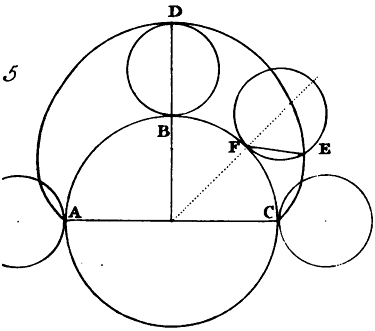

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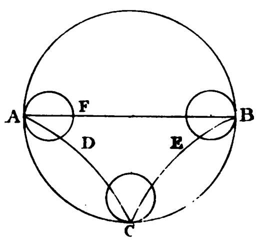
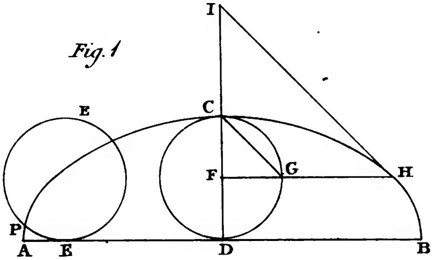


ATE. A.

Fig. 1



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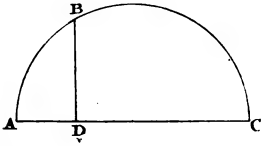

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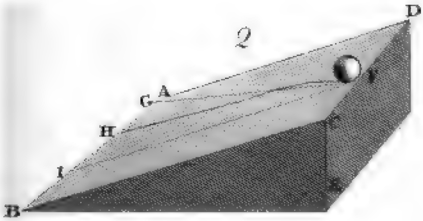

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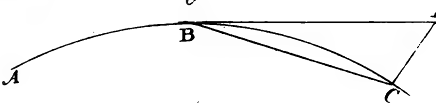

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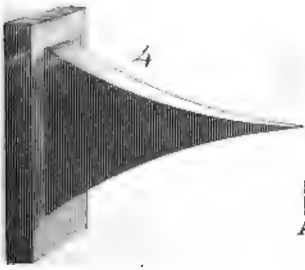

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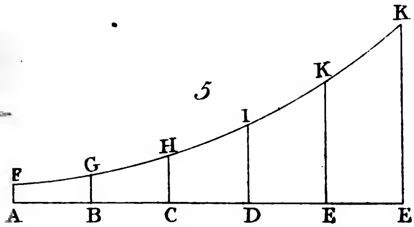

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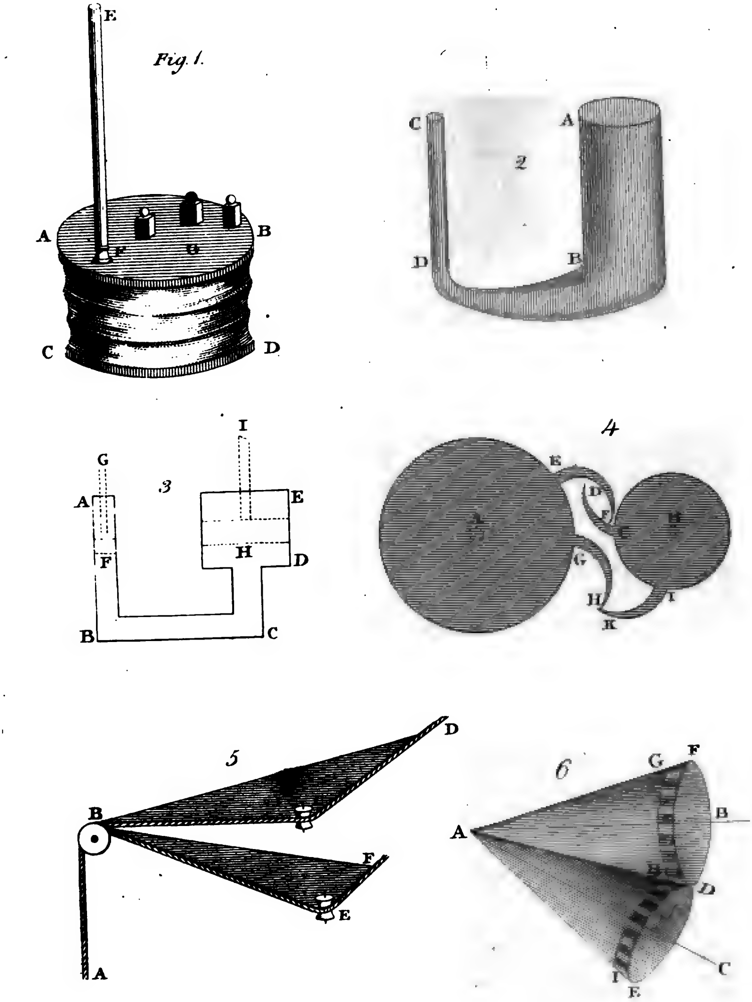
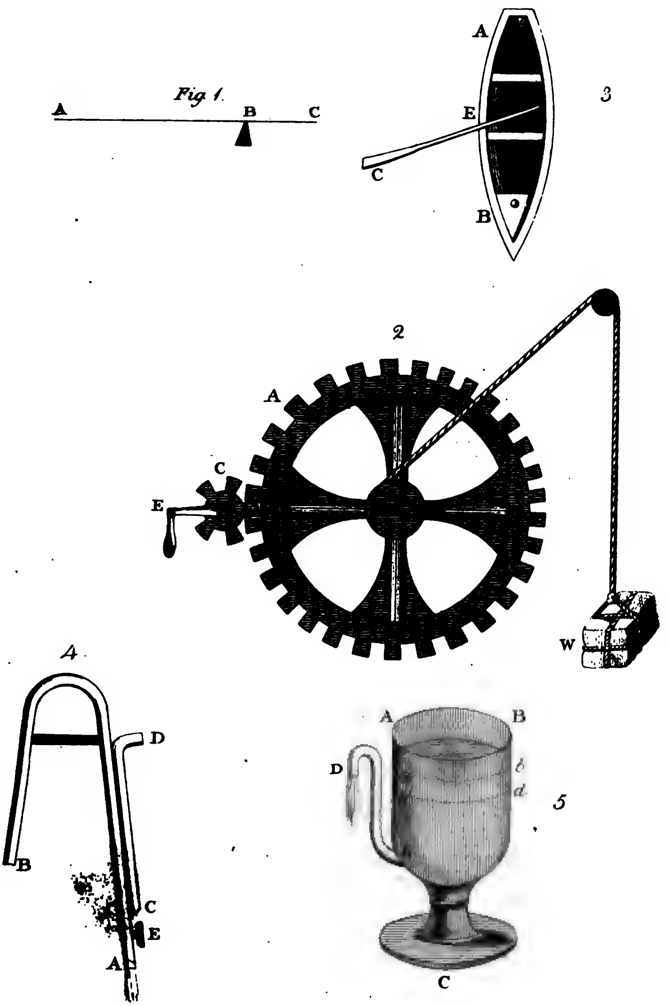
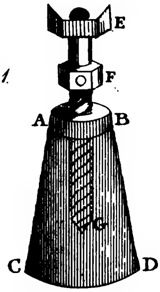


PLATE. C.



PL. D.

Fig. 1.



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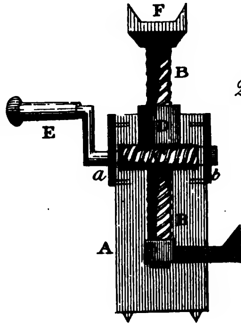

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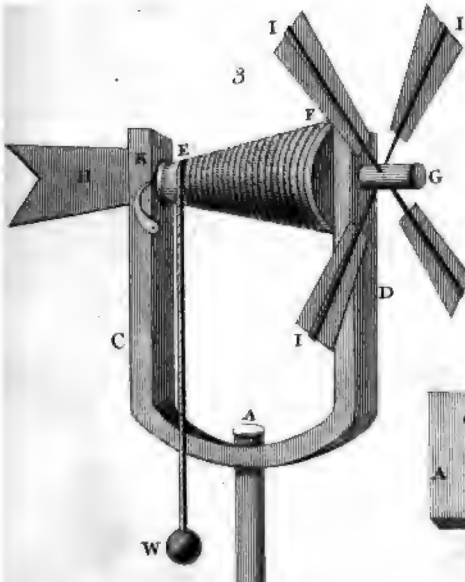

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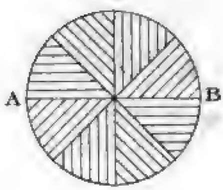

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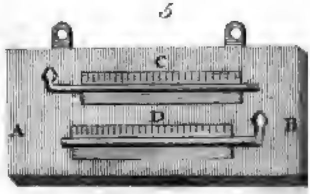
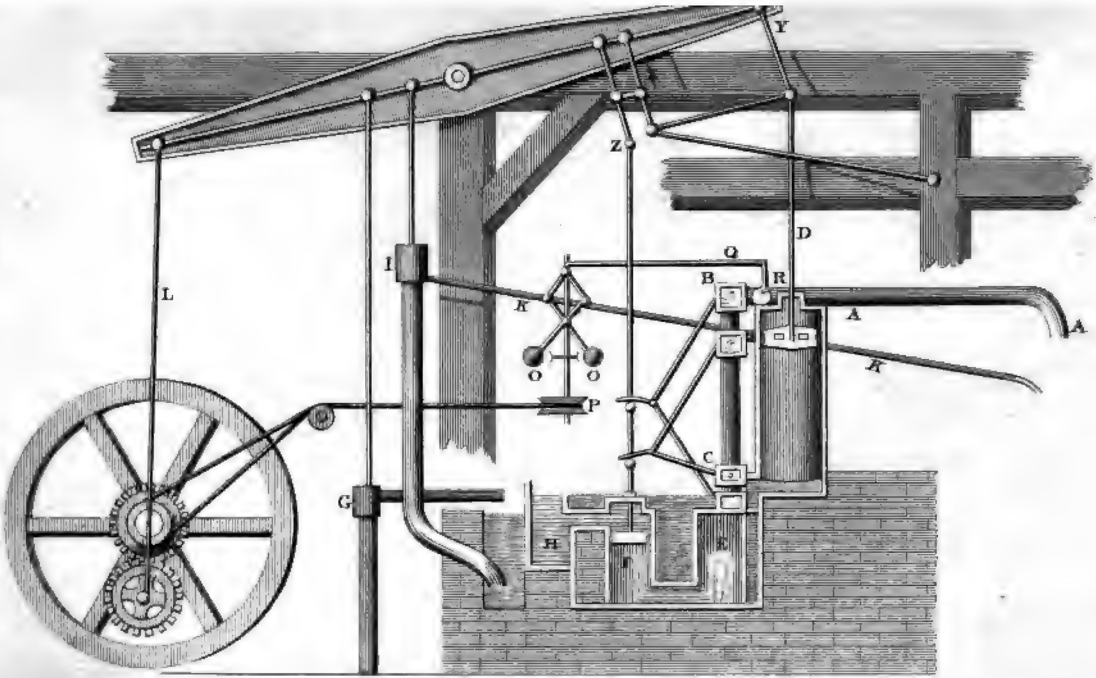


Pl. E.

Fig. 1.



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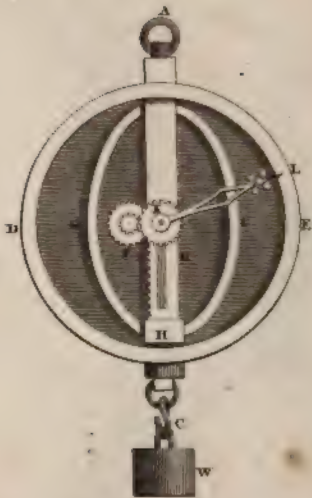

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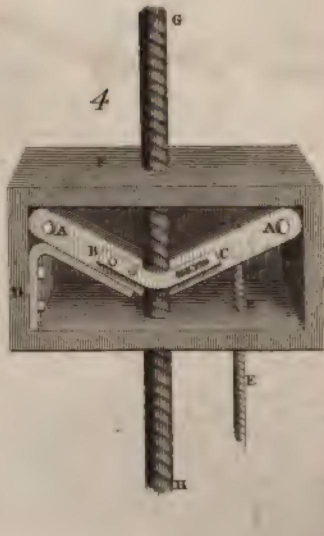

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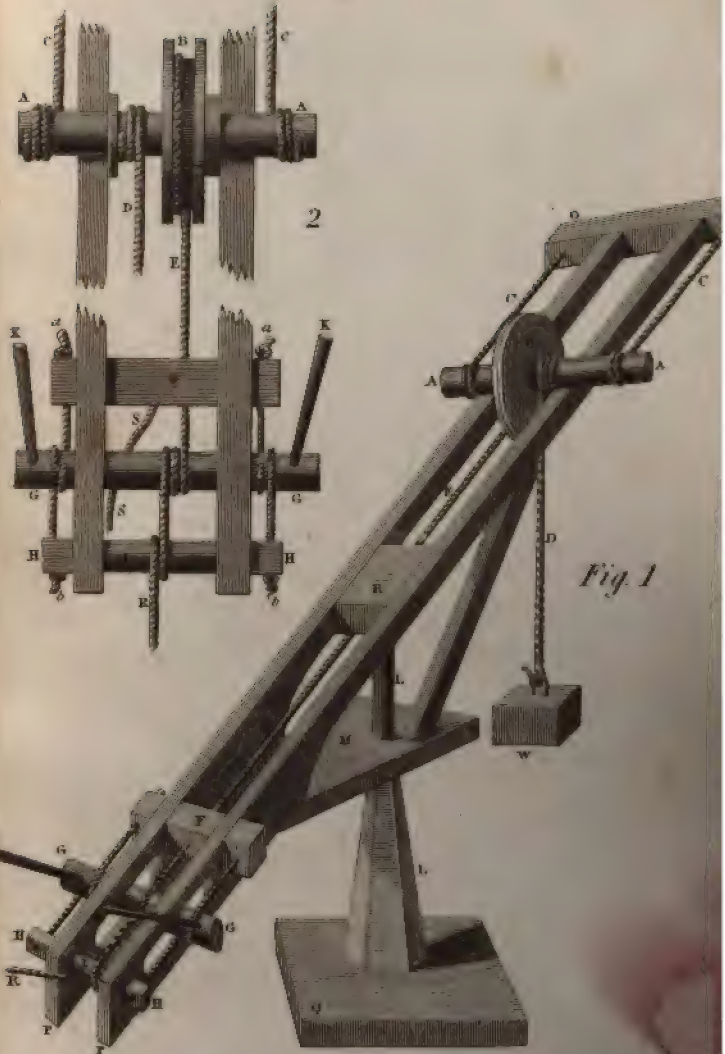
Pl. G.



Pl. II.

Fig. 1

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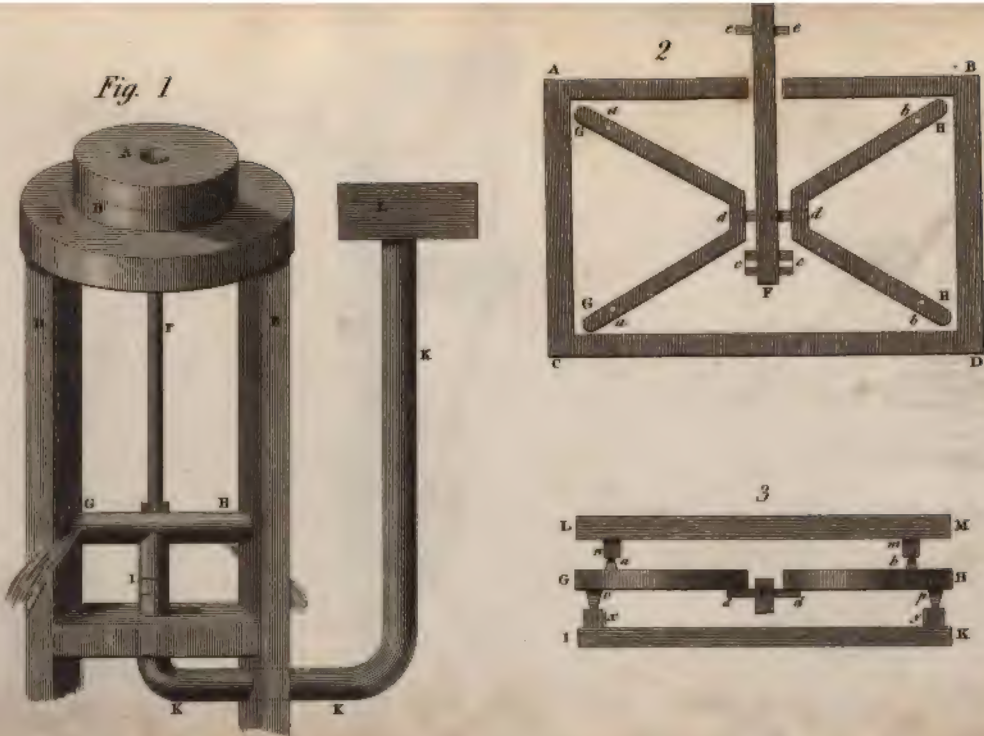
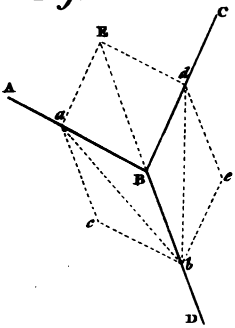
3



Pl. I.

Fig. 1



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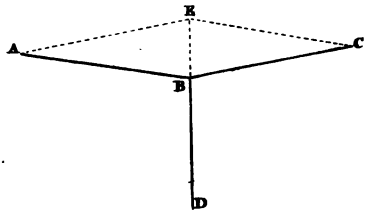

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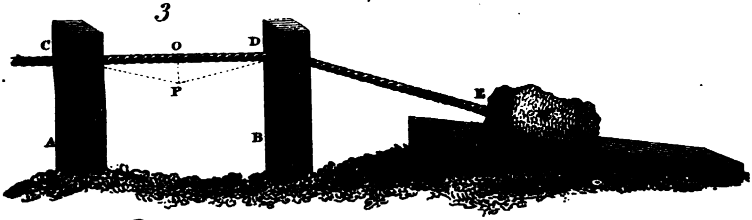

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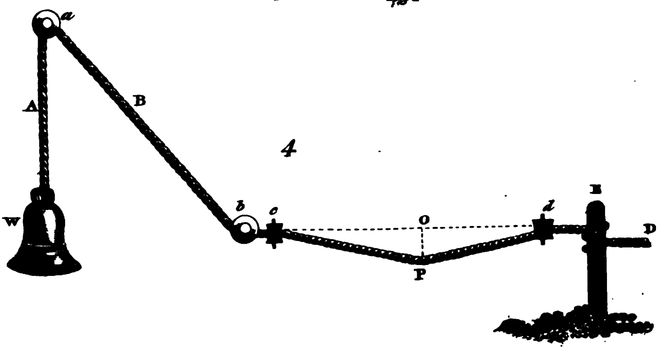
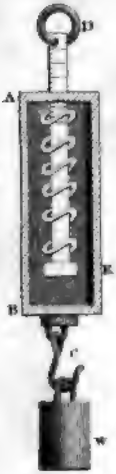


Pl. K.

Fig. 1



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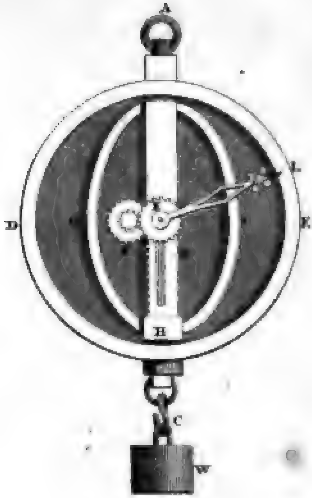

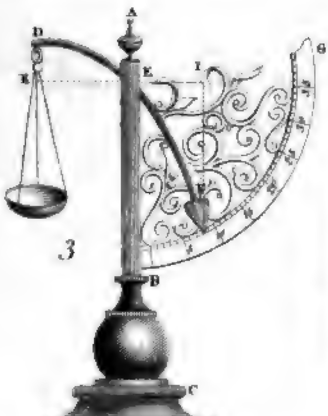

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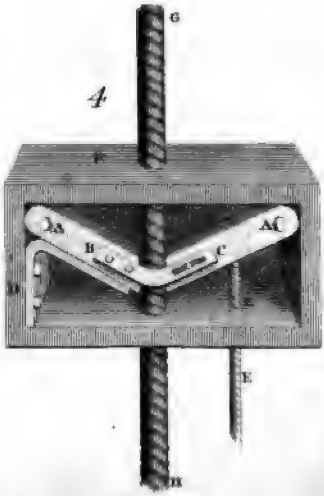

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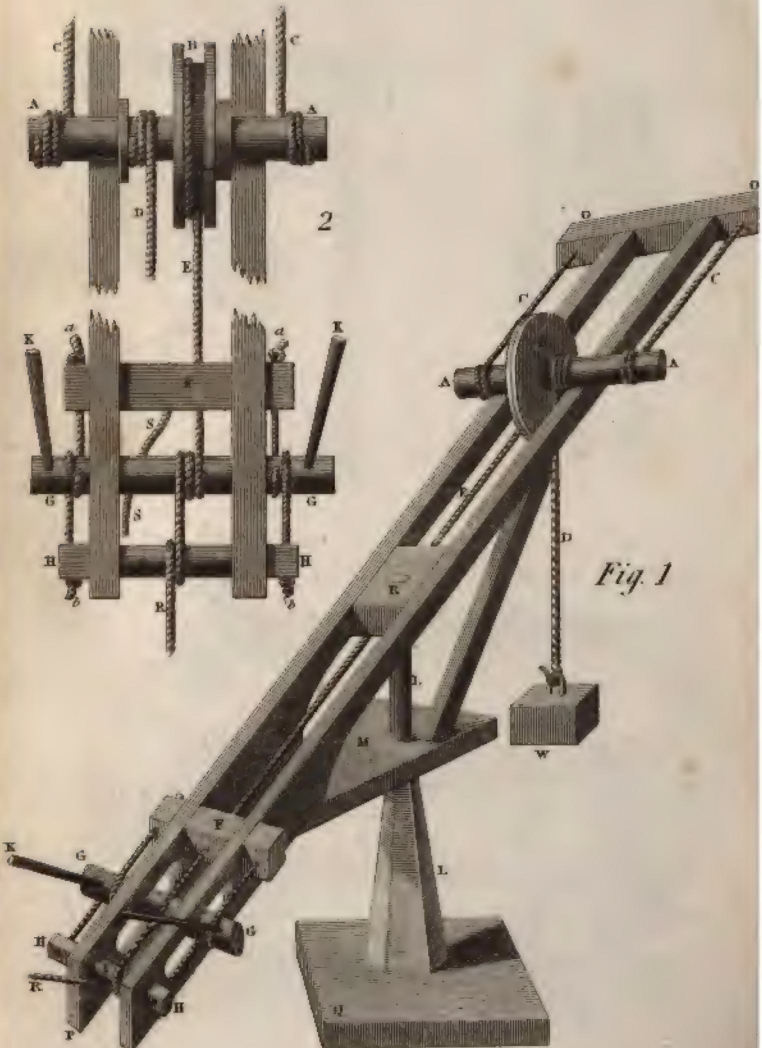
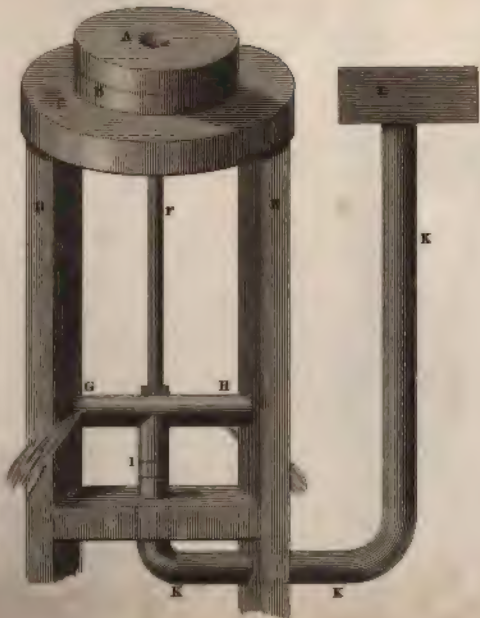
Pl. G.



Pl. II.

Fig. 1



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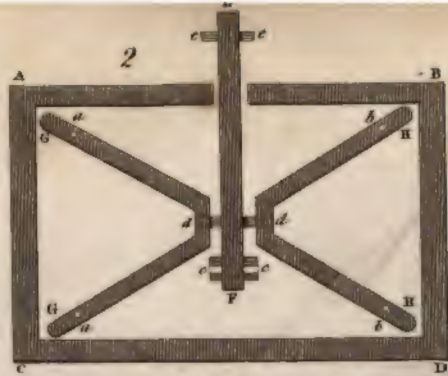

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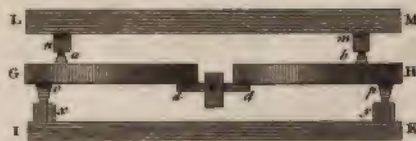
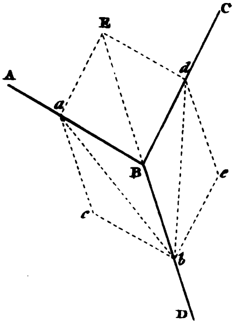


Pl. I.

Fig. 1



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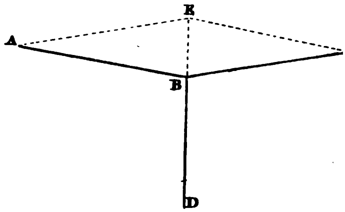

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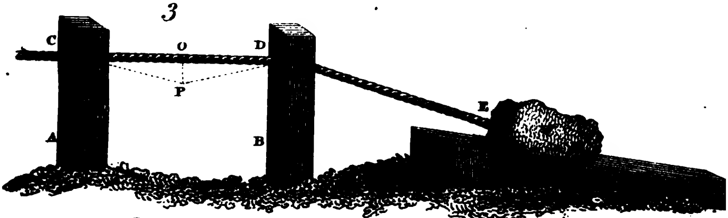

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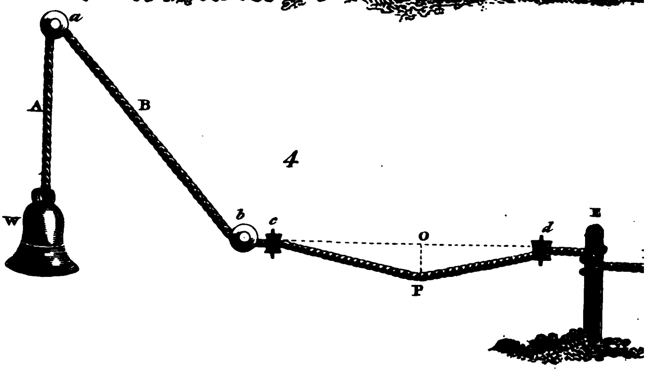

Pl. K.